LESSON 9.2 – RATIONAL EXPONENTS

OVERVIEW

Here's what you'll learn in this lesson:

Roots and Exponents

a. The n^{th} root of a number

b. Definition of $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$

c. Properties of rational exponents

Simplifying Radicals

a. Simplifying radicals

Operations on Radicals

a. Adding and subtracting radical expressions

b. Multiplying radical expressions

c. Dividing radical expressions

A farmer is experimenting with different fertilizers and varieties of corn in order to find ways to boost crop production. A researcher is studying the effects of pollutants and disease on fish populations around the world. A student volunteer is analyzing surveys to help increase donation levels for his organization.

Each of these people—Emerson Sarawop, the farmer; Sharon Ming, the researcher; and Vince Poloncic, the student volunteer—does work for the Center for World Hunger. And, each works every day with equations that involve radicals.

In this lesson, you will learn about radicals. You will learn how to simplify expressions that contain radicals. You will also learn how to add, subtract, multiply, and divide such expressions.

EXPLAIN

ROOTS AND EXPONENTS

Summary

Square Roots

When you square the square root of a number, you get back the original number.

For example:

$$\left(\sqrt{16}\right)^2 = 16$$

The square root of 16 is written as $\sqrt{16}$.

Every positive integer has both a positive and a negative square root. The symbol $\sqrt{a}$ denotes the positive square root of *a*. The symbol $-\sqrt{a}$ denotes the negative square root of *a*.

For example:

$\sqrt{25} = 5$ $\quad -\sqrt{25} = -5$

$\sqrt{16} = 4$ $\quad -\sqrt{16} = -4$

$\sqrt{81} = 9$ $\quad -\sqrt{81} = -9$

The positive square root is called the principal square root.

You can't take the square root of a negative number and get a real number because no real number times itself equals a negative number.

Cube Roots

When you cube the cube root of a number, you get back the original number.

For example:

$$\left(\sqrt[3]{64}\right)^3 = 64$$

The cube root of 64 is written as $\sqrt[3]{64}$.

Both positive and negative numbers have real cube roots.

For example:

$\sqrt[3]{125} = 5$ because $5 \cdot 5 \cdot 5 = 125$

$\sqrt[3]{-125} = -5$ because $(-5) \cdot (-5) \cdot (-5) = -125$

*n*th Roots

Numbers also have 4th roots, 5th roots, 6th roots, and so on.

For example:

$\sqrt[4]{81} = 3$	because $3 \cdot 3 \cdot 3 \cdot 3 = 81$
$\sqrt[5]{-1} = -1$	because $(-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) = -1$
$\sqrt[6]{64} = 2$	because $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$

When you raise the *n*th root of a number to the *n*th power, you get back the original number.

For example:

$$\left(\sqrt[n]{10}\right)^n = 10$$

index ↓ $\sqrt[n]{a}$ ← *radicand*

In general, the *n*th root of a number *a* is written

$$\sqrt[n]{a}$$

where *n* is a positive integer.

When there isn't any number written as the index, it is understood to be 2. So $\sqrt{a}$ is the same as $\sqrt[2]{a}$.

Here, *a* is called the radicand and *n* is called the index. The index is the number of times that the root has to be multiplied in order to get the radicand.

When finding real roots:

- If *n* is odd, then $\sqrt[n]{a}$ is a real number.
- If *n* is even, then $\sqrt[n]{a}$ is a real number if $a \geq 0$.

For example:

If the index is even, then the radicand must be positive in order to get a real number. This is because there is no real number that, when multiplied by itself an even number of times, gives a negative number.

$\sqrt[3]{8} = 2$	because $2^3 = 2 \cdot 2 \cdot 2 = 8$
$\sqrt[3]{-8} = -2$	because $(-2)^3 = (-2) \cdot (-2) \cdot (-2) = -8$
$\sqrt{36} = 6$	because $6^2 = 6 \cdot 6 = 36$

$\sqrt{-36}$ is not a real number

Rational Exponents

All roots can also be written as rational, or fractional, exponents.

In general:

You may find it easier to solve problems if you first rewrite the exponent with a radical sign. For example,

$$16^{\frac{1}{2}} = \sqrt{16} = 4$$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

For example:

$$\sqrt[3]{8} = 8^{\frac{1}{3}}$$

Since you can rewrite rational exponents as roots, the same rules that apply to roots also apply to rational exponents:

- If n is odd, then $a^{\frac{1}{n}}$ is a real number.
- If n is even, then $a^{\frac{1}{n}}$ is a real number when $a \geq 0$.

If the numerator of the rational exponent is not equal to 1, you can still rewrite the problem using radicals.

In general:

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Notice that you get the same answer whether you first take the root of the number and then raise it to the appropriate power, or whether you first raise the radicand to the appropriate power and then take the root.

To simplify an expression when the rational exponent is not equal to 1:

1. Rewrite the problem using radicals.
2. Take the appropriate root.
3. Raise the result to the correct power.

For example, to find $32^{\frac{2}{5}}$:

1. Rewrite the problem using radicals. $= \left(\sqrt[5]{32}\right)^2$
2. Take the 5th root. $= 2^2$
3. Simplify. $= 4$

When dealing with large numbers, you may find it easier to first take the root of the number and then raise it to the correct power.

Always reduce a rational exponent to lowest terms or you may get the wrong answer.

For example:

$(-16)^{\frac{1}{2}} = \sqrt{-16}$, which is not a real number

$(-16)^{\frac{2}{4}} \neq \sqrt[4]{(-16)^2} \neq \sqrt[4]{256} \neq 4$

Since $\frac{2}{4}$ is not reduced to lowest terms, the answer, 4, is incorrect.

The basic properties for integer exponents also hold for rational exponents as long as the expressions represent real numbers.

