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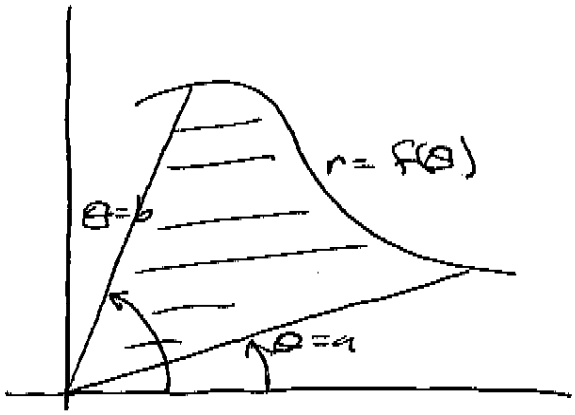
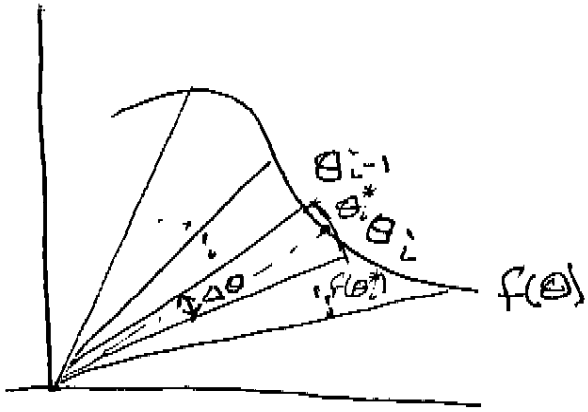
10.4: Areas & Arclength in Polar

(1) Area

recall

$A = \frac{1}{2} r^2 \theta$ (why?)





$$\Delta A_i \approx \frac{1}{2}\left[f(\theta_i^*)\right]^2 \Delta\theta$$

$$\Rightarrow A \approx \sum_{i=1}^{N} \frac{1}{2}\left[f(\theta_i^*)\right]^2 \Delta\theta$$

AND $$A = \lim_{N\to\infty} \sum_{i=1}^{N} \frac{1}{2}\left[f(\theta_i^*)\right]^2 \Delta\theta = \int_a^b \frac{1}{2}\left[f(\theta)\right]^2 d\theta$$

OR $$A = \int_a^b \frac{1}{2} r^2 \, d\theta$$

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Ex1: Find the area of the inner loop of $r = 1 - 2\cos\theta$.

(perhaps $\pi - \frac{3\sqrt{3}}{2}$)

(2) Arclength

Like the tangent problem for 10.3 — consider it as parametric.

show $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \left(\frac{dr}{d\theta}\right)^2 + r^2$

And if $r = f(\theta)$ has cont. f'.

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Ex2: Set up the integral to find the arclength of the outer loop of $r = 1 - 2\cos\theta$.