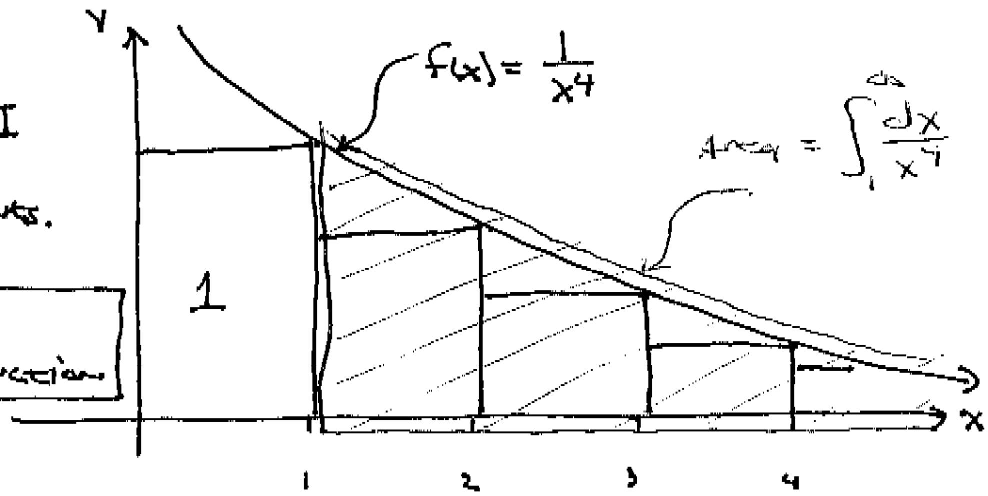
11.3: Integral Test & Estimating Sums

Ex1: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \dots$

Reminds us of the convergent $\displaystyle\int_1^{\infty} \frac{dx}{x^4} = \lim_{t\to\infty} -\frac{1}{3x^3}\Big|_1^t \;\frac{1}{3}$

since I believe it will converge, I use right endpoints.

The series is below the function

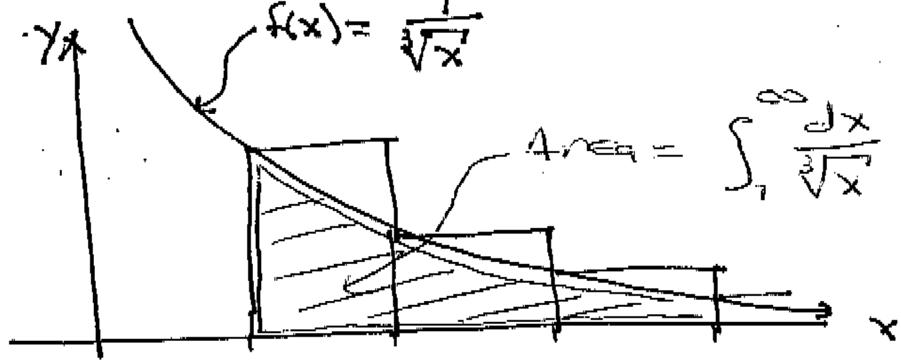


AND we have $\displaystyle 0 < \sum_{n=1}^{\infty} \frac{1}{n^4} < 1 + \frac{1}{3}$. Hence it converges. Using Mathematica, we know that it converges to $\pi^4/90$. But, most importantly it converges.

Ex2: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} = 1 + \frac{1}{\sqrt[3]{2}} + \frac{1}{\sqrt[3]{3}} + \dots$

This reminds us of the divergent $\displaystyle\int_1^{\infty} \frac{dx}{\sqrt[3]{x}}$.

Since I believe the series will diverge, I use left endpoints.



The ~~series~~ sequence is above the function

and $\int_1^\infty \frac{dx}{\sqrt[3]{x}} < \sum_{n=1}^\infty \frac{1}{\sqrt[3]{n}}$. So the series diverges.

<u>The Integral Test</u> Suppose f is a cont, pos, decreasing fct on $1 \leq x$ and let $a_n = f(n)$. Then $\sum_{n=1}^\infty a_n$ is convergent iff $\int_1^\infty f(x)\,dx$ is convergent.

<u>Ex 3:</u> Does $\sum_{n=1}^\infty n e^{-n}$ converge?

