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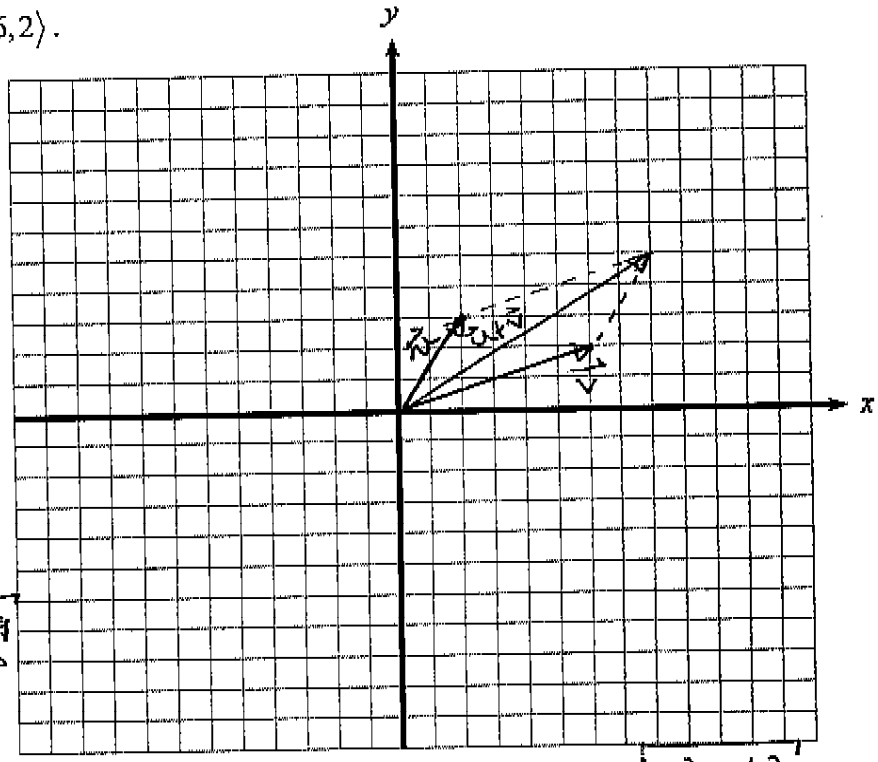
Math 126: Calculus III
12.02: Vectors

1.) What is a vector?

A vector is a quantity w/ both magnitude and direction.

2.) How do we visualize vectors and add them graphically?

a.) Graph: $\vec{u} = \langle 2,3 \rangle$ and $\vec{v} = \langle 6,2 \rangle$.



b.) Find $\vec{u} + \vec{v}$ graphically.

c.) Find $\vec{u} + \vec{v}$ algebraically.

$\langle 2,3 \rangle + \langle 6,2 \rangle$

$= \langle 8,5 \rangle$

d.) Find $\|\vec{u}\|$.

$\|\vec{u}\| = \sqrt{4+9} = \sqrt{13}$

e.) Find $\|\vec{u} + \vec{v}\|$.

$\|\vec{u} + \vec{v}\| = \sqrt{64+25} = \sqrt{89}$

NOTE: $\|\langle a,b \rangle\| = \sqrt{a^2 + b^2}$

$\|\langle a,b,c \rangle\| = \sqrt{a^2 + b^2 + c^2}$

3.) What is the direction of the zero vector $\vec{0}$?

the $\vec{0}$ has no specific direction

Properties of Vectors

If $\vec{u}, \vec{v}, \vec{w}$ are vectors in V_n & c,d are scalars

(1) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

(2) $\vec{u} + \vec{0} = \vec{u}$

(3) $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$

(4) $c \cdot d(\vec{u}) = c(d\vec{u})$

(5) $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$

(6) $\vec{u} + (-\vec{u}) = \vec{0}$

(7) $(c+d)\vec{u} = c\vec{u} + d\vec{u}$

(8) $1\vec{u} = \vec{u}$

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4.) If $\vec{u} = \langle -3, -4, -1 \rangle$ and $\vec{v} = \langle 6, 2, -3 \rangle$, find $\|\vec{u}\|$, $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, $2\vec{u}$, and $3\vec{u} + 4\vec{v}$.

a.) $\|\vec{u}\| = \sqrt{9+16+1} = \sqrt{26}$

b.) $\vec{u} + \vec{v} = \langle -3, -4, -1 \rangle + \langle 6, 2, -3 \rangle = \langle 3, -2, -4 \rangle$

c.) $\vec{u} - \vec{v} = \langle -3, -4, -1 \rangle - \langle 6, 2, -3 \rangle = \langle -9, -6, 2 \rangle$

d.) $2\vec{u} = 2\langle -3, -4, -1 \rangle = \langle -6, -8, -2 \rangle$

e.) $3\vec{u} + 4\vec{v} = 3\langle -3, -4, -1 \rangle + 4\langle 6, 2, -3 \rangle = \langle 15, -4, -15 \rangle$

5.) What is a unit vector and what are the unit vectors $\vec{i}, \vec{j}, and\ \vec{k}$?

A unit vector has magnitude 1.

