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solution key.
11.4

6: $\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$

$\frac{1}{n-\sqrt{n}} > \frac{1}{n}$ for all $n \geq 2$, and it diverges by comparison test,

because $\sum_{n=2}^{\infty} \frac{1}{n}$ is a harmonic series.

12: $\sum_{n=0}^{\infty} \frac{1+\sin n}{10^n}$

$\frac{1+\sin n}{10^n} \leq \frac{2}{10^n}$ and $\sum_{n=0}^{\infty} \frac{2}{10^n} = 2 \sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^n$

so, it converges by comparison test with a constant multiple of a convergent geometric series.

26: $\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$ $a_n = \frac{n+5}{\sqrt[3]{n^7+n^2}}$ $b_n = \frac{n}{\sqrt[3]{n^7}} = \frac{n}{n^{7/3}} = \frac{1}{n^{4/3}}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\frac{n+5}{\sqrt[3]{n^7+n^2}}}{\frac{1}{n^{4/3}}} = \frac{\frac{n+5}{n^{7/3}+n^{2/3}}}{\frac{1}{n^{4/3}}} = \frac{(n+5)(n^{4/3})}{n^{7/3}+n^{2/3}} = \frac{n^{7/3}+5n^{4/3}}{n^{7/3}+n^{2/3}}$$

$$\Rightarrow \lim_{n\to\infty} \frac{n^{7/3}+5n^{4/3}}{n^{7/3}+n^{2/3}} \cdot \frac{n^{-7/3}}{n^{-7/3}} = \lim_{n\to\infty} \frac{1+5/n}{(1+1/n^5)^{1/3}} = \frac{1+0}{(1+0)^{1/3}} = 1$$

so $\Rightarrow$ $1>0$ is convergent and also by p-series $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ $p>0$

30 : $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

$\frac{1 \cdot 2 \cdot 3 \cdot \cdots \cdot (n-1)n}{n \cdot n \cdot \cdots \cdots} \leq \frac{1}{n} \cdot \frac{2}{n} \cdot 1 \cdot 1 \cdots 1$ For $n \geq 2$

So $\sum_{n=1}^{\infty} \frac{2}{n^2}$ Converges by P-series $P = 2 > 1$

there fore $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ Converges also by camparison test.