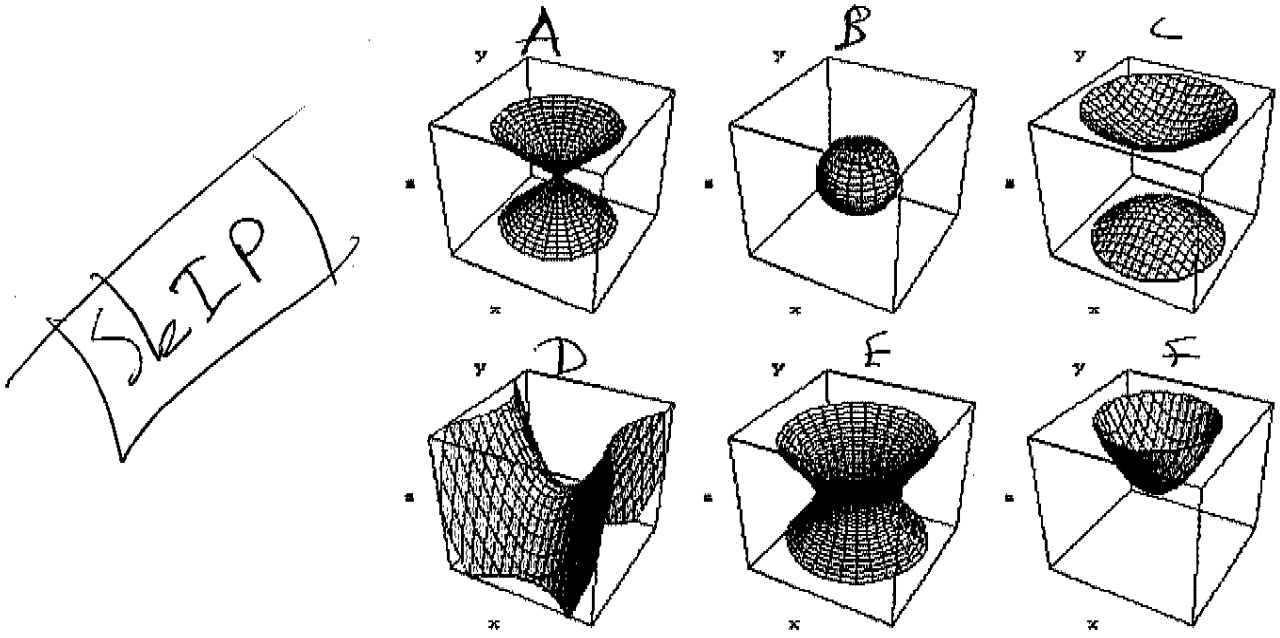
Overview: The following practice problems are from sections 12.6, 14.1, 14.3, and 14.4. There is not enough space given to solve each problem on the handout.

1.) Match each function to its graph. Give reasons for your choices.

B - $x^2+y^2+\frac{1}{4}z^2=1$ $z=-\sqrt{4-x^2-y^2}$ $y^2+\frac{1}{4}z^2=1$

$\frac{1}{9}z^2-\frac{1}{4}y^2=1$ $\frac{1}{4}x^2-y^2-z^2=1$ $|z|=\sqrt{x^2+y^2}$



SELF

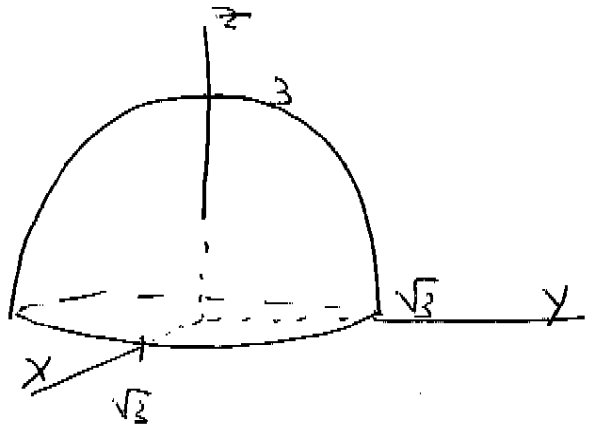
2.) Show that the curve of intersection of the surfaces $x^2+2y^2-z^2+3x=1$ and $2x^2+4y^2-2z^2-5y=0$ lies in a plane.

$* 2$

$2x^2+4y^2-2z^2-5y = 2x^2+4y^2-2z^2+6x-2$

$\Rightarrow \quad 2 = 5y+6x$ (a plane)

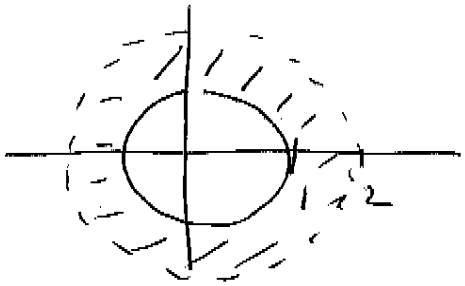
3.) Sketch the graph of $f(x,y)=3-x^2-y^2$



4.) Find and sketch the domain of the functions $f(x,y)=\sqrt{x^2+y^2-1}+\ln(4-x^2-y^2)$

$x^2+y^2-1 \geq 0$ & $4-x^2-y^2>0$

$x^2+y^2 \geq 1$ & $4 > x^2+y^2$



Domain.