$\vec{i} = \langle 1, 0, 0 \rangle$; $\vec{j} = \langle 0, 1, 0 \rangle$; and $\vec{k} = \langle 0, 0, 1 \rangle$

6.) If $\vec{u} = 3\vec{i} - 2\vec{k}$ and $\vec{v} = \vec{i} - \vec{j} + \vec{k}$, find $\|\vec{u}\|$, $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, $2\vec{u}$, and $3\vec{u} + 4\vec{v}$.

a.) $\|\vec{u}\| = \sqrt{9+4} = \sqrt{13}$

b.) $\vec{u} + \vec{v} = 4\vec{i} - \vec{j} - \vec{k}$

c.) $\vec{u} - \vec{v} = 2\vec{i} + \vec{j} - 3\vec{k}$

d.) $2\vec{u} = 6\vec{i} - 4\vec{k}$

e.) $3\vec{u} + 4\vec{v} = 13\vec{i} - 4\vec{j} - 2\vec{k}$

7.) Find an exact unit vector in the same direction as the vector $\vec{w} = 12\vec{i} - 5\vec{j}$.

$\|\vec{w}\| = \sqrt{144 + 25} = \sqrt{169} = 13$

$\frac{\vec{w}}{\|\vec{w}\|} = \frac{12}{13}\vec{i} - \frac{5}{13}\vec{j}$

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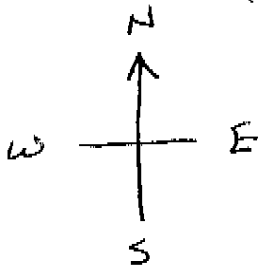
8.) Joe walks due west on the deck of the *Shepherd Moon* at 3 knots. The vessel is sailing south at a speed of 6 knots. Find the speed and direction Joe is walking relative to the surface of the water. (Note: This is a very short walk).

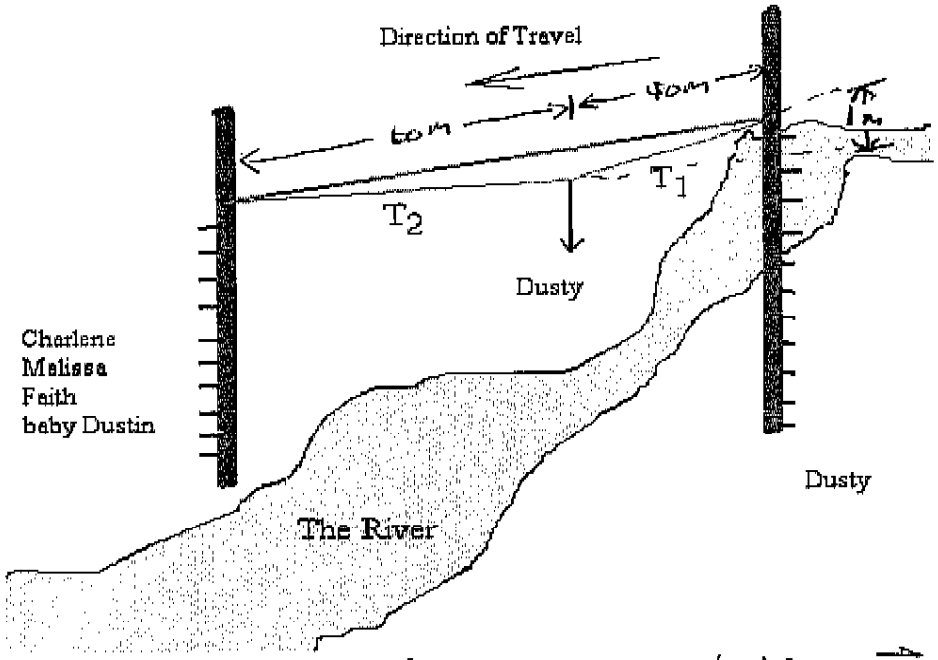


$\langle -3, 0 \rangle + \langle 0, -6 \rangle = \langle -3, -6 \rangle$ Joe's direction relative to the surface

$\| \langle -3, -6 \rangle \| = \sqrt{9 + 36} = \sqrt{45}$

Joe's speed is $\sqrt{45}$ knots/hr.

9.) A 100m cable is tightly suspended across a river (assume that it is a massless cable). The river floods and Dusty finds himself on the wrong side of the river with the cable as his only means to get back to safety, Charlene, Melissa, Faith, and baby Dustin. So, he decides to go across the cable hand over hand (Dusty is both stronger and more foolish than you might think). When Dusty is 40% of the way across, the cable is pulled down 1 meter. If we assume $g = 10\frac{m}{s^2}$ and that Dusty's mass is 61kg, what are the tensions on each side of the cable?



$\| T_1 \| \langle 40, 1 \rangle + \| T_2 \| \langle -60, 1 \rangle + \langle 0, -61(10) \rangle = \vec{0}$

$\Rightarrow \begin{bmatrix} 40 & -60 & 0 \\ 1 & 1 & 610 \end{bmatrix} \Rightarrow$ $\| \vec{T}_1 \| = 366 \sqrt{161} \approx 14644$ lbs

$\| \vec{T}_2 \| = 244 \sqrt{361} = 14642$ lbs

RREF on calc.