LESSON 10.2 – QUADRATIC EQUATIONS II

OVERVIEW

Here's what you'll learn in this lesson:

Completing the Square

a. Solving quadratic equations of the form $x^2 + bx + c = 0$ by completing the square

b. Solving quadratic equations of the form $ax^2 + bx + c = 0$, $a \neq 1$, by completing the square

The Quadratic Formula

a. Introduce the quadratic formula

b. Using the quadratic formula to solve quadratic equations of the form $ax^2 + bx + c = 0$

c. Using the discriminant of a quadratic equation to determine the nature of the solutions of the equation

An astronomer is measuring the distance to a star. A family wants to determine the best investments to help pay for their child's education. A paperclip manufacturer is trying to figure out the selling price that will produce the greatest revenue for the company.

Each of these people can find the information they are looking for by solving equations known as quadratic equations, or equations in one variable of degree two.

In this lesson you will learn how to solve quadratic equations by completing the square and by using the quadratic formula.

EXPLAIN

COMPLETING THE SQUARE

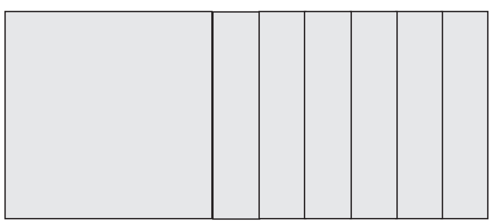
Summary

You can solve some quadratic equations by factoring and you can solve some using the square root property. What do you do when you have an equation that can't be solved using either of these methods? You use a method called completing the square. In this method, you write the equation in the form $x^2 = a$, then you use the square root property. Completing the square can be used to solve any quadratic equation.

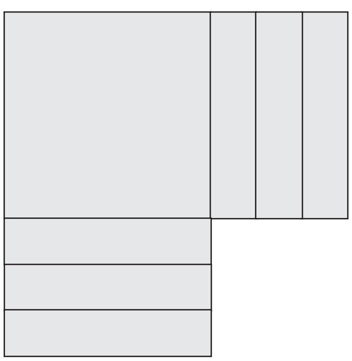
Completing the Square

To write an equation in the form $x^2 = a$, you must write the left side of the equation as a perfect square. You can learn how to do this with the example $x^2 + 6x = 0$. It sometimes helps to use algebra tiles.

Here's a model of the left side, $x^2 + 6x$:



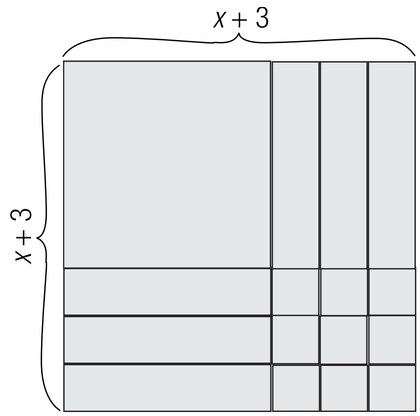
Try to rearrange the tiles to form a square by moving half the x tiles:



You see that the square is not complete.

Notice that the area of the square is $(x + 3)(x + 3)$.

To complete the square, you need to add 9 unit tiles.



By adding 9 to both sides of the equation $x^2 + 6x = 0$, you have formed a square.

$$x^2 + 6x + 9 = 9$$
$$(x + 3)(x + 3) = 9$$
$$(x + 3)^2 = 9$$

The equation is now in the form $x^2 = a$.

What if you were trying to complete the square without using tiles. How would you do it?

1. Make sure the coefficient of x^2 is 1,then multiply the coefficient of x by $\frac{1}{2}$.
2. Square the result.
3. Add this number to both sides of the equation.

So, for example, to complete the square given $x^2 + 6x = 0$:

1. Make sure the coefficient of x^2 is 1, then multiply the coefficient of x by $\frac{1}{2}$.	$6 \cdot \frac{1}{2} = 3$
2. Square the result.	$3^2 = 9$
3. Add this number to both sides.	$x^2 + 6x + 9 = 9$

The left side of the equation is a perfect square: $(x + 3)^2$.