5.) Draw a contour map of the function $f(x,y)=\dfrac{y}{x^2+y^2}$ showing several level curves.

$2=\dfrac{y}{x^2+y^2}$

$0=\dfrac{y}{x^2+y^2} \Rightarrow y=0$

$1=\dfrac{y}{x^2+y^2} \Rightarrow x^2+y^2=y \Rightarrow x^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{4}$

$2=\dfrac{y}{x^2+y^2} \Rightarrow x^2+y^2-\frac{1}{2}y=0$

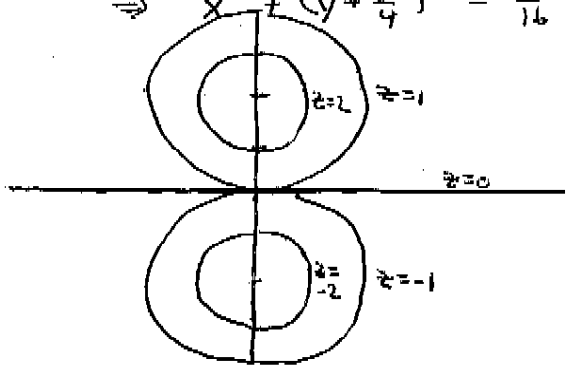
$x^2+\left(y-\frac{1}{4}\right)^2=\frac{1}{16}$

$-1=\dfrac{y}{x^2+y^2} \Rightarrow x^2+y^2+y=0$

$x^2+\left(y+\frac{1}{2}\right)^2=\frac{1}{4}$

$-2=\dfrac{y}{x^2+y^2} \Rightarrow x^2+y^2+\frac{y}{2}=0$

$\Rightarrow x^2+\left(y+\frac{1}{4}\right)^2=\frac{1}{16}$



6.) Find the partial derivatives of $u = x^{\frac{y}{2}}$.

$= (x^{\frac{1}{2}})^y = (x^y)^{\frac{1}{2}}$

$u_x = \frac{y}{2} x^{\frac{y}{2}-1}$

$u_y = (x^{\frac{1}{2}})^y \cdot \ln(x^{\frac{1}{2}})$

$= x^{y/2} \ln(x^{\frac{1}{2}})$

$= \frac{1}{2} x^{y/2} \ln(x)$

$u_z = -(x^y)^{\frac{1}{2}-1} \cdot \ln(x^y)$

$= -y \cdot x^{y/2} \ln(x)$

7.) Verify that Clairaut's Theorem holds for $u = x \cdot y \cdot e^y$.

$u_x = y e^y$

$u_{xy} = e^y + y e^y$

$u_y = x e^y + x y e^y$

$u_{yx} = e^y + y e^y$

So, $u_{xy} = u_{yx}$

8.) Show that $u = \sin(x - a \cdot t) + \ln(x + a \cdot t)$ satisfies that wave equation $u_{tt} = a^2 u_{xx}$.

$u_t = -a \cos(x - at) + \frac{a}{x + at}$

$u_{tt} = -a^2 \sin(x - at) + \frac{-a^2}{(x+at)^2}$

$= -a^2 \left(\sin(x - at) + \frac{1}{(x+at)^2} \right)$

$u_x = \cos(x - at) + \frac{1}{x + at}$

$u_{xx} = -\sin(x - at) - \frac{1}{(x+at)^2}$

So, $u_{tt} = a^2 u_{xx}$

9.) Find the equation of the tangent plane to the surface $z = e^{x^2-y^2}$ at the point $(1,-1,1)$.

$$z_x = 2x e^{x^2-y^2}\Big|_{(1,-1)} \quad 2$$

$$z_y = -2y e^{x^2-y^2}\Big|_{(1,-1)} \quad -2$$

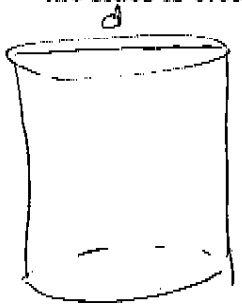
$$z = 1 + 2(x-1) - 2(y+1)$$

10.) Find the differential of the function $u = \frac{r}{s+2t}$.

$$u_r = \frac{1}{s+2t} \; ; \; u_s = \frac{-r}{(s+2t)^2} \; ; \; u_t = \frac{-2r}{(s+2t)^2}$$

$$du = \frac{dr}{s+2t} - \frac{r\,ds}{(s+2t)^2} - \frac{2r\,dt}{(s+2t)^2}$$

11.) Use differentials to estimate the amount of metal in a closed cylindrical can that is 10cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick



$$SA = \pi d h + 2\pi\left(\frac{d}{2}\right)^2 = \pi d h + \frac{\pi d^2}{2}$$

$$V = \pi\left(\frac{d}{2}\right)^2 h = \frac{\pi d^2 h}{4}$$

$$SA_d = \pi h + \pi d \qquad V_d = \pi d h/2$$

$$SA_h = \pi d \qquad V_h = \frac{\pi d^2}{4}$$

Actual volume of metal 9.000 cm³

$$dSA = (\pi h + \pi d)\,dd + \pi d\,dh$$

change in SA.