Property of Exponents	Integer Exponents	Rational Exponents
Multiplication	$7^3 \cdot 7^5 = 7^{3+5} = 7^8$	$7^{\frac{1}{2}} \cdot 7^{\frac{1}{4}} = 7^{\frac{1}{2}+\frac{1}{4}} = 7^{\frac{3}{4}}$
Division	$\frac{3^2}{3^6} = 3^{2-6} = 3^{-4}$	$\frac{3^{\frac{1}{2}}}{3^{\frac{1}{4}}} = 3^{\frac{1}{2}-\frac{1}{4}} = 3^{\frac{1}{4}}$
Power of a Power	$(2^3)^4 = 2^{3 \cdot 4} = 2^{12}$	$(5^{\frac{1}{2}})^{\frac{1}{3}} = 5^{\frac{1}{2} \cdot \frac{1}{3}} = 5^{\frac{1}{6}}$
Power of a Product	$(5 \cdot 7)^2 = 5^2 \cdot 7^2$	$(5 \cdot 7)^{\frac{2}{9}} = 5^{\frac{2}{9}} \cdot 7^{\frac{2}{9}}$
Power of a Quotient	$\left(\frac{3}{8}\right)^4 = \frac{3^4}{8^4}$	$\left(\frac{3}{8}\right)^{\frac{1}{4}} = \frac{3^{\frac{1}{4}}}{8^{\frac{1}{4}}}$

The properties of exponents can help you simplify some expressions.

For example, to simplify $(8 \cdot 27)^{\frac{1}{3}}$:

1. Apply the power of a product property. $= (8)^{\frac{1}{3}} \cdot (27)^{\frac{1}{3}}$
2. Rewrite the problem using radicals. $= \sqrt[3]{8} \cdot \sqrt[3]{27}$
3. Take the cube roots. $= 2 \cdot 3$
4. Simplify. $= 6$

Sample Problems

Answers to Sample Problems

1. Find: $\sqrt[3]{-1331}$

   □ a. Simplify $\sqrt[3]{-1331}$. ________

   a. –11

2. Rewrite as a radical and evaluate: $\left(\frac{81}{100}\right)^{\frac{2}{4}}$

   ✔ a. Reduce the exponent to lowest terms. $= \left(\frac{81}{100}\right)^{\frac{1}{2}}$

   ✔ b. Apply the power of a quotient property of exponents. $= \frac{81^{\frac{1}{2}}}{100^{\frac{1}{2}}}$

   □ c. Rewrite as radicals. = ________

   c. $\frac{\sqrt{81}}{}$

   □ d. Take the square root of the numerator and the denominator. = ________

   d. $\frac{9}{10}$

3. Evaluate: $(8 \cdot 125)^{\frac{1}{3}}$

   ✔ a. Raise each term to the $\frac{1}{3}$ power. $8^{\frac{1}{3}} \cdot 125^{\frac{1}{3}}$

   ✔ b. Express exponents as radicals. $= \sqrt[3]{8} \cdot \sqrt[3]{125}$

   ✔ c. Simplify the radicals. $= 2 \cdot 5$

   □ d. Simplify. = ______

   d. 10

SIMPLIFYING RADICALS

Summary

Equations often contain radical expressions. In order to simplify these expressions, you have to know how to simplify radicals.

The Multiplication Property of Radicals

The rule for multiplying square roots is:

The square root of a product = the product of the square roots.

For example:

$$\sqrt{144 \cdot 121}$$

$$= \sqrt{144} \cdot \sqrt{121}$$

$$= 12 \cdot 11$$

$$= 132$$

In general:

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Here, a and b are real numbers, $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and n is a positive integer.

Division Property of Radicals

The rule for dividing square roots is:

The square root of a quotient = the quotient of the square roots.

For example:

$$\sqrt{\frac{16}{169}}$$

$$= \frac{\sqrt{16}}{\sqrt{169}}$$

$$= \frac{4}{13}$$

In general:

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Here, a and b are real numbers, $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and n is a positive integer.

Sums and Differences of Roots

The *n*th root of a sum is not equal to the sum of the *n*th roots.

For example:

Is $\sqrt{9+16} = \sqrt{9} + \sqrt{16}$?

Is $\sqrt{25} = 3 + 4$?

Is $5 = 7$? No.

Similarly, the *n*th root of a difference is not equal to the difference of the *n*th roots.

The Relationship Between Powers and Roots

If you start with a number, cube it, then take its cube root, you end up with the same number that you started with.

For example,

$$\sqrt[3]{8^3} = 8$$

However, if you start with a number, square it, then take its square root, you only get back the original number if the original number is greater than or equal to 0. If the original number is less than 0, taking the root will give you (–1) times the number.

For example:

$$\sqrt{9^2} = 9$$

$$\sqrt{(-9)^2} = -(-9) = (9)$$

When taking roots:

- If the radicand is positive, $\sqrt[n]{a^n} = a$
- If a is negative and n is odd, $\sqrt[n]{a^n} = a$
- If a is negative and n is even, $\sqrt[n]{a^n} = -a$

Simplifying Radicals

A radical expression is in simplest terms if it meets the following conditions:

- In the expression $\sqrt[n]{a}$, the radicand, a, contains no factors that are perfect *n*th powers.
- There are no fractions under the radical sign.
- There are no radicals in the denominator of the expression.

To simplify a radical expression that contains factors which are powers of the index, n:

1. Write the radicand as a product of its prime factors.
2. Rewrite the factors using exponents.
3. Where possible, rewrite factors as a product having the index, n, as an exponent.
4. Bring all possible factors outside the radical.
5. Simplify.