Here's another example. Complete the square given $x^2 + 5x = 0$:

1. Make sure the coefficient of x^2 is 1, then multiply the coefficient of x by $\frac{1}{2}$.	$5 \cdot \frac{1}{2} = \frac{5}{2}$
2. Square the result.	$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$
3. Add this number to both sides.	$x^2 + 5x + \frac{25}{4} = \frac{25}{4}$

The left side of the equation is a perfect square: $\left(x + \frac{5}{2}\right)^2$.

As a final example, complete the square given $4x^2 + 16x = 0$:

1. Make sure the coefficient of x^2 is 1, then multiply the coefficient of x by $\frac{1}{2}$. $\quad x^2 + 4x = 0$, $\quad 4 \cdot \frac{1}{2} = 2$
2. Square the result. $\quad 2^2 = 4$
3. Add this number to both sides. $\quad x^2 + 4x + 4 = 4$

The left side of the equation is a perfect square: $(x + 2)^2$.

Now that you know how to complete the square, you can write any equation in the form $x^2 = a$, and then use the square root property to find the solutions.

Solving Quadratic Equations by Completing the Square

To solve a quadratic equation by completing the square:

1. Isolate the x^2- and x-terms on the left side of the equation.
2. Make sure the coefficient of x^2 is 1.
   (You may have to divide both sides by the coefficient of x^2.)
3. Complete the square.
   a. Multiply the coefficient of x by $\frac{1}{2}$.
   b. Square the result.
   c. Add this number to both sides of the equation.
4. Factor the left side of the equation. (It will factor as a perfect square.)
5. Finish solving for the variable.

For example, to solve $x^2 + 10x = 9$ for x:

1. Isolate the x^2- and x-terms on the left. They are already isolated.
2. Make sure the coefficient of x^2 is 1. The coefficient of x^2 is 1.
3. Complete the square.
   a. Multiply the coefficient of x by $\frac{1}{2}$. $\quad 10 \cdot \frac{1}{2} = 5$
   b. Square the result. $\quad 5^2 = 25$
   c. Add this number to both sides. $\quad x^2 + 10x + 25 = 9 + 25$
4. Factor the left side of the equation. $\quad (x + 5)^2 = 34$
5. Finish solving using the square root property.

$$x + 5 = \sqrt{34} \quad \text{or} \quad x + 5 = -\sqrt{34}$$
$$x = -5 + \sqrt{34} \quad \text{or} \quad x = -5 - \sqrt{34}$$

You can use a shortcut and write these two solutions as $x = -5 \pm \sqrt{34}$.

The "±" symbol is read as "plus or minus".

Some quadratic equations don't have real solutions.

For example, try to solve $2x^2 + 3x + 4 = 0$ by completing the square:

1. Isolate the x^2- and x-terms on the left. $2x^2 + 3x = -4$

2. Make sure the coefficient of x^2 is 1.
   (Divide both sides by 2.) $x^2 + \frac{3}{2}x = -2$

3. Complete the square.

   a. Multiply the coefficient of x by $\frac{1}{2}$. $\frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}$

   b. Square the result. $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

   c. Add this number to both sides. $x^2 + \frac{3}{2}x + \frac{9}{16} = -2 + \frac{9}{16}$

4. Factor the left side of the equation. $\left(x + \frac{3}{4}\right)^2 = -\frac{23}{16}$

5. Finish solving using the square root property. STOP!

To use the square root property you would have to take the square root of a negative number. This doesn't make sense for real numbers, so this quadratic equation has no real solutions.

Sample Problems

1. Solve $x^2 + 6x - 13$ for x:

☐ a. Isolate the x^2- and x-terms on the left. $\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_$

✔ b. Make sure the coefficient of x^2 is 1.The coefficient of x^2 is 1.

