10.1
1/1

Math 126: Calc III

syllabus
calendar
hw
} print from the web.

NOTE: OH on W, not T.

In math 124 & 125, we almost exclusively studied calculus on fcts of the form $y = f(x)$. That is, 1 variable & 1 output, continuous, & 2-D outputs.

In math 126 we study calc on

- parametric
- polar
- 3D
- discrete
- vector fcts.

} derivatives & integrals on $y = f(x)$ only work on simply fcts.
The new skills will broaden the power of calc.

Parametric Equations.

ex1: The gist. Dusty's walk to work.

Parametric Equation: $f: t \longmapsto (x, y)$

or the pts on $(x(t), y(t))$.

MATCH THE PARAMETRIC EQUATIONS w/ THE CORRESPONDING GRAPH

(1) $x = t^2 - 1$; $y = t^4$

(2) $x = t^2 - 1$; $y = \sin t$

(3) $x = \cos 3t$; $y = \sin 2t$

(4) $x = t - 1$; $y = t^3$

(5) $x = t^2 - 1$; $y = \sin 2t$

(6) $x = 3\cos t$; $y = 2\sin t$

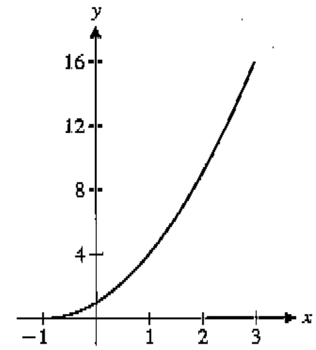


Figure C

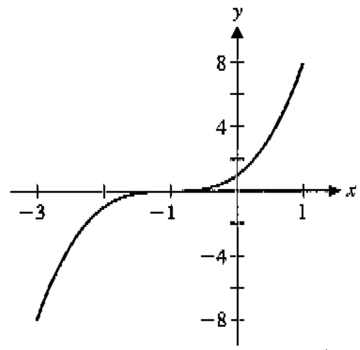


Figure D

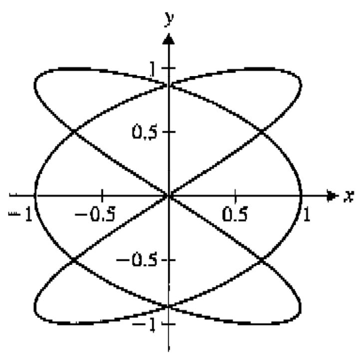


Figure A

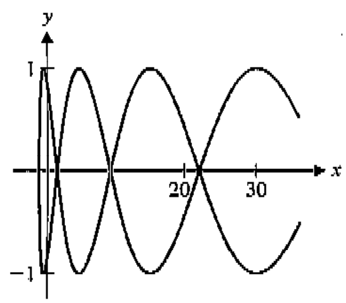


Figure E

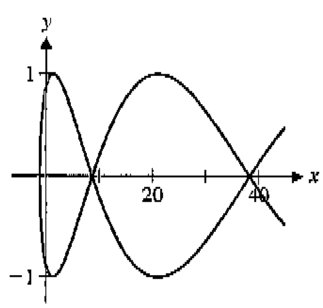


Figure B

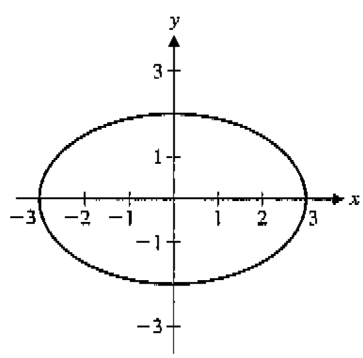


Figure F

CALCULUS, 2ND by Smith & Minton, p 729

10.1
2/2

Ex 2: sketch $x = 1 + \sqrt{t}$; $y = t^2 - 4t$ on $t \in [0,5]$ by plotting pts.

Ex 3: sketch $x = 1 + 3t$; $y = 2 - t^2$ by eliminating the parameter.

Ex 4: MATCHING HANDOUT.

Ex 5: eliminate the parameter $x = \sec\theta$; $y = \tan\theta$ and $-\frac{\pi}{2} \le \theta < \frac{\pi}{2}$. note: $\sec^2\theta - \tan^2\theta = 1$

$x^2 - y^2 = 1$ (hyperbola).

Parametric Graphing w/ calc. (Graph problems on the board).

parametrizations matter.

Ex 6: $t \longmapsto (\cos t, \sin t)$ unit circle.

the line $y = x$ w/ diff domains.

$$\begin{cases} t \longmapsto (t, t) \\ t \longmapsto \langle \sqrt{t}, \sqrt{t} \rangle \\ t \longmapsto (\sin t, \sin t) \\ t \longmapsto (\pi + (1-\pi)t, \pi + (1-\pi)t), \quad 0 \le t \le 1 \end{cases}$$

working w/ parametric eqts is generally not too difficult. The sticky part is finding a parametrization.

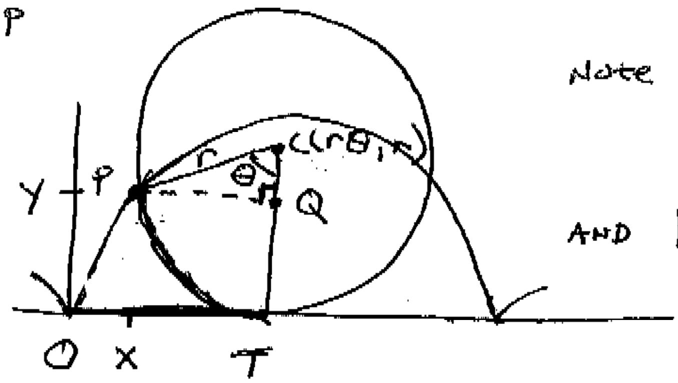
A famous example is the cycloid.

1st: recall that if 

, then $s = r\theta$.

now, the cycloid is like...

AND



Note $|PQ| = r\sin\theta$

$|CQ| = r\cos\theta$

AND $|OT| = \text{arc } PT = r\theta$

so $x = r\theta - r\sin\theta = r(\theta - \sin\theta)$

$y = r - r\cos\theta = r(1 - \cos\theta)$

AND we can parametrize the curve w/ the parameter θ (radians the circle rotates)

$$\theta \longmapsto \big(r(\theta - \sin\theta), r(1 - \cos\theta)\big).$$