12.2: Vectors.

Solutions to problems (1) thru (7).

- Know the notation of vectors.
- Know the algebraic and graphical interpretations of vectors.
- Understand how to break vectors into components.
- Be able to find the magnitude or norm of a vector.
- Know the properties of vectors.
- Know the basic unit vectors i, j, and k.
- Know how to find a unit vector parallel to a given vector.
- Be able to solve basic static equilibrium problems using vectors.

As I have no trouble sleeping, you are on your own for (8)-(12)

12.3: The Dot Product

- Definition and properties of the dot product.
- Geometric interpretation/definition of the dot product.
- Projections.

12.4: The Cross Product

- Definition and properties of the cross product.
- Geometric definition of the cross product.
- Parallelogram law and the volume of the parallelepiped.

12.5: Equations of Lines and Planes

- Parametric equations for a line.
- Symmetric equations for a line.
- Line segment between two points.
- Scalar equation of r a plane through a point.
- Line of intersection between two planes.
- Distance between a point and line.

Practice Problems

These are meant to be a sample and should not be considered as an exhaustive list of the problems in the sections covered on this exam.

1.) For $x = 3te^{-t}$, $y = e^{2t}$, find the exact equation of the tangent line to the curve when $t = \ln(5)$.

$$\frac{dy}{dt} = 2e^{2t} \qquad y(\ln 5) = 25$$

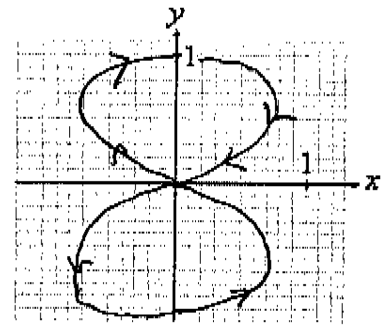
$$\frac{dx}{dt} = 3e^{-t} - 3te^{-t} \qquad x(\ln 5) = \frac{3\ln 5}{5}$$

$$\frac{dy}{dx} = \frac{2e^{2t}}{3e^{-t}(1-t)}$$

line: $y - 25 = \frac{250}{3(1-\ln 5)}\left(x - \frac{3\ln 5}{5}\right)$

$$\left.\frac{dy}{dx}\right|_{t=\ln(5)} \quad \frac{2(e^{\ln 5})^2}{\frac{3}{e^{\ln 5}}(1-\ln 5)} = \frac{50}{\frac{3}{5}(1-\ln 5)}$$

2.) Given $x = \sin(2t)$ and $y = \cos(t)$, (a.) eliminate the parameter to find the Cartesian equation of the curve. (b.) Sketch the curve and indicate with an arrow the direction that the curve is traced as the parameter increases.



$x = 2\sin t\cos t$ and $t = \cos^{-1}(y)$

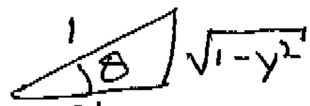
$\cancel{x^2 + y^2 = 4\sin^2 t\cos^2 t + \cos^2 t}$

$\cancel{= \cos^2 t\,(4\sin^2 t + 1)}$

$\Rightarrow x = 2\sin(\cos^{-1}(y))\cos(\cos^{-1}(y))$

$= 2\sqrt{1-y^2}\,y$

$\Rightarrow x^2 = 4(1-y^2)y^2$



3.) Find the length of the curve $x = e^t \sin t$ and $y = e^t \cos t$ for $0 \le t \le \frac{\pi}{2}$ and graph the curve.

$\frac{dx}{dt} = e^t\sin t + e^t\cos t \Rightarrow \left(\frac{dx}{dt}\right)^2 = e^{2t}\sin^2 t + 2e^{2t}\sin t\cos t + e^{2t}\cos^2 t$

$\frac{dy}{dt} = e^t\cos t - e^t\sin t \Rightarrow \left(\frac{dy}{dt}\right)^2 = e^{2t}\cos^2 t - 2e^{2t}\sin t\cos t + e^{2t}\sin^2 t$

$L = \int_0^{\pi/2} \sqrt{2e^{2t}(\sin^2 t + \cos^2 t)}\,dt$

$= \int_0^{\frac{\pi}{2}} \sqrt{2}\,e^t\,dt$

$= \sqrt{2}\,e^t \Big|_0^{\pi/2}$

$= \sqrt{2}(e^{\pi/2} - 1)$.

4.) If $\bar{u} = i + 2j + 3k$ and $\bar{v} = -2i + j - 5k$, find $\|\bar{u}\|$, $\bar{u} + \bar{v}$, $\bar{u} - \bar{v}$, $3\bar{v}$, and $2\bar{u} - 3\bar{v}$.

unit vector in the direction of $\vec{u}$: $\underbrace{\left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle}_{\text{unit vector}}$

$\|\bar{u}\|$	$\bar{u}+\bar{v}$	$\bar{u}-\bar{v}$	$3\bar{v}$	$2\bar{u}-3\bar{v}$
$\sqrt{14}$	$-\vec{i} + 3\vec{j} - 2\vec{k}$	$\langle 3, 1, 8 \rangle$	$\langle -6, 3, -15 \rangle$	$\langle 8, 1, 21 \rangle$

5.) Find the "angle" between the vectors $\langle 1,2,3,4 \rangle$ and $\langle -2,3,-5,7 \rangle$.

$$\cos\theta = \frac{-2+6-15+28}{\sqrt{1+2^2+3^2+4^2}\cdot\sqrt{(-2)^2+3^2+(-5)^2+7^2}}$$

$$\cos\theta = \frac{17}{\sqrt{30}\cdot\sqrt{87}}$$

$$\Rightarrow \theta \approx 70.5^\circ$$

6.) Find the "angle" between the vectors $\langle 1,2,3,4 \rangle$ and $\langle -2,3,-5,7 \rangle$.

7.) Find the line through the points $A(5,0,-1)$ and $B(2,1,-3)$.

sol 1:

$\vec{r}_0 = \langle 5,0,-1 \rangle$

$\vec{v} = \langle 3,-1,2 \rangle$

$\vec{r} = \vec{r}_0 + t\vec{v}$

sol 2:

$\vec{r}_1 = \langle 5,0,-1 \rangle$ & $\vec{r}_2 = \langle 2,1,-3 \rangle$

$\vec{r} = (1-t)\vec{r}_1 + t\vec{r}_2$

8.) Find the plane through the points $A(-1,-2,-3)$, $B(3,5,-2)$, and $C(3,2,7)$.

$\vec{u} = \langle 4,7,1 \rangle$

$\vec{v} = \langle 4,4,10 \rangle$

$$\Rightarrow \vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 7 & 1 \\ 4 & 4 & 10 \end{vmatrix} = \langle 66,-36,-12 \rangle$$

plane: $66(x-3) + -36(y-2) - 12(z-7) = 0$

from the normal & pt C