ERROR BOUNDS (11.4)

I only have 1 tool: $\int_{k+1}^{\infty} f(x)\,dx \leq R_k \leq \int_k^{\infty} f(x)\,dx.$

example For what smallest k is $\sum_{n=k+1}^{\infty} \frac{1}{n^4+1} < 0.00005$?

$$\int_{k+1}^{\infty} \frac{dx}{x^4+1} \leq R_k \leq \underbrace{\int_k^{\infty} \frac{dx}{x^4+1}}_{\text{no obvious antiderivative.}} \leq 0.00005$$

so squeeze a comparison term in

$$R_n \leq \int_k^{\infty} \frac{dx}{x^4+1} < \overbrace{\underbrace{\int_k^{\infty} \frac{dx}{x^4}}_{\frac{1}{3}\cdot\frac{1}{k^3}}}^{} \leq 0.00005$$

$$\Rightarrow R_n < \frac{1}{3k^3} \leq 0.00005$$

$$\text{and } k \geq \sqrt[3]{\frac{1}{3(0.00005)}} \approx 18.82$$

so, using a comparison, we can approximate $\sum \frac{1}{n^4+1}$ to w/in 0.00005 by using S_{19}.