For example, to simplify $\sqrt[3]{80}$:

1. Write 80 as a product of its prime factors. $= \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}$
2. Rewrite the factors using exponents. $= \sqrt[3]{2^4 \cdot 5}$
3. Rewrite 2^4 as a product including 2^3. $= \sqrt[3]{2^3 \cdot 2 \cdot 5}$
4. Bring $\sqrt[3]{2^3}$ outside the radical. $= 2\sqrt[3]{2 \cdot 5}$
5. Simplify. $= 2\sqrt[3]{10}$

To simplify a radical expression that has a fraction under the radical sign:

1. Rewrite the fraction with two radical signs—one in the numerator and one in the denominator.
2. Multiply the numerator and denominator of the fraction by the same number to eliminate the radical in the denominator of the fraction.
3. Simplify.

When you multiply the numerator and denominator of a fraction by the same number, it is the same as multiplying the expression by 1, so the value of the rational expression doesn't change.

For example, to simplify $\sqrt{\frac{2}{3}}$:

1. Rewrite the fraction with two radical signs. $= \frac{\sqrt{2}}{\sqrt{3}}$
2. Multiply the numerator and denominator by $\sqrt{3}$. $= \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$
3. Simplify. $= \frac{\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$

   $= \frac{\sqrt{6}}{3}$

To simplify a radical expression that has a radical in the denominator:

1. Multiply the numerator and denominator of the fraction by the same number to eliminate the radical in the denominator of the fraction.
2. Simplify.

For example, to simplify $\frac{7}{\sqrt[3]{5}}$:

Why do you multiply $\sqrt[3]{5}$ by $\sqrt[3]{5^2}$? Because this gives $\sqrt[3]{5^3}$, which equals 5.

1. Multiply the numerator and denominator by $\sqrt[3]{5^2}$. $\quad = \frac{7}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5^2}}{\ }$

2. Simplify. $\quad = \frac{7\sqrt[3]{5^2}}{\ }$

$= \frac{7\sqrt[3]{5^2}}{\ }$

$= 7\sqrt[3]{25}$

When simplifying radicals, it is helpful to recognize some perfect squares and perfect cubes. You may want to remember the numbers in this table:

Number (n)	Square (n^2)	Cube (n^3)
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000

Sample Problems

Answers to Sample Problems

1. Simplify: $\sqrt{\frac{49}{64}}$

✔ a. Rewrite the fraction using two radical signs. $\quad = \frac{\sqrt{49}}{\sqrt{64}}$

☐ b. Simplify the square roots. $\quad =$ ________

b. $\frac{7}{8}$

2. Simplify: $\sqrt[3]{-125x^6y^5}$

✔ a. Rewrite the radicand as a product of its prime factors. $\quad = \sqrt{(-5)(-5)(-5)x^6y^5}$

✔ b. Rewrite the factors using cubes, where possible. $\quad = \sqrt[3]{(-5)^3(x^2)^3y^3y^2}$

☐ c. Bring all perfect cubes outside the radical. $\quad =$ ____________

c. $-5x^2y\sqrt[3]{y^2}$

OPERATIONS ON RADICALS

Summary

Identifying Like Radical Terms

To add or subtract radical expressions or to eliminate a radical sign in the denominator of a fraction, you will need to identify similar, or like, radical terms.
Similar, or like, radical terms have the same index and the same radicand.

For example, here are two terms that are like terms:

$\sqrt[3]{7}$	index: 3; radicand: 7
$4\sqrt[3]{7}$	index: 3; radicand: 7

Here are two terms that are not like terms:

$2\sqrt[4]{5}$	index: 4; radicand: 5
$\sqrt{5}$	index: 2; radicand: 5

Here are two more terms that are not like terms:

$3\sqrt[5]{9}$	index: 5; radicand: 9
$6\sqrt[5]{8}$	index: 5; radicand: 8

Adding and Subtracting Radical Expressions

Now that you can identify like terms you can add and subtract radical expressions.

For example, to find $5\sqrt[3]{54} + 8\sqrt[3]{2} - 5\sqrt[3]{250}$:

1. Factor the radicands into their prime factors. $= 5\sqrt[3]{2 \cdot 3 \cdot 3 \cdot 3} + 8\sqrt[3]{2} - 5\sqrt[3]{2 \cdot 5 \cdot 5 \cdot 5}$
2. Rewrite the factors using cubes, where possible. $= 5\sqrt[3]{2 \cdot 3^3} + 8\sqrt[3]{2} - 5\sqrt[3]{2 \cdot 5^3}$
3. "Undo" the perfect cubes. $= \left(3 \cdot 5\sqrt[3]{2}\right) + 8\sqrt[3]{2} - \left(5 \cdot 5\sqrt[3]{2}\right)$
4. Simplify. $= 15\sqrt[3]{2} + 8\sqrt[3]{2} - 25\sqrt[3]{2}$
5. Combine like terms. $= -2\sqrt[3]{2}$

All the radicals in step (3) are like terms because they have the same index, 3, and the same radicand, 2.

Multiplying Radical Expressions

You can use the multiplication property of radicals to simplify complex radical expressions.

For example, to simplify $3\sqrt[4]{8} \cdot 5\sqrt[4]{4}$:

1. Apply the multiplication property of radicals. $= 3 \cdot 5\sqrt[4]{8 \cdot 4}$

2. Factor the radicands into their prime factors. $= 3 \cdot 5\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

3. Write the 2's as a product involving a factor of 2^4. $= 15\sqrt[4]{2^4 \cdot 2}$

4. Bring $\sqrt[4]{2^4}$ outside the radical. $= 15 \cdot 2\sqrt[4]{2}$

5. Simplify. $= 30\sqrt[4]{2}$

Sometimes when you multiply polynomials, you use the distributive property. This property is also useful when you multiply radicals.