☐ c. Complete the square.

Multiply the coefficient of x by $\frac{1}{2}$. $\_\_\_\_ \cdot \frac{1}{2} = \_\_\_\_$

Square the result. $(\_\_\_)^2 = \_\_\_\_$

Add this number to both sides. $\_\_\_\_\_\_\_\_\_\_ = 13 + \_\_\_\_$

☐ d. Factor the left side of the equation. $(x + \_\_\_\_)^2 = \_\_\_\_$

☐ e. Finish solving for x. $x + \_\_\_\_ = \_\_\_\_\_\_$ or $x + \_\_\_\_ = \_\_\_\_\_\_$

So, the solutions are: $x = \_\_\_\_\_\_$

2. Solve $3x^2 + 7x + 3 = 0$ by completing the square:

✔ a. Isolate the x^2- and x-terms on the left. $3x^2 + 7x = -3$

✔ b. Make sure the coefficient of x^2 is 1. $x^2 + \frac{7}{3}x = -1$

(Divide both sides by 3.)

☐ c. Complete the square.

Multiply the coefficient of x by $\frac{1}{2}$. $\_\_\_\_ \cdot \frac{1}{2} = \_\_\_\_$

Square the result. $(\_\_\_)^2 = \_\_\_\_$

Add this number to both sides. $x^2 + \frac{7}{3}x + \_\_\_\_ = -1 + \_\_\_\_$

☐ d. Factor the left side of the equation. $(x + \_\_\_\_\_)^2 = \_\_\_\_\_$

☐ e. Finish solving for x. $x + \_\_\_\_\_ = \_\_\_\_\_$ or $x + \_\_\_\_\_ = \_\_\_\_\_$

So, the solutions are: $x = \_\_\_\_\_\_\_\_$

Answers to Sample Problems

a. $x^2 + 6x$, 13

c. 6, 3

3, 9

$x^2 + 6x + 9$, 9

d. 3, 22

e. 3, $\sqrt{22}$, 3, $-\sqrt{22}$

$-3 \pm \sqrt{22}$

c. $\frac{7}{3}$, $\frac{7}{6}$

$\frac{7}{6}$, $\frac{49}{36}$

$\frac{49}{36}$, $\frac{49}{36}$

d. $\frac{7}{6}$, $\frac{13}{36}$

e. $\frac{7}{6}$, $\sqrt{\frac{13}{36}}$, $\frac{7}{6}$, $-\sqrt{\frac{13}{36}}$

$-\frac{7}{6} \pm \frac{\sqrt{13}}{6}$

THE QUADRATIC FORMULA

Summary

If you start with the quadratic equation $ax^2 + bx + c = 0$ and complete the square, you will get a general formula that will solve any quadratic equation. This formula is called the quadratic formula.

The quadratic formula is: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So to find the solutions of any quadratic equation, you only have to substitute the values of a, b, and c into the quadratic formula.

Solving Quadratic Equations using the Quadratic Formula

To solve a quadratic equation using the quadratic formula:

1. Write the equation in standard form.
2. Identify the values of a, b, and c.
3. Substitute these values into the quadratic formula.
4. Simplify.

For example, to solve $3x^2 + 2x - 8 = 0$:

1. Write the equation in standard form. $3x^2 + 2x - 8 = 0$
2. Identify the values of a, b, and c. $a = 3$, $b = 2$, $c = -8$

Notice $c = -8$, not 8.

3. Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-8)}}{2(3)}$$

4. Simplify.

$$x = \frac{-2 \pm \sqrt{4 + 96}}{6}$$

$$x = \frac{-2 \pm \sqrt{100}}{6}$$

$$x = \frac{-2 \pm 10}{6}$$

$$x = \frac{-2 + 10}{6} \quad \text{or} \quad x = \frac{-2 - 10}{6}$$

$$x = \frac{8}{6} \quad \text{or} \quad x = \frac{-12}{6}$$

$$x = \frac{4}{3} \quad \text{or} \quad x = -2$$

As another example, to solve $x^2 - 5x = 11$:

1. Write the equation in standard form. $x^2 - 5x - 11 = 0$
2. Identify the values of a, b, and c. $a = 1$, $b = -5$, $c = -11$
3. Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-11)}}{2(1)}$$

4. Simplify.

$$x = \frac{5 \pm \sqrt{25 + 44}}{2}$$

$$x = \frac{5 \pm \sqrt{69}}{2}$$

As a third example, to solve $25x^2 + 9 = 30x$:

1. Write the equation in standard form. $25x^2 - 30x + 9 = 0$
2. Identify the values of a, b, and c. $a = 25$, $b = -30$, $c = 9$
3. Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(25)(9)}}{2(25)}$$

4. Simplify.

$$x = \frac{30 \pm \sqrt{900 - 900}}{50}$$

$$x = \frac{30 \pm \sqrt{0}}{50}$$

$$x = \frac{3}{5} \quad \text{or} \quad x = \frac{3}{5}$$

This equation has a solution of multiplicity two.

As a final example, to solve $x^2 + 3x + 9 = 0$:

1. Write the equation in standard form. $x^2 + 3x + 9 = 0$
2. Identify the values of a, b, and c. $a = 1$, $b = 3$, $c = 9$
3. Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(9)}}{2(1)}$$

4. Simplify.

$$x = \frac{-3 \pm \sqrt{9 - 36}}{2}$$

$$x = \frac{-3 \pm \sqrt{-27}}{2}$$

Taking the square root of a negative number does not make sense for real numbers. So, this equation has no real solutions.

Discriminants

The equations above have solutions that look very different. You can tell what the solutions will look like without solving the entire quadratic formula by looking at the discriminant. The discriminant is $b^2 - 4ac$, the part of the quadratic formula under the square root sign.

Discriminant	Quadratic Formula	Solutions
$b^2 - 4ac > 0$	$x = \frac{-b \pm \sqrt{\text{positive number}}}{2a}$	two distinct real
$b^2 - 4ac < 0$	$x = \frac{-b \pm \sqrt{\text{negative number}}}{2a}$	no real
$b^2 - 4ac = 0$	$x = \frac{-b \pm \sqrt{0}}{2a}$ or $x = \frac{-b}{2a}$	two equal real

Sample Problems

1. Solve $3x^2 + 2x - 5 = 0$:

☑ a. Write the equation in standard form. $3x^2 + 2x - 5 = 0$

☐ b. Identify the values of a, b, and c. $a = 3$, $b = 2$, $c =$ ____

☐ c. Substitute these values into the quadratic formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-\_\_\_ \pm \sqrt{2^2 - 4(\_\_\_)(\_\_\_)}}{2(3)}$

☐ d. Simplify. $x = \frac{-\_\_\_ \pm \sqrt{\_\_\_}}{6}$

$x = \frac{-\_\_\_ \pm \_\_\_}{6}$

$x =$ ____ or $x =$ ____

2. Solve $x^2 - 12x + 3 = 0$:

☑ a. Write the equation in standard form. $x^2 - 12x + 3 = 0$

☐ b. Identify the values of a, b, and c. $a =$ ___, $b =$ ___, $c = 3$

☐ c. Substitute these values into the quadratic formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-\_\_\_ \pm \sqrt{(\_\_\_)^2 - 4(\_\_\_)(3)}}{2(\_\_\_)}$

☐ d. Simplify. $x =$ ______ or $x =$ ______

Answers to Sample Problems

b. –5

c. 2, 3, –5

d. 2, 64,

2, 8

1; $-\frac{5}{3}$ *or* $\frac{-10}{6}$

b. 1, –12

c. top: –12, –12, 1
bottom: 1

d. $6 + \sqrt{33}, 6 - \sqrt{33}$

Here is one way to simplify:

$x = \frac{12 \pm \sqrt{132}}{2}$

$x = \frac{12 \pm \sqrt{4} \cdot \sqrt{33}}{2}$

$x = \frac{12 \pm 2\sqrt{33}}{2}$

$x = 6 + \sqrt{33}$ or $x = 6 - \sqrt{33}$

3. Solve $4x^2 + 28x = -49$:

☑ a. Write the equation in standard form. $4x^2 + 28x + 49 = 0$

☑ b. Identify the values of a, b, and c. $a = 4,\ b = 28,\ c = 49$

☐ c. Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-\_\_\_\_ \pm \sqrt{\_\_\_\_^2 - 4(4)(49)}}{2(\_\_\_\_)}$$

☐ d. Simplify. $x = \_\_\_\_$ or $x = \_\_\_\_$

4. Solve $2x^2 + 4 = -2x$:

☑ a. Write the equation in standard form. $2x^2 + 2x + 4 = 0$

☐ b. Identify the values of a, b, and c. $a = \_\_\_\_,\ b = \_\_\_\_,\ c = \_\_\_\_$

☐ c. Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-\_\_\_ \pm \sqrt{\_\_\_^2 - 4(\_\_)(\_\_\_)}}{4}$$

☐ d. Simplify. $x = \_\_\_\_\_\_\_\_$ or $x = \_\_\_\_\_\_\_\_$

Answers to Sample Problems

Problem 3

c. top: 28, 28
bottom: 4

d. $-\frac{7}{2}, -\frac{7}{2}$

Here is one way to simplify:

$$x = \frac{-28 \pm \sqrt{0}}{8}$$

$$x = \frac{-28 \pm 0}{8}$$

$$x = -\frac{7}{2} \text{ or } x = -\frac{7}{2}$$

This quadratic equation has a solution of multiplicity two.

Problem 4

b. 2, 2, 4

c. 2, 2, 2, 4

d. $\frac{-2 + \sqrt{-28}}{4}, \frac{-2 - \sqrt{-28}}{4}$

This quadratic equation has no real solutions.

Answers to Sample Problems

EXPLORE

Sample Problems

On the computer you examined the solutions of quadratic equations and discriminants. Below are some additional exploration problems.

1. The solutions of a quadratic equation are $\frac{7}{3}$ and –2. Work backwards to find a quadratic equation with these solutions.

☑ a. Start with the solutions. $x = \frac{7}{3}$ $x = -2$

☑ b. Write the equation in the form $ax + b = 0$. $3x = 7$, $3x - 7 = 0$ $x + 2 = 0$

☑ c. Multiply. $(3x - 7)(x + 2) = 0$

☐ d. Use the FOIL method. $3x^2 + 6x$ ____ + ____ = 0

d. –7x, –14

☐ e. Simplify. ________________ = 0

e. $3x^2 - x - 14$

2. The sum of the solutions of a quadratic equation is $-\frac{b}{a}$. The product of the solutions of a quadratic equation is $\frac{c}{a}$.

If you know the solutions of a quadratic equation you can use their sum and product to find an equation with those solutions.

The solutions of a quadratic equation are $-\frac{1}{2}$ and –3. Use their sum and product to find an equation with those solutions.

☑ a. Add the solutions. $-\frac{1}{2} + -3 = -\frac{7}{2} = -\frac{b}{a}$

☑ b. Multiply the solutions. $-\frac{1}{2} \cdot -3 = \frac{3}{2} = \frac{c}{a}$

☐ c. Find possible values of a, b, and c. $a = 2$, $b =$ ____, $c = 3$

c. 7

☐ d. Write a quadratic equation using these values. ____x^2 + ____x + ____ = 0

d. 2, 7, 3

3. The quadratic equation $2x^2 - 7x + c = 0$ has a discriminant of 9. What is the value of c?

☑ a. Set the discriminant equal to 9. $b^2 - 4ac = 9$

☐ b. Substitute the values of a and b. $(\_\_)^2 - 4(\_\_)(c) = 9$

☐ c. Simplify.

