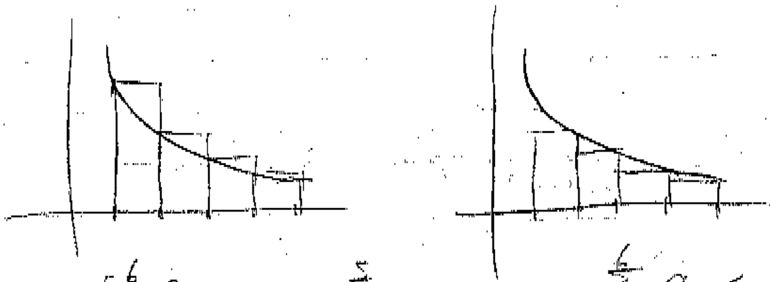
Jane Cho
2/14/2007
Sec 11.3

20/20

#2. 5/5



$\int_1^6 f(x)\,dx < \sum_{i=1}^{5} a_i$ $\qquad \sum_{i=2}^{6} a_i < \int_1^6 f(x)\,dx$

$\therefore \sum_{i=2}^{6} a_i < \int_1^6 f(x)\,dx < \sum_{i=1}^{5} a_i$ ✓

#3. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

$\int_1^\infty \frac{1}{x^4}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x^4}\,dx = \lim_{t\to\infty}\left[\frac{-1}{3x^3}\right]_1^t$

$= \lim_{t\to\infty}\left[-\frac{1}{3t^3}+\frac{1}{3}\right] = \frac{1}{3}$

$\therefore \sum_{n=1}^{\infty} \frac{1}{n^4}$ converges ✓

#5 $\sum_{n=1}^{\infty} \frac{1}{3n+1}$ $\qquad \int_1^\infty \frac{1}{3x+1}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{3x+1}\,dx = \lim_{b\to\infty}\frac{1}{3}\ln(3x+1)\Big]_1^b$

$= \frac{1}{3}\lim_{b\to\infty}(\ln(3b+1)-\ln 4] = \infty$

$\therefore \sum_{n=1}^{\infty} \frac{1}{3n+1}$ diverges

5/5 #6 $\sum_{n=1}^{\infty} e^{-n}$ $\qquad \int_1^\infty e^{-x}\,dx = \lim_{t\to\infty}\int_1^t e^{-x}\,dx = \lim_{t\to\infty} -e^{-x}\Big]_1^t$

$= \lim_{t\to\infty}(-e^{-t}+e^{-1}) = e^{-1}$

$\therefore \sum_{n=1}^{\infty} e^{-n}$ converges ✓

#11. $1+\frac{1}{8}+\frac{1}{27}+\frac{1}{64}+\frac{1}{125}+\cdots = \sum_{n=1}^{\infty}\frac{1}{n^3}$ $\qquad p=3>1$ converges

5/5 #12. $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}+\cdots = \sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ $\qquad p=\frac{3}{2}>1$ converges ✓

#19 $\sum_{n=1}^{\infty} ne^{-n^2}$ $\qquad f(x) = xe^{-x^2}$ $\qquad f'(x) = e^{-x^2}-2x^2e^{-x^2} = e^{-x^2}(1-2x^2)$

$\int_1^\infty xe^{-x^2}\,dx = \lim_{t\to\infty}\left[-\frac{1}{2}e^{-x^2}\right]_1^t = 0-\left(-\frac{1}{2}e^{-1}\right) = 1/2e$

$\therefore \sum_{n=1}^{\infty} ne^{-n^2}$ converges ✓

#21 $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$ $\qquad f(x)=\frac{1}{x\ln x}$ $\qquad f'(x) = -\frac{1+\ln x}{x^2(\ln x)^2} < 0$ for $x>2$

$\int_2^\infty \frac{1}{x\ln x}\,dx = \lim_{t\to\infty}\left[\ln(\ln x)\right]_2^t = \lim_{t\to\infty}(\ln(\ln t)-\ln(\ln 2)) = \infty$

$\therefore \sum_{n=2}^{\infty}\frac{1}{n\ln n}$ diverges ✓

7/5 #28 $\sum_{n=1}^{\infty} \frac{\ln n}{n^p}$ If $p \le 0$ $\lim_{n\to\infty} \frac{\ln n}{n^p} = \infty$ and $\sum_{n=1}^{\infty} \frac{\ln n}{n^p}$ diverges

so $\boxed{p > 0}$

$f(x) = \frac{\ln x}{x^p}$

$$f'(x) = \frac{x^{p}\frac{1}{x} - px^{p-1}\ln x}{x^{2p}} = \frac{x^{p-1} - px^{p-1}\ln x}{x^{2p}}$$

$$= \frac{x^{p-1}(1 - p\ln x)}{x^{2p}} < 0$$

$$\int_1^{\infty} \frac{\ln x}{x^p}\,dx = \lim_{t\to\infty}\left[\frac{x^{1-p}[(1-p)\ln x - 1]}{(1-p)^2}\right]_1^t \quad (p \neq 1)$$

$$= \frac{1}{(1-p)^2}\left(\lim_{t\to\infty}[t^{1-p}((1-p)\ln t - 1) + 1]\right) \quad 1-p < 0 \Rightarrow p > 1$$

$\sum_{n=1}^{\infty} \frac{\ln n}{n^p}$ converges $(p > 1)$ ✓

#29. ??