For example, to simplify $\sqrt{8}\left(4\sqrt{2} + 3\sqrt{24}\right)$:

1. Apply the distributive property. $= \sqrt{8}(4\sqrt{2}) + \sqrt{8}\left(3\sqrt{24}\right)$

2. Apply the multiplication property of radicals. $= 4\sqrt{8 \cdot 2} + 3\sqrt{8 \cdot 24}$

3. Simplify.

$$= 4\sqrt{16} + 3\sqrt{192}$$
$$= 4\sqrt{4 \cdot 4} + 3\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$$
$$= 4\sqrt{4^2} + 3\sqrt{(2^3)^2 \cdot 3}$$
$$= 4\sqrt{4^2} + 3\sqrt{8^2 \cdot 3}$$
$$= (4 \cdot 4) + \left(3 \cdot 8\sqrt{3}\right)$$
$$= 16 + 24\sqrt{3}$$

Remember FOIL?

$(x + 1)(x + 2)$

$= (x \cdot x) + (x \cdot 2) + (1 \cdot x) + (1 \cdot 2)$

$= x^2 + 2x + x + 2$

$= x^2 + 3x + 2$

You can also use the FOIL method to multiply radicals.

For example, to find $(\sqrt{x} + 2)(3\sqrt{x} - 7)$:

1. Find the sum of the products of the first terms, the outer terms, the inner terms, and the last terms. $= \left(\sqrt{x} \cdot 3\sqrt{x}\right) - \left(\sqrt{x} \cdot 7\right) + \left(2 \cdot 3\sqrt{x}\right) - (2 \cdot 7)$

2. Simplify.

$$= 3x - 7\sqrt{x} + 6\sqrt{x} - 14$$
$$= 3x - \sqrt{x} - 14$$

Conjugates

Sometimes when you multiply two irrational numbers you end up with a rational number.

For example, to find $\left(2 + \sqrt{7}\right)\left(2 - \sqrt{7}\right)$:

1. Find the sum of the products of the first terms, the outer terms, the inner terms, and the last terms. $= (2)^2 - 2\sqrt{7} + 2\sqrt{7} - \left(\sqrt{7}\right)^2$

$= 4 - 7$

$= -3$

The expressions $2 + \sqrt{7}$ and $2 - \sqrt{7}$ are called conjugates of each other. When conjugates are multiplied, the result is a rational number.

In general, these expressions are conjugates of one another:

$\left(a + \sqrt{b}\right)$ and $\left(a - \sqrt{b}\right)$

$\left(\sqrt{a} + \sqrt{b}\right)$ and $\left(\sqrt{a} - \sqrt{b}\right)$

Here, $\sqrt{a}$ and $\sqrt{b}$ are real numbers.

When you multiply conjugates, here's what happens:

$$\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right)$$
$$= \left(\sqrt{a} \cdot \sqrt{a}\right) - \left(\sqrt{a} \cdot \sqrt{b}\right) + \left(\sqrt{b} \cdot \sqrt{a}\right) - \left(\sqrt{b} \cdot \sqrt{b}\right)$$
$$= \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2$$
$$= a - b$$

As another example, to find $\left(\sqrt{6} + \sqrt{14}\right)\left(\sqrt{6} - \sqrt{14}\right)$:

1. Find the sum of the products of the first, outer, inner, and last terms. $= \left(\sqrt{6}\right)^2 - \left(\sqrt{14}\right)^2$
2. Simplify. $= 6 - 14$

   $= -8$

Dividing Radical Expressions

You can use the division property of radicals to simplify radical expressions.

For example, to simplify $\sqrt{\frac{7}{2y}}$:

Since the expression in the denominator is a square root, to eliminate it you must multiply it by itself one time (so there are a total of two factors of 2y under the square root sign). If the expression in the denominator had been $\sqrt[3]{2y}$, to eliminate it you would have had to multiply it by itself two times (so there would be a total of three factors of 2y under the cube root sign). In general, you need n factors to clear an nth root.

The process of eliminating a root in the denominator is called rationalizing the denominator.

1. Apply the division property of radicals. $= \frac{\sqrt{7}}{\sqrt{2y}}$
2. Rationalize the denominator. $= \frac{\sqrt{7}}{\quad} \cdot \frac{\sqrt{2y}}{\quad}$
3. Perform the multiplication. $= \frac{\sqrt{7 \cdot 2y}}{\quad}$
4. Simplify. $= \frac{\sqrt{14y}}{\quad}$

As another example, to simplify $\sqrt[4]{\frac{2x}{5}}$:

1. Apply the division property of radicals. $= \sqrt[4]{2x}$
2. Rationalize the denominator. $= \frac{\sqrt[4]{2x}}{} \cdot \frac{\sqrt[4]{5^3}}{}$
3. Perform the multiplication. $= \frac{\sqrt[4]{5^3 \cdot 2x}}{\sqrt[4]{5^4}}$
4. Simplify. $= \frac{\sqrt[4]{125 \cdot 2x}}{5}$

   $= \frac{\sqrt[4]{250x}}{}$

When there is a sum or difference involving roots in the denominator of a radical expression, you can often simplify the expression by multiplying the numerator and denominator by the conjugate of the denominator.

For example, to simplify $\frac{3}{\sqrt{x}+5}$:

1. Multiply the numerator and denominator by the conjugate of $\left(\sqrt{x}+5\right)$, $\left(\sqrt{x}-5\right)$. $= \frac{3}{\sqrt{x}+5} \cdot \frac{(\sqrt{x}-5)}{}$

   $= \frac{3(\sqrt{x}-5)}{}$
2. Simplify. $= \frac{3 \cdot \sqrt{x} - 3 \cdot 5}{\left(\sqrt{x}\right)^2 - (5)^2}$

   $= \frac{3\sqrt{x}-15}{}$

Sample Problems

1. Find: $5\sqrt{49} + 6\sqrt{3} + 15\sqrt{3} + 2\sqrt{48}$

   - ☑ a. Factor the radicals into their prime factors. $= 5\sqrt{7 \cdot 7} + 6\sqrt{3} + 15\sqrt{3} + 2\sqrt{3 \cdot 4 \cdot 4}$
   - ☑ b. Where possible, rewrite factors as perfect squares. $= 5\sqrt{7^2} + 6\sqrt{3} + 15\sqrt{3} + 2\sqrt{3 \cdot 4^2}$
   - ☐ c. Take perfect squares out from under the radical signs. = ______________________________
   - ☐ d. Simplify. = ______________________________
   - ☐ e. Combine like terms. = ______________________________