$$\_\_\_\_ - \_\_c = 9$$
$$\_\_\_\_c = \_\_\_\_$$
$$c = \_\_\_\_$$

4. The quadratic equation $2x^2 - 7x + c = 0$ has a discriminant of 9. What are the solutions?

☑ a. Substitute values into the quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{9}}{2(2)}$$

☑ b. Simplify.

$$x = \frac{7 \pm 3}{4}$$
$$x = \frac{10}{4} \text{ or } x = \frac{4}{4}$$
$$x = \frac{5}{2} \text{ or } x = 1$$

Answers to Sample Problems

b. $-7, 2$

c. $49, 8$

$-8, -40$

5

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain
Completing the Square

1. Complete the square: $x^2 + 13x$. What is the area of the completed square?
2. Solve $x^2 + 2x = 8$ by completing the square.
3. Solve $x^2 - 4x = 1$ by completing the square.
4. Solve $x^2 + 9x = -2$ by completing the square.
5. Solve $x^2 - 36x = 40$ by completing the square.
6. Solve $x^2 + 3x - 7 = 0$ by completing the square.
7. Solve $2x^2 + 6x = 2$ by completing the square.
8. Solve $3x^2 - 5x - 9 = 0$ by completing the square.
9. Seana is competing in the bicycle Race Across America. She rode 62 miles before lunch and 69 miles after lunch. She rode for one hour more after lunch than before lunch, but her speed after lunch was 2 mph slower. What were her speeds before and after lunch?

   Hint: Let t be her time spent riding before lunch. Then $t + 1$ is her time spent riding after lunch.

   Since speed = $\frac{\text{distance}}{\text{time}}$, we have $\frac{62}{t} = \frac{69}{t+1} + 2$.

   Now, solve for t.
10. Clair takes 1 hour longer than Jenna to mow the lawn. If they can mow the lawn together in 5 hours, how long would it take each of them to mow the lawn alone?

    (Hint: Let t be the time in hours it takes Jenna to mow the lawn. Then Clair can mow the lawn in $t + 1$ hours.

    To find t, solve $\frac{5}{t} + \frac{5}{t+1} = 1$.)
11. Solve $9x^2 - 15x = 32$ by completing the square.
12. Solve $5x^2 + 7x + 13 = 0$ by completing the square.

The Quadratic Formula

13. Solve $x^2 + 10x + 25 = 0$ using the quadratic formula.
14. Solve $x^2 - 6x - 16 = 0$ using the quadratic formula.
15. Solve $2x^2 + 3x + 1 = 0$ using the quadratic formula.
16. Solve $20x^2 - 42x - 26 = 0$ using the quadratic formula.
17. Solve $x^2 + 8x + 3 = 0$ using the quadratic formula.
18. Solve $x^2 - 5x = 9$ using the quadratic formula.
19. Solve $3x^2 - 15x = 20$ using the quadratic formula.
20. For each quadratic equation in the list below, calculate the discriminant, then write the letter of the statement that best describes its solutions.

    a. Two unequal real solutions

    b. Two equal real solutions

    c. No real solutions

Equation	**Discriminant**	**Solutions**
$x^2 - 2x + 8 = 0$	________	____________
$x^2 - 5x - 16 = 0$	________	____________
$49x^2 + 70x + 25 = 0$	________	____________
$4x^2 - 6x - 9 = 0$	________	____________
$-2x^2 - 7x + 10 = 0$	________	____________

21. Pediatricians use formulas to convert adult dosages for medication to child dosages. Most pediatricians use formulas based on the child's weight. However, some use one of the formulas below, where a is the age of the child and d is the adult dosage.

child's dosage $= \frac{a}{a + 12}d$ child's dosage $= \frac{a + 1}{24}d$

At approximately what age(s) do these two formulas yield the same child dosage?

