Mathematica picture

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14.3: Partial Derivatives

Ex1: Find the partials of $f(x,y,z,t) = xyz^2\tan(y^2t)$.

Ex2: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of $yz = \ln(x+z)$.

$\frac{\partial z}{\partial x}$: $y\frac{\partial z}{\partial x} = \frac{1}{x+z}\left(1+\frac{\partial z}{\partial x}\right)$

$$\Rightarrow y\frac{\partial z}{\partial x} = \frac{1}{x+z} + \frac{\partial z}{\partial x}\left(\frac{1}{x+z}\right)$$

$$\Rightarrow \frac{\partial z}{\partial x}\left(y - \frac{1}{x+z}\right) = \frac{1}{x+z}$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{1}{x+z} \cdot \frac{1}{y-\frac{1}{x+z}}.$$

Ex3: Find all second partials of $v = \sqrt{x+y^2}$

Clairaut's Thm: Suppose f is defined on a disk D that contains the point (a,b). If f_{xy} and f_{yx} are both cont on D, then

$$f_{xy}(a,b) = f_{yx}(a,b).$$

Ex4: If $u = e^{r\theta}\sin\theta$, find $\frac{\partial^3 u}{\partial r^2\partial\theta}$

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Ex5: Verify that $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$ is a solution of the 3D Laplace equation
$$u_{xx} + u_{yy} + u_{zz} = 0.$$

Ex6: If f and g are twice differentiable fcts of a single variable, show that the fct
$$u(x,t) = f(x+at) + g(x-at)$$
is a solution to the wave eqt. $u_{tt} = a^2 u_{xx}$.