2. Find: $(\sqrt{x} + 5)(6\sqrt{x} - 8)$

   - ☐ a. Find the sum of the products of the first terms, the outer terms, the inner terms, and the last terms. $= 6x - 8\sqrt{x} +$ ______ $-$ ______
   - ☐ b. Combine like terms. = ______________________

3. Simplify: $\frac{5}{\sqrt[3]{x} \cdot \sqrt{y}}$

   - ☑ a. Multiply the numerator and denominator by $\left(\sqrt[3]{x^2}\right) \cdot \left(\sqrt{y}\right)$. $= \frac{5}{\sqrt[3]{x} \cdot \sqrt{y}} \cdot \frac{(\sqrt[3]{x^2}) \cdot (\sqrt{y})}{}$
   - ☐ b. Simplify the radicals. = ______________________

Answers to Sample Problems

c. $5 \cdot 7 + 6\sqrt{3} + 15\sqrt{3} + 2 \cdot 4\sqrt{3}$

d. $35 + 6\sqrt{3} + 15\sqrt{3} + 8\sqrt{3}$

e. $35 + 29\sqrt{3}$

a. $30\sqrt{x}$, 40

b. $6x + 22\sqrt{x} - 40$

b. $\frac{5(\sqrt[3]{x^2}) \cdot (\sqrt{y})}{}$

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Roots and Exponents

1. Rewrite using a radical, then evaluate: $8^{\frac{1}{3}}$
2. Evaluate: $\sqrt[3]{-216}$
3. Evaluate: $2^{\frac{1}{4}} \cdot 2^{\frac{2}{5}}$
4. Rewrite using a radical, then evaluate: $4^{\frac{3}{2}}$
5. Evaluate: $\sqrt[5]{1024}$
6. Simplify the expression below. Write your answer using only positive exponents.

   $\left(x^{\frac{1}{3}} \cdot y^{\frac{1}{2}}\right)^{-2}$
7. Evaluate: $\sqrt[4]{-81}$
8. Simplify: $x^{\frac{4}{3}} \cdot x^{\frac{1}{3}}$
9. The number of cells of one type of bacteria doubles every 5 hours according to the formula $n_f = n_i \cdot 2^{\frac{t}{5}}$ where n_f is the final number of cells, n_i is the initial number of cells, and t is the initial number of hours since the growth began. If a biologist starts with a single cell of the bacteria, how many cells will she have after 50 hours?
10. Alan invests $100 in a savings account. How much money would he have after a year if the interest rate for this account is 3% compounded every 4 months?

    The amount A in a savings account can be expressed as

    $A = P\left(1 + \frac{r}{n}\right)^{nt}$

    where P is the amount of money initially invested, t is the number of years the money has been invested, r is the annual rate of interest, and n is the number of times the interest is compounded each year.
11. Evaluate the expression below. Express your answer using only positive exponents.

    $\left(x^{\frac{5}{7}} \cdot x^{-\frac{3}{4}} \cdot y^{\frac{5}{4}}\right)^{-4}$
12. Evaluate the expression below. Express your answer using only positive exponents.

    $\left(\sqrt[3]{-1331}\right)\left(x^{\frac{2}{9}}\right)^{-3}\left(\frac{1}{x^{-\frac{1}{3}}}\right)$

Simplifying Radicals

Simplify the expressions in problems (13)–(20). Assume x, y, and z are positive numbers.

13. $\sqrt[3]{\frac{54}{250}}$
14. $\sqrt{\frac{49x}{4}}$
15. $\frac{\sqrt{5x^3}}{}$
16. $\sqrt{\frac{-1048}{75}}$
17. $\sqrt[3]{-27x^6y^3}$
18. $\frac{\sqrt{x}}{\sqrt{y^4z^{12}}}$
19. $\sqrt[3]{16x^9y^4z^2}$
20. $\sqrt{\frac{242xy^{12}}{288y^2z^4}}$
21. One of the three unsolved problems of antiquity was to "double a cube"—that is, to construct a cube with twice the volume of a given cube. What would be the length of a side of a cube with twice the volume of 1 m^3? (Hint: The volume, V, of a cube with sides of length L is $V = L \cdot L \cdot L = L^3$.)

22. In a cube with surface area, A, the length, s, of each side is given by this formula:

$$s = \sqrt{\frac{A}{6}}$$

The volume, V, of the cube is:

$$V = s^3$$

What is the volume of a cube with a surface area of 48 ft^2?

Simplify the expressions in problems (23) and (24). Assume x, y, and z are positive numbers.

23. $\sqrt{4x^5y^7z^2}$

24. $\frac{\sqrt[3]{-8x^6y^3z^3}}{\sqrt{36x^2y^6z^2}}$

Operations on Radicals

25. Circle the like terms:

$7\sqrt[4]{360}$

$72\sqrt{360}$

$\frac{\sqrt[4]{360}}{72}$

$360\sqrt[4]{72}$

$-22\sqrt[3]{360}$

$-\frac{11}{4}\sqrt[4]{360}$

$\sqrt[4]{36}$

26. Simplify: $7\sqrt{125} + 2\sqrt{500} - \frac{3}{2}\sqrt{20} - 2\sqrt{10}$

27. Simplify: $5\sqrt{12}\left(6\sqrt{3} - 7\sqrt{27}\right)$

28. Circle the like terms:

79	$\sqrt{79}$
$\sqrt{79^3}$	$\sqrt[2]{79}$
$\frac{1}{44}\sqrt{799}$	$-\sqrt{79}$
$2\sqrt{3} \cdot \sqrt{79}$	$-\sqrt[2]{79^2}$

