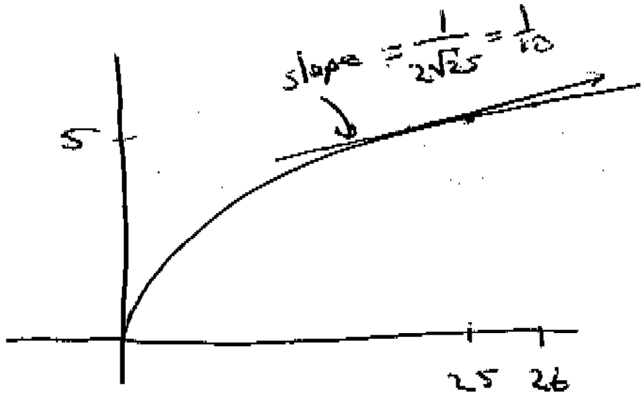
14.4: Tangent Planes & Linear Approx

$\boxed{\begin{array}{c}14.4\\ 1/5\end{array}}$

Ex1: Approx $\sqrt{26}$ w/ a linear approximations.

$$f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$$



$$y = \frac{1}{10}(x-25) + 5$$

$$\sqrt{26} \approx \frac{1}{10}(1) + 5 = 5.1$$

Ex 2: Approx $\sqrt{26}$ w/a tangent plane.

$$\cancel{f(x,y) = \sqrt{x^2+y^2} \Rightarrow f_x(x,y) = \frac{1}{2\sqrt{x^2+y^2}}}$$

$$\cancel{f_y(x,y) = \frac{1}{2\sqrt{x^2+y^2}}}$$

$$\cancel{f(\text{[illegible]}) \approx 5 + \frac{1}{10}(x - \text{[illegible]}) + \frac{1}{10}(y - \text{[illegible]})}$$

$$f(x,y) = \sqrt{x^2+y^2} \Rightarrow f_x(x,y) = \frac{2x}{2\sqrt{x^2+y^2}}$$

$$f_y(x,y) = \frac{y}{\sqrt{x^2+y^2}}$$

$$f(3,4) \approx 5 + \frac{3}{5}(x-3) + \frac{4}{5}(y-4).$$

$$f(1,5) \approx 5 + \frac{3}{5}(-2) + \frac{4}{5}(1) = 4.8$$

14.4: Tangent Planes

The equation of a plane thru $P(x_0, y_0, z_0)$ can be written $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$.

Solving for z: $z = z_0 - \frac{A}{C}(x - x_0) - \frac{B}{C}(y - y_0)$

and letting $a = -\frac{A}{C}$ and $b = -\frac{B}{C}$, we have

$$z = z_0 + a(x - x_0) + b(y - y_0).$$

Now, as we approximated $y = f(x)$ w/ tangent lines, we approximate $z = f(x, y)$ w/ tangent planes.

Intuitively, given a pt (x_0, y_0), the partials on $f(x, y)$ must be equal to those on the plane.

$$\Rightarrow L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y -)$$

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Ex 2: Find the ~~tangent~~ plane to $z = y\ln(x)$ when $x=1$ and $y=4$.

Ex 3: Approx $4.1 \ln(1.1)$ w/ ↗

Does every fct have a tangent plane approx @ every pt?

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{else.} \\ 0 & ,\ (x,y) = (0,0) \end{cases}$$

Thm: If f_x, f_y exist near (a,b) and are cont at (a,b), then f is differentiable at (a,b)

$$f_x = \frac{y(y^2-x^2)}{(x^2+y^2)^2} \longleftarrow \text{not continuous at the origin.}$$

Ex 4: Is $f(x,y) = \arctan(x+2y)$ differentiable at $(1,0)$? If so, find the linear approximation.

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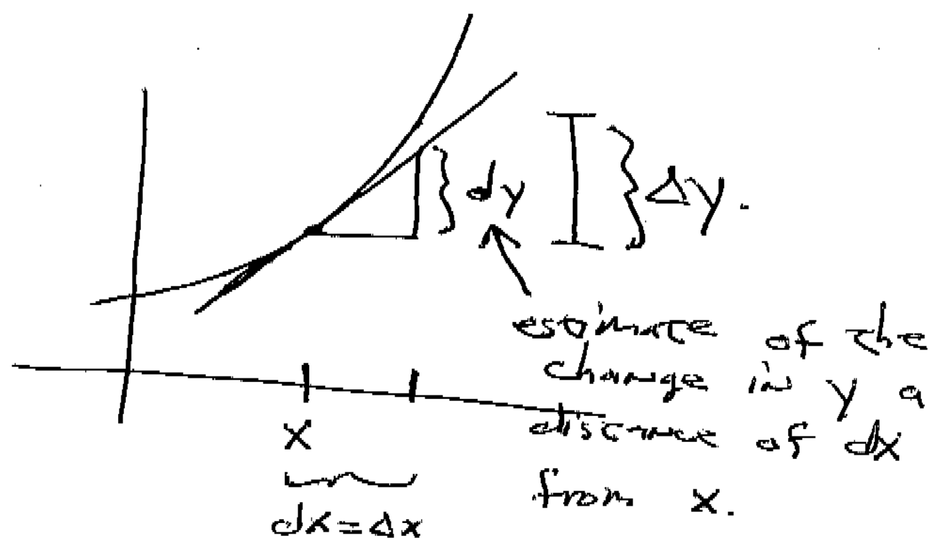
Ex5: Using AGI table, estimate the tax owed by a family w/2.2 children and an AGI of \$44,252.

Differentials.

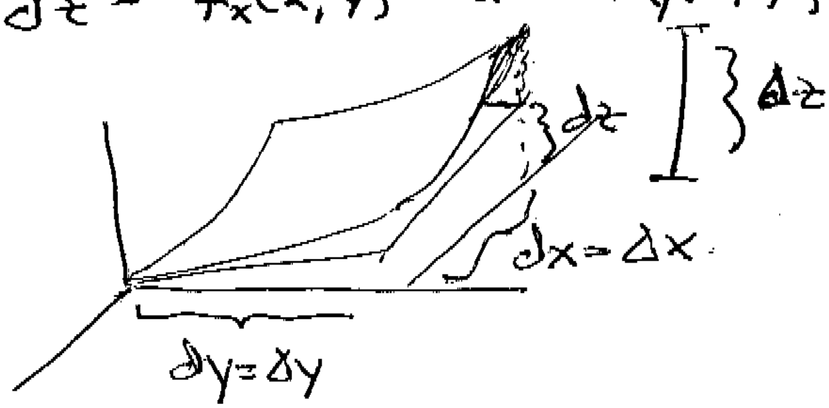
If $f'(x) = \frac{dy}{dx} \Rightarrow dy = f'(x)\,dx$.

graphically.



Dfn: The Total Differential

$$dz = f_x(x,y)\,dx + f_y(x,y)\,dy.$$



Ex 6: If $z = x^2 - xy + 3y^2$ and (x, y) changes from $(3, -1)$ to $(2.96, -0.95)$, compare the values of Δz and dz.