(Hint: Begin by setting the expressions equal to each other: $\frac{a}{a + 12}d = \frac{a + 1}{24}d$

Then divide by d: $\frac{a}{a + 12} = \frac{a + 1}{24}$

Now find the LCD of the denominators, multiply by the LCD, and solve for a.)

22. Joe has a rectangular deck in his backyard. Its length measures 1 foot more than its width. He is planning to extend the length of the deck by 3 additional feet. If the new deck would have an area of 165 square feet, what is the width of the deck?

23. Solve $4x(x + 1) - 9 = 6x^2 - x - 18$ using the quadratic formula.

24. Solve $x^2 + 5 = 0$ using the quadratic formula.

Explore

25. The solutions of a quadratic equation are $-\frac{4}{3}$ and 5. What is an equation with these solutions?

26. The solutions of a quadratic equation are 3 and –2. Use their sum and product to find the equation.

27. The quadratic equation $3x^2 + 4x + c = 0$ has a discriminant of 196. What is the value of c? What are the solutions of the equation?

28. The solutions of a quadratic equation are $\frac{5 \pm 3\sqrt{13}}{2}$. What is an equation with these solutions?

29. The solutions of a quadratic equation are $\frac{5}{6}$ and $-\frac{2}{3}$. Use their sum and product to find an equation with these solutions.

30. The quadratic equation $ax^2 - 5x + 2 = 0$ has a discriminant of 17. What is the value of a? What are the solutions of the equation?

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Completing the Square

1. Solve $x^2 - 6x = 27$ by completing the square.
2. Solve $x^2 + 2x = 15$ by completing the square.
3. Solve $x^2 - 4x = 45$ by completing the square.
4. Solve $x^2 + 10x = 56$ by completing the square.
5. Solve $x^2 + 8x = 20$ by completing the square.
6. Solve $x^2 + 12x = -11$ by completing the square.
7. Solve $x^2 - 8x = -7$ by completing the square.
8. Solve $x^2 + 6x = -5$ by completing the square.
9. Solve $x^2 + 6x = 12$ by completing the square.
10. Solve $x^2 - 16x = 13$ by completing the square.
11. Solve $x^2 + 4x = 21$ by completing the square.
12. Solve $x^2 + 12x = -14$ by completing the square.
13. Solve $x^2 - 18x = -57$ by completing the square.
14. Solve $x^2 - 8x = -5$ by completing the square.
15. Solve $x^2 + 3x = 16$ by completing the square.
16. Solve $x^2 + 9x = 15$ by completing the square.
17. Solve $x^2 + 7x = 9$ by completing the square.
18. Solve $x^2 - 7x = 3$ by completing the square.
19. Solve $x^2 - x = 14$ by completing the square.
20. Solve $x^2 - 5x = 2$ by completing the square.
21. Solve $4x^2 + 16x = 84$ by completing the square.
22. Solve $2x^2 + 16x = 40$ by completing the square.
23. Solve $3x^2 + 12x = 36$ by completing the square.
24. Solve $5x^2 - 30x = 200$ by completing the square.
25. Solve $3x^2 - 30x = 432$ by completing the square.
26. Solve $2x^2 - 2x = 112$ by completing the square.
27. Solve $3x^2 - 5x = 7$ by completing the square.
28. Solve $4x^2 + 3x = 6$ by completing the square.