29. Simplify: $8\sqrt[3]{24} - \frac{\sqrt{16}}{2} + \frac{1}{4}\sqrt[3]{2} - \sqrt[3]{-3^3}$

30. Simplify: $\left(7\sqrt{2} - 8\sqrt{3}\right)\left(5\sqrt{2} + 6\sqrt{3}\right)$

31. Circle the like terms:

$\sqrt[3]{24}$

$2\sqrt[3]{3}$

$\sqrt[3]{8} \cdot \sqrt[3]{3}$

$\sqrt{(-3)^2}$

$\frac{\sqrt[3]{3}}{27}$

3^{-3}

$3^{\frac{1}{3}}$

32. Simplify: $\frac{1-\sqrt{5}}{\sqrt{5}-9}$

33. The period of a simple pendulum is given by the formula $t = 2\pi\sqrt{\frac{L}{32}}$ where t is the period of the pendulum in seconds, and L is the length of the pendulum in feet. What is the period of a 16 foot pendulum?

34. The Pythagorean Theorem, $a^2 + b^2 = c^2$, gives the relationship between the lengths of the two legs of a right triangle, a and b, and the length of the hypotenuse of the triangle, c. If the lengths of the legs of a right triangle are $\sqrt{2}$ cm and $\sqrt{6}$ cm, how long is the hypotenuse?

35. Simplify: $\frac{\sqrt[3]{22}}{\sqrt[3]{77}}$

36. Simplify: $\frac{6\sqrt[3]{2} - 2\sqrt[3]{4}\left(2\sqrt[3]{32} - 2\sqrt[3]{4}\right)}{2\sqrt[3]{2} - \sqrt{2} \cdot \sqrt[3]{2}}$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Roots and Exponents

1. Rewrite using a radical, then evaluate: $9^{\frac{5}{2}}$
2. Rewrite using a radical, then evaluate: $16^{\frac{3}{2}}$
3. Rewrite using a radical, then evaluate: $27^{\frac{2}{3}}$
4. Rewrite using a radical, then evaluate: $32^{\frac{4}{5}}$
5. Rewrite using a radical, then evaluate: $81^{\frac{3}{4}}$
6. Evaluate: $\sqrt[4]{625}$
7. Evaluate: $\sqrt[5]{7776}$
8. Evaluate: $\sqrt[5]{1024}$
9. Evaluate: $-\sqrt[4]{81}$
10. Evaluate: $\sqrt[5]{-32}$
11. Evaluate: $\sqrt[3]{-216}$
12. Rewrite using rational exponents: $\sqrt[4]{245^3}$
13. Rewrite using rational exponents: $\sqrt[5]{312^4}$
14. Rewrite using rational exponents: $\sqrt[3]{315^2}$
15. Rewrite using rational exponents: $\sqrt[7]{200^5}$
16. Rewrite using rational exponents: $\sqrt[8]{400^3}$
17. Find: $y^{\frac{2}{3}} \cdot y^{\frac{1}{4}}$
18. Find: $x^{\frac{1}{3}} \cdot z^{\frac{2}{5}}$
19. Find: $x^{\frac{1}{6}} \cdot x^{\frac{1}{5}}$
20. Find: $x^{\frac{1}{7}} \cdot x^{\frac{3}{7}} \cdot x^{\frac{2}{7}}$
21. Find: $x^{\frac{2}{9}} \cdot x^{\frac{5}{9}} \cdot x^{\frac{2}{9}}$
22. Find: $x^{\frac{3}{4}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{3}{4}}$
23. Evaluate the expression below. Express your answer using only positive exponents.

$$\left(\frac{3a^{\frac{3}{4}}}{2b^2}\right)^{-4}$$

24. Evaluate the expression below. Express your answer using only positive exponents.

$$\left(\frac{x^{-\frac{4}{5}}}{2y}\right)^{5}$$

25. Evaluate the expression below. Express your answer using only positive exponents.

$$\left(\frac{4x^{-\frac{2}{3}}}{3y}\right)^{3}$$

26. Evaluate the expression below. Express your answer using only positive exponents.

$$\left(a^{\frac{3}{7}} \cdot b^{-\frac{2}{5}}\right)^{3}$$

27. Evaluate the expression below. Express your answer using only positive exponents.

$$\left(x^{-\frac{4}{9}} \cdot y^{\frac{6}{11}}\right)^{2}$$

28. Evaluate the expression below. Express your answer using only positive exponents.

$$\left(x^{-\frac{2}{3}} \cdot y^{\frac{3}{5}} \cdot z^{-\frac{4}{7}}\right)^{3}$$

Simplifying Radicals

29. Simplify: $\sqrt{\frac{121}{64}}$
30. Simplify: $\sqrt{\frac{289}{361}}$
31. Simplify: $\sqrt{\frac{169}{576}}$
32. Simplify: $\sqrt[3]{\frac{27}{8}}$
33. Simplify: $\sqrt[3]{\frac{-64}{125}}$

34. Simplify: $\sqrt[3]{\frac{-343}{27}}$

35. Simplify: $\sqrt[5]{\frac{-32}{243}}$

36. Simplify: $\sqrt[4]{\frac{625}{1296}}$

37. Simplify: $\sqrt[6]{\frac{729}{64}}$

38. Calculate: $\sqrt{(-35^2)}$

39. Calculate: $\sqrt{(-56)^2}$

40. Calculate: $\sqrt[3]{(13^3)}$

41. Calculate: $\sqrt[5]{(-47^5)}$

42. Calculate: $\sqrt[3]{(-29)^3}$

43. Which of the radical expressions below is in simplified form?

$\underline{\sqrt{81}}$ $\quad \sqrt{\frac{25}{49}}$ $\quad \frac{6}{\sqrt{30}}$ $\quad \underline{\sqrt[4]{7}}$

44. Which of the radical expressions below is in simplified form?

$\frac{\sqrt[3]{5}}{x}$ $\quad \frac{4}{\sqrt{20}}$ $\quad \sqrt{\frac{16}{9}}$ $\quad \sqrt{49}$