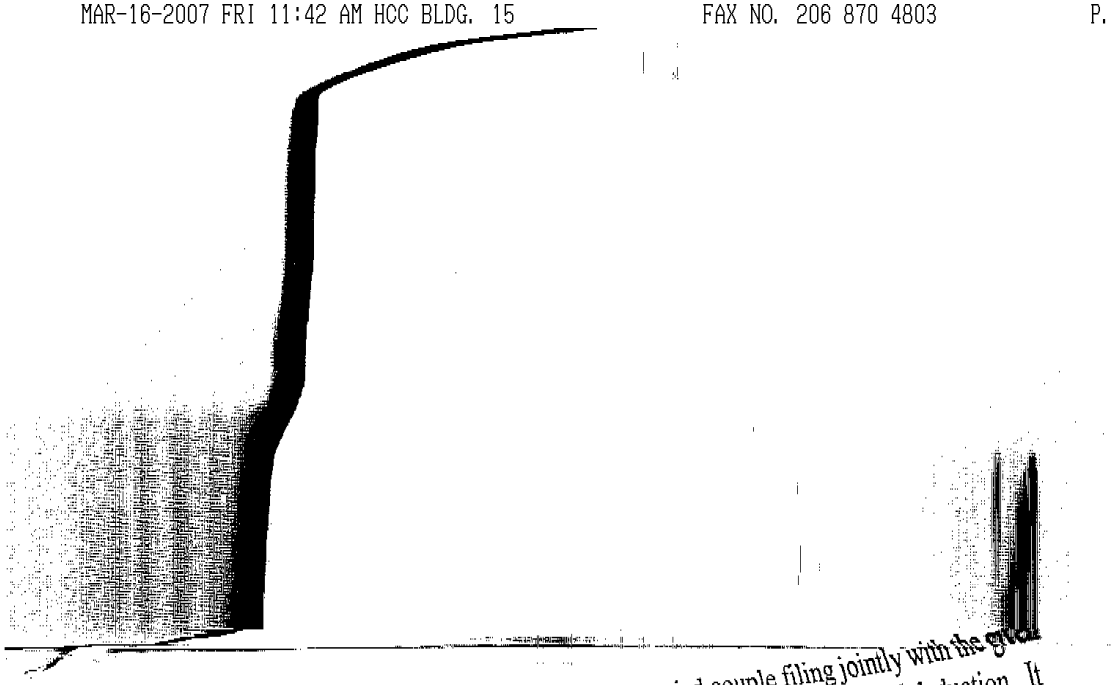
$$dV = \frac{\pi d h}{2}\,dd + \frac{\pi d^2}{4}\,dh$$

$h=10$, $d=4$, $dh=0.2$, $dd=0.1$

change in volume $dV = 8.796$ cm³

$h=10$, $d=4$, $dh=0.2$, $dd=0.1$

6.91 cm²



The table below lists the 2004 tax owed for a married couple filing jointly with the grad
AGI and number of children. It assumes that the filers claim the standard deduction. It
ignores all variables other than number of children. This includes the Alternative Minimum
Tax and the Earned Income Credit.

Number of Children / AGI	0	1	2	3
$ -	$ -	$ (1,000.00)	$ (2,000.00)	$ (3,000.00)
$ 5,000.00	$ -	$ (1,000.00)	$ (2,000.00)	$ (3,000.00)
$ 10,000.00	$ -	$ (1,000.00)	$ (2,000.00)	$ (3,000.00)
$ 15,000.00	$ -	$ (1,000.00)	$ (2,000.00)	$ (3,000.00)
$ 20,000.00	$ 410.00	$ (900.00)	$ (2,000.00)	$ (3,000.00)
$ 25,000.00	$ 910.00	$ (400.00)	$ (1,710.00)	$ (3,000.00)
$ 30,000.00	$ 1,410.00	$ 100.00	$ (1,210.00)	$ (2,520.00)
$ 35,000.00	$ 2,150.00	$ 685.00	$ (710.00)	$ (2,020.00)
$ 40,000.00	$ 2,900.00	$ 1,435.00	$ (30.00)	$ (1,495.00)
$ 45,000.00	$ 3,650.00	$ 2,185.00	$ 720.00	$ (745.00)
$ 50,000.00	$ 4,400.00	$ 2,935.00	$ 1,470.00	$ 5.00
$ 55,000.00	$ 5,150.00	$ 3,685.00	$ 2,220.00	$ 755.00

12.) Use the given table to estimate the tax owed by a married couple filing jointly with an AGI of $44,252 that has 2.2 children that are claimed as dependents.

$T_N(N=2,\ AGI=45000) = -1465$

$T_a(N=2,\ AGI=45000) = 750/5000 = 0.15$

$T(N=2.2,\ AGI=44252) \approx$

$\approx 720 - 1465(2.2-2) + 0.15(44252-4$

$= \$314.8$ Approx tax owed by this family.