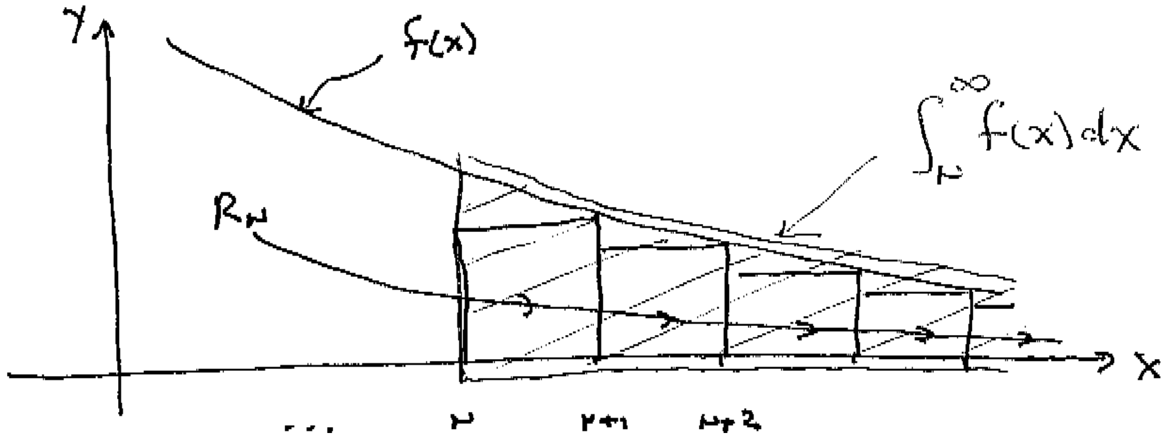
<u>Ex4:</u> For what values of p does $\sum_{n=1}^\infty \frac{1}{n^p}$ converge?

<u>Estimating Sums</u>

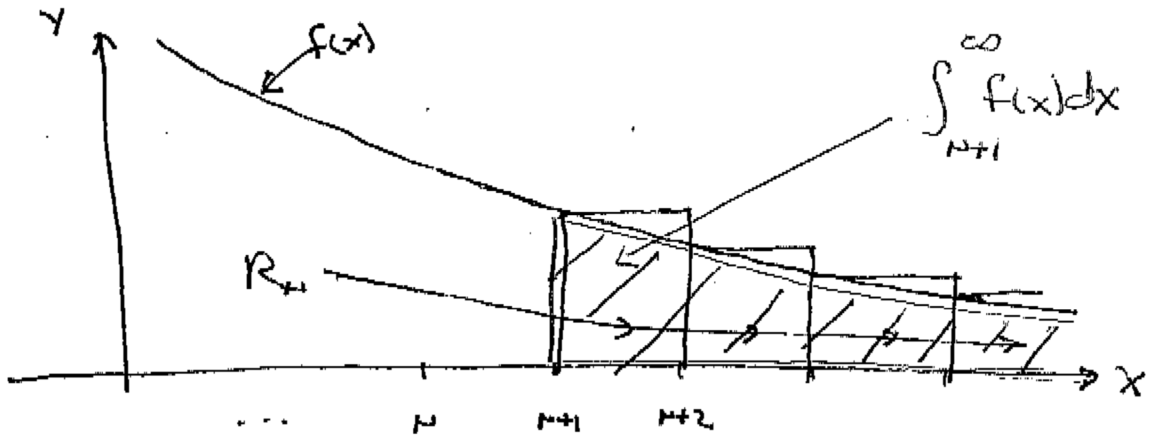
Suppose we know $\sum a_n$ converges but we cannot evaluate it directly. So, we want to estimate the series w/ the n^{th} partial sum S_n. But, this leaves us w/ an important question.

What is the error associated w/ each partial sum?

Remainder: $R_n = a_{n+1} + a_{n+2} + \ldots$
in S_n.



and $R_n \leq \int_n^\infty f(x)dx$. similarly



and $\int_{n+1}^\infty f(x)dx \leq R_n$.

<u>Remainder Estimate for the Integral Test</u>

If $\sum a_n$ converges by the I.T. and $R_n = S - S_n$,
then $\int_{n+1}^\infty f(x)dx \leq R_n \leq \int_n^\infty f(x)dx$.

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Ex5: Suppose $\sum_{n=1}^{\infty} \frac{1}{n^5} = S$

a) Find $S_5 = \frac{1}{1^5} + \ldots + \frac{1}{5^5}$

b) Bound R_5: $\underbrace{\int_6^{\infty} \frac{dx}{x^5}} \leq R_5 \leq \underbrace{\int_5^{\infty} \frac{dx}{x^5}}$

Integrate $\rightarrow \frac{1}{4 \cdot 6^4} \leq R_5 \leq \frac{1}{4 \cdot 5^4}$

c) For what n's is $R_n \leq 0.00005$

$$R_n \leq \int_n^{\infty} \frac{dx}{x^5} \leq 0.00005$$

$$\Rightarrow R_n \leq \frac{1}{4n^4} \leq 0.00005$$

$$\Rightarrow 8.41 \leq n$$

or so long as $n \geq 9$, $R_n \leq 0.00005$.