45. Simplify: $\sqrt{36a^2b^6}$

46. Simplify: $\sqrt{100m^6n^4}$

47. Simplify: $\sqrt{64x^4y^6z^{10}}$

48. Simplify: $\sqrt{54a^3b^8}$

49. Simplify: $\sqrt{108m^5n^9}$

50. Simplify: $\sqrt{72x^4y^7}$

51. Simplify: $\sqrt[3]{192a^3b^5c^9}$

52. Simplify: $\sqrt[3]{-250x^4y^6z^8}$

53. Simplify: $\sqrt[5]{160m^2n^7p^{12}}$

54. Simplify: $\frac{\sqrt{49a^3b^8}}{\sqrt{7ab^7}}$

55. Simplify: $\frac{\sqrt[3]{64m^7n^5}}{\sqrt[3]{2mn^3}}$

56. Simplify: $\underline{\sqrt[4]{81x^9y^6}}$

Operations on Radicals

57. Circle the like terms:

$\frac{\sqrt{-5}}{4}$

$\sqrt{5}$

$\frac{1}{3}\sqrt{5}$

$-9\sqrt{50}$

$\underline{7\sqrt{5}}$

$6\sqrt[5]{50}$

$-\sqrt[5]{2}$

58. Circle the like terms:

$\frac{5}{2}\sqrt{3}$

$-\sqrt[3]{2}$

$\frac{\sqrt{-3}}{3}$

$-6\sqrt{30}$

$4\sqrt[3]{30}$

$\sqrt{3}$

$\frac{6\sqrt{3}}{5}$

59. Simplify: $7\sqrt{5} + \sqrt{20} - 3\sqrt{80}$

60. Simplify: $10\sqrt{2} - \sqrt{128} + 3\sqrt{32}$

61. Simplify: $8\sqrt{3} + \sqrt{12} - 4\sqrt{27}$

62. Simplify: $\sqrt{40} + 3\sqrt{10} - \sqrt{18}$

63. Simplify: $4\sqrt{50} - 5\sqrt{27} + 2\sqrt{75}$

64. Simplify: $\sqrt{20} - 2\sqrt{18} + \sqrt{8}$

65. Simplify: $\sqrt[3]{250x^2} + 3\sqrt[3]{16x^5} - 3\sqrt[3]{432x^2}$

66. Simplify: $5\sqrt[4]{32y} + \sqrt[4]{162y^5} - \sqrt[4]{1250y}$

67. Simplify: $\sqrt[3]{128x} + 2\sqrt[3]{16x^4} - \sqrt[3]{54x}$

68. Simplify: $5\sqrt{6}\left(3\sqrt{8} - 9\sqrt{21}\right)$

69. Simplify: $2\sqrt[3]{9}\left(5\sqrt[3]{3} - 7\sqrt[3]{5}\right)$

70. Simplify: $3\sqrt[4]{8}\left(4\sqrt[2]{2} + 6\sqrt[4]{3}\right)$

71. Simplify: $3\sqrt{2y}\left(7\sqrt{10y} + 4\sqrt{3}\right)$

72. Simplify: $6\sqrt{3z}\left(2\sqrt{6z} - 3\sqrt{5}\right)$

73. Simplify: $2\sqrt[3]{4z}\left(5\sqrt[3]{2z^2} - 7\sqrt[3]{11z}\right)$

74. Simplify: $\left(\sqrt{5} + \sqrt{3}\right)\left(\sqrt{6} + \sqrt{11}\right)$

75. Simplify: $\left(\sqrt{5} - \sqrt{10}\right)\left(\sqrt{2} - \sqrt{15}\right)$

76. Simplify: $\left(\sqrt{6} + \sqrt{5}\right)\left(\sqrt{3} - \sqrt{10}\right)$

77. Simplify: $\left(3\sqrt{5z} + \sqrt{6}\right)\left(3\sqrt{5z} - \sqrt{6}\right)$

78. Simplify: $\left(2\sqrt{3y} + \sqrt{7}\right)\left(2\sqrt{3y} - \sqrt{7}\right)$

79. Simplify: $\left(5\sqrt{2y} - \sqrt{3x}\right)\left(5\sqrt{2y} + \sqrt{3x}\right)$

80. Simplify: $\frac{3\sqrt{y}}{y\sqrt{6}}$

81. Simplify: $\frac{2\sqrt{x}}{x\sqrt{2}}$

82. Simplify: $\frac{3\sqrt{5}}{x + \sqrt{5}}$

83. Simplify: $\frac{5\sqrt{2}}{x - \sqrt{2}}$

84. Simplify: $\frac{x - \sqrt{3}}{x + \sqrt{3}}$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

Assume that x, y, and z are positive numbers.

1. Simplify: $\sqrt[5]{x} \cdot \sqrt{x}$

2. Rewrite the expression using rational exponents.

   $\sqrt[5]{243^3}$

3. Circle the real number(s) in the list below:

   $\sqrt{-100}$

   $\sqrt[3]{-125}$

   $\sqrt[4]{-16}$

   $\sqrt[6]{-729}$

4. Simplify: $\left(\dfrac{8y^{-\frac{1}{2}}}{7^{\frac{3}{2}}x}\right)^2$

5. Simplify: $\sqrt{\dfrac{169}{225}}$

6. Calculate: $\sqrt{(-29)^2}$

7. Which of the radical expressions below is simplified?

   $\sqrt{\dfrac{3}{16}}$

   $\dfrac{xy}{\sqrt{8}}$

   $\sqrt{25}$

   $\dfrac{\sqrt[3]{3}}{2}$

8. Simplify: $\sqrt{\dfrac{81x^2y^2}{121z}}$

9. Simplify: $6\sqrt{5x} + 3\sqrt{125x} - 3$

10. Find: $(3\sqrt{5} - 8)(3\sqrt{5} + 8)$

11. Find: $(3\sqrt{2} + 3)(2\sqrt{2} - 6)$

12. Find: $\dfrac{\sqrt{y}}{\sqrt[3]{y}}$

TOPIC 9 CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic. Or you may wish to do these problems to review for a test.

