Strategy for Testing Series
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Math 126 – Winter 2005

KEY

No work = no credit.

I cannot compute C_m for $m > 3$. This may be due to old age, stupidity, and laziness

Paul Erdős (1913 - 1996)
Hungarian Mathematician

Instructions: Determine convergence or divergence of each infinite series. Support you're answer in a solid mathematical manner such as a test, partial sum argument, or limit.

1.) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+7}$ Alternating Series Test.

1) show decreasing. $f(x) = \frac{\sqrt{x}}{x+7}$, so $f'(x) = \frac{\frac{1}{2\sqrt{x}}(x+7) - \sqrt{x}}{(x+7)^2}$

$\Rightarrow f'(x) = \frac{x+7-2x}{2\sqrt{x}(x+7)^2} = \frac{7-x}{2\sqrt{x}(x+7)^2}$. So $f' < 0$ when $x > 7$ and $\sqrt{n}/(n+7)$ is decreasing on $n > 7$.

2) show $\lim_{n\to\infty} \frac{\sqrt{n}}{n+7} = 0$. $\lim_{n\to\infty} \frac{\sqrt{n}}{n+7} \overset{H}{=} \lim_{n\to\infty} \frac{1}{2\sqrt{n}} = 0$.

So convergent by the AST.

2.) $\sum_{n=3}^{\infty} \frac{4}{n\ln(n)\cdot\ln(\ln n)}$

Integral Test.

$\int_3^{\infty} \frac{4dx}{x\ln(x)\cdot\ln(\ln(x))}$ Let $u = \ln(\ln(x))$, $du = \frac{dx}{x\ln(x)}$

$= \int_{\ln(\ln(3))}^{\infty} \frac{4du}{u}$ which is divergent by the p-test.

So, the series diverges.

3.) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ Telescoping Series

$= [\ln(1) - \ln(2)] + [\ln(2) - \ln(3)] + \ldots + [\ln(n) - \ln(n+1)] + \ldots$

$= \ln(1) + \lim_{n\to\infty} -\ln(n+1)$

$= 0 - \infty$

$= -\infty$

Divergent.

<u>Limit Comparison Test</u>

4.) $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n(n-1)(n-2)}}$

$$\lim_{n \to \infty} \frac{2/n^{3/2}}{2/\sqrt{n^3 - 3n^2 + 2n}} = \lim_{n \to \infty} \frac{\sqrt{n^3 - 3n^2 + 2n}}{\sqrt{n^3}}$$

$$= \lim_{n \to \infty} \frac{\sqrt{1 - 3/n + 2/n^2}}{1}$$

$$= 1.$$

since $\sum \frac{2}{n^{3/2}}$ is a convergent p-series, our series converges.

5.) $\sum_{n=1}^{\infty} \frac{n^n}{3^{1+3n}}$ <u>Root Test</u>

$$\lim_{n \to \infty} \sqrt[n]{\left|\frac{n^n}{3^{1+3n}}\right|} = \lim_{n \to \infty} \frac{n}{3^{1/n+3}} \to \frac{\infty}{27} = \infty > 1.$$

so, by the root test the series diverges.

6.) $\sum_{n=1}^{\infty} (-1)^n \frac{3n}{n!} = \sum_{1} \frac{(-1)^n \cdot 3}{(n-1)!}$ can use Ratio test.

<u>Alternating Series Test.</u>

1) $\frac{3}{(n-1)!}$ is decreasing

2) $\lim_{n \to \infty} \frac{3}{(n-1)!} = 0$

so, convergent by the AST.