The Quadratic Formula

29. Solve $4x^2 - 5x + 1 = 0$ using the quadratic formula.
30. Solve $3x^2 - 5x - 2 = 0$ using the quadratic formula.
31. Solve $x^2 - 4x + 1 = 0$ using the quadratic formula.
32. Solve $x^2 + 8x - 5 = 0$ using the quadratic formula.
33. Solve $x^2 - 6x + 4 = 0$ using the quadratic formula.
34. Solve $2x^2 - 7x + 2 = 0$ using the quadratic formula.
35. Solve $5x^2 + 3x - 4 = 0$ using the quadratic formula.
36. Solve $3x^2 + 7x - 7 = 0$ using the quadratic formula.
37. Solve $x^2 - 2x = 7$ using the quadratic formula.
38. Solve $x^2 + 8x = 5$ using the quadratic formula.
39. Solve $x^2 + 3x = 8$ using the quadratic formula.
40. Solve $4x^2 + 19x = -17$ using the quadratic formula.
41. Solve $5x^2 - 46x = -48$ using the quadratic formula.
42. Solve $3x^2 - 25x = -28$ using the quadratic formula.
43. Solve $x^2 = 3x + 7$ using the quadratic formula.
44. Solve $x^2 = -3x + 5$ using the quadratic formula.
45. Solve $x^2 = x + 1$ using the quadratic formula.
46. Solve $2x^2 = 5x - 3$ using the quadratic formula.

47. Solve $4x^2 = 9x - 3$ using the quadratic formula.

48. Solve $3x^2 = -2x + 7$ using the quadratic formula.

49. Calculate the discriminant of the quadratic equation $5x^2 + 8x - 9 = 0$ and determine the nature of the solutions of the equation.

50. Calculate the discriminant of the quadratic equation $3x^2 + 2x + 5 = 0$ and determine the nature of the solutions of the equation.

51. Calculate the discriminant of the quadratic equation $4x^2 - 12x + 9 = 0$ and determine the nature of the solutions of the equation.

52. Calculate the discriminant of the quadratic equation $x^2 + 7x - 6 = 0$ and determine the nature of the solutions of the equation.

53. Calculate the discriminant of the quadratic equation $9x^2 + 30x + 25 = 0$ and determine the nature of the solutions of the equation.

54. Calculate the discriminant of the quadratic equation $x^2 + 3x + 4 = 0$ and determine the nature of the solutions of the equation.

55. Calculate the discriminant of the quadratic equation $7x^2 - 6x + 5 = 0$ and determine the nature of the solutions of the equation.

56. Calculate the discriminant of the quadratic equation $4x^2 + 8x - 5 = 0$ and determine the nature of the solutions of the equation.

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Complete the square for this expression.

   $x^2 + 9x + ?$

   What is the perfect square?

2. Solve $4x^2 + 8x = 152$ by completing the square.

3. After completing the square by adding 16 to both sides, the result is $(x + 4)^2 = 2$. What was the original equation?

4. Solve $4x^2 - 5x + 1 = 0$ by completing the square.

5. Circle the equation below that has the solution $x = \frac{-2 \pm 3\sqrt{2}}{2}$.

   $x^2 + 4x - 7 = 0$

   $2x^2 + 4x - 7 = 0$

   $2x^2 + 4x + 7 = 0$

   $x^2 - 4x - 7 = 0$

6. Use the quadratic formula to solve this quadratic equation:
   $6x = 1 - 5x^2$

7. Circle the quadratic equations below that have no real solutions.

   $x^2 + 5x - 9 = 0$

   $x^2 + 4x + 11 = 0$

   $x^2 - x + 1 = 0$

   $4x^2 + 5x + 1 = 0$

   $2x^2 - 10x - 3 = 0$

   $x^2 + 2x + 5 = 0$

8. Find the two values for b for which the quadratic equation $9x^2 + bx + 36 = 0$ has two equal real solutions.

9. The quadratic equation $x^2 - 7x + c = 0$ has a discriminant of 45. What is the value of c? What are the solutions of the equation?

10. The sum of the solutions of a quadratic equation is $\frac{3}{2}$. The product of its solutions is 3. What is the equation?

11. Find a quadratic equation whose two solutions are –3 and $\frac{1}{5}$.

12. Find the greatest possible value of c in the quadratic equation $2x^2 - 7x + c = 0$ for which there are two real solutions.