1. Find: $(x^2 + 12x)(x + 3y^2 + 1)$

2. Solve for x: $\frac{2x + 5}{2 - x} = 3$

3. Solve for y: $\frac{1}{y} - \frac{2}{3} = \frac{y}{3}$

4. Find:

   a. $(125)^{\frac{1}{3}}(16)^{\frac{3}{4}}$

   b. $(x^3y)^{\frac{1}{3}}$

   c. $\frac{a^{\frac{1}{2}}b^2a^{\frac{1}{3}}}{b^{-2}}$

5. Last year Scott earned 5% in interest on his savings account and 13% in interest on his money market account. If he had \$14,125 in the bank and earned a total of \$1706.25 in interest, how much did he have in each account?

6. Graph the line that passes through the point (0, –3) with slope 2.

7. For what values is the rational expression $\frac{x^3 - 3x + 29}{x^2 + 13x + 36}$ undefined?

8. Solve $-10 < 9x - 7 < 11$ for x.

9. Solve this system of equations:

$$y = -\frac{2}{7}x + 3$$
$$14y + 4x = 14$$

10. Factor: $2ab + 14a + 5b + 35$

11. Angela and Casey were asked to clean their classroom. Working alone, Angela could clean the room in 20 minutes. It would take Casey 25 minutes to clean the room by herself. How long would it take them to clean the room together?

12. Simplify: $\left(\frac{27}{x}\right)^{-\frac{1}{3}}\left(\sqrt{x}\right)^{\frac{4}{3}}$

13. Find the equation of the line that passes through the point (–7, 3) and has slope $\frac{5}{3}$. Write your answer in point-slope form, in slope-intercept form, and in standard form.

14. Simplify this expression: $2r^2s + 3t + 4s^2 - 5r^2s - 6s^2 + 7t$

15. Find:
    $(3a^2b^2 + 2a^2b - 7ab + a) - (a^2b^2 - 12ab + 2a^2b + b)$

16. Simplify: $\frac{2 + \sqrt{2}}{\sqrt{2} + \sqrt{6}}$

17. Find:

    a. $\frac{3^0 \cdot 10^2}{2^3}$

    b. $(-3a^2)^3$

    c. $[(x^3y^2)^2z]^4$

18. Graph the inequality $2y - 10x \leq 32$.

19. Solve for x: $3(x + 2) - x = 3x - 8$

20. Graph the line $y - \frac{1}{2} = \frac{1}{6}(x + 2)$.

21. Factor: $-16y^2 + 24y - 9$

22. Rewrite using radicals, then simplify: $\frac{\left(24^{\frac{1}{3}} + 100^{\frac{1}{2}}\right)}{16^{\frac{1}{4}}}$

23. Circle the true statements.

22 + 32 = 52

$|3 - 4| = |3| - |4|$

The GCF of 52 and 100 is 4.

$\frac{9}{25} = \frac{3}{5}$

The LCM of 30 and 36 is 180.

24. Find the slope of the line that is perpendicular to the line that passes through the points (8, 2) and (–4, 9).

25. Factor: $x^2 + 3x - 130$

26. In a bin, the ratio of red apples to green apples is 10 to 3. If there is a total of 15 green apples, how many red ones are there?

27. Find the slope and y-intercept of this line: $4y + 3x = -18$

28. Graph the system of linear inequalities below to find its solution.

$3y - 5x < 3$

$5y + 3x > -10$

29. Simplify: $\sqrt{\frac{72x^3y^2}{(2y^2)^2}}$

30. Find the slope of the line through the points $\left(\frac{3}{4}, 7\right)$ and $\left(\frac{1}{4}, -\frac{1}{2}\right)$.

31. Simplify: $\frac{8}{3 + \sqrt{11}}$

32. Graph the line $2y + 1 = 1 - 3x$.

33. Find: $(2x^3 + 21x^2 - 27x + 8) \div (2x - 1)$

34. Evaluate the expression $3a^2 + ab - b^2$ when $a = 2$ and $b = 8$.

35. Factor: $7x^2y^2 + 14xy^2 + 7y^2$

36. Factor: $5x^2 - 80$

37. Solve for y: $-10 < 5 - 3y \leq 2$

38. Rewrite using only positive exponents: $\frac{a^3b^{-2}}{(c^{-1})^{-2}}$

39. Find:

a. $\sqrt{20} + \sqrt{80}$

b. $\sqrt{6}\left(\sqrt{6x^2} + \sqrt{3x^2}\right)$

c. $\left(a + \sqrt{b}\right)\left(a - \sqrt{b}\right)$

40. Factor: $a^2 + 6a + 6b + ab$

41. A juggler has 10 more balls than juggling pins. If the number of balls is 1 more than twice the number of pins, how many pins and balls are there?

42. Find: $\frac{x^2 - 9}{x^2 + 3x} \cdot \frac{x^2 - 7x}{x^2 - 10x + 21}$

43. Simplify: $\frac{x - y}{\sqrt{x} - \sqrt{y}}$

44. Solve for y: $3\left[7y + 5(1 - 2y)\right] = -27$

45. Factor: $8x^3 - 1$

46. Solve for x: $\frac{1}{2}(x + 7) = 12$

47. Solve for a: $\frac{a}{a - 5} + 1 = \frac{a - 3}{a - 5}$

48. Evaluate the expression $a^3b + 3 - ab^3 + 2ab$ when $a = -2$ and $b = 4$.

49. Simplify: $\left(\frac{2y^3}{3x^4}\right)^2$

50. Factor: $2x^2 - 40x + 198$