LESSON 5.3 – SYSTEMS OF INEQUALITIES

OVERVIEW

Here's what you'll learn in this lesson:

Solving Linear Systems

a. Solving systems of linear inequalities by graphing

As a conscientious dog owner, you like to make sure that your best friend gets all the vitamins and minerals he needs. When given the choice between canned dog food and fresh meat, you consider two things: price and nutrition. You want to find combinations of fresh meat and canned food that you can afford to buy that will give your dog all the vitamins and minerals he needs.

The nutritional content of each kind of food and the amount of each you can afford to buy describe a system of linear inequalities which may have many solutions. To decide how much of each kind of food to buy, you can graph this system of linear inequalities, and consider all of the possibilities.

In this lesson you will learn how to solve systems of linear inequalities.

EXPLAIN

SOLVING LINEAR SYSTEMS

Summary

Solving Systems of Linear Inequalities

Just as you can use graphing to find the solution of a system of linear equations, you can also use graphing to find the solution of a system of linear inequalities.

To find the solution of a system of linear inequalities by graphing:

1. Graph the linear equation that corresponds to the first inequality. Use a dotted line.
2. Shade the region whose points satisfy the inequality.
3. Graph the linear equation that corresponds to the second inequality. Use a dotted line.
4. Shade the region whose points satisfy the inequality.
5. The solution is the points in the region where the shading overlaps.

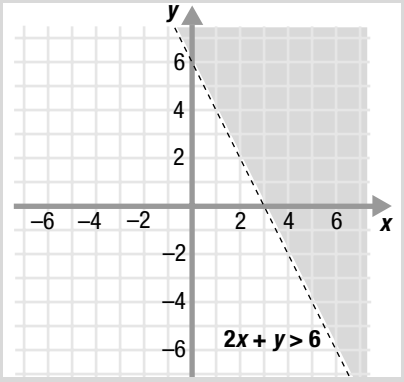
For example, to find the solution of this linear system:

$$2x + y > 6$$
$$x - y < 1$$

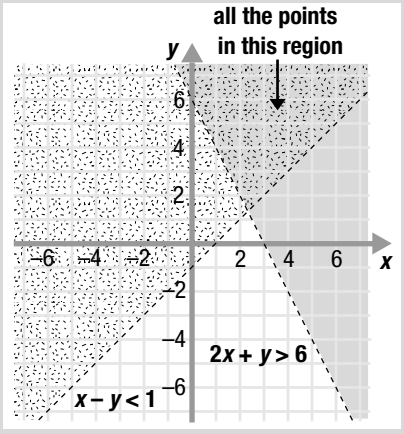
1. Graph the linear equation $2x + y = 6$.
2. Shade the region above the line.
3. Graph the linear equation $x - y = 1$.
4. Shade the region above the line.
5. The solution is the points in the region where the shading overlaps.

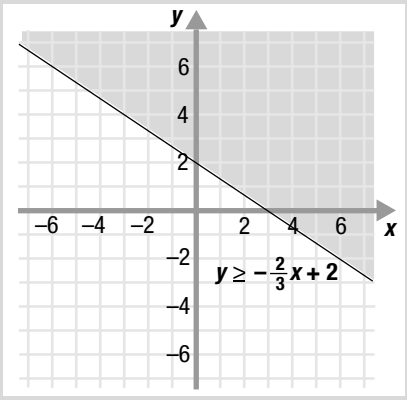
Don't forget to use a solid line if the inequality is ≤ or ≥.

Any point that makes both inequalities true is called a solution of the system.



*How do you know to shade above the line? Substitute the test point (0, 0) into the inequality 2x + y > 6. Since 2(0) + 0 > 6 is **not** true, shade on the side of the line which **doesn't** contain (0, 0).*

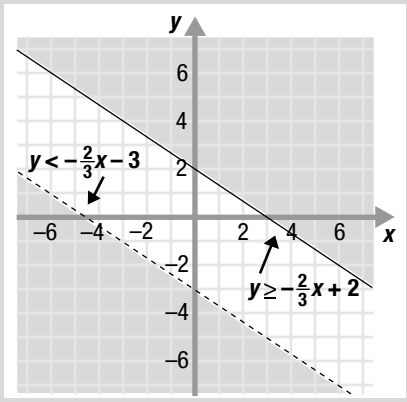




As another example, find the solution of this system:

$$y \geq -\frac{2}{3}x + 2$$
$$y < -\frac{2}{3}x - 3$$

1. Graph the linear equation $y = -\frac{2}{3}x + 2$.
2. Shade the region above and including the line.



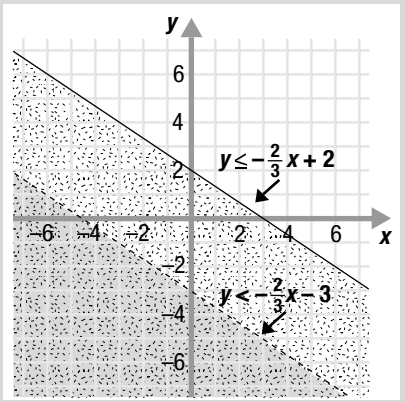
3. Graph the linear equation $y = -\frac{2}{3}x - 3$.
4. Shade the region below the line.
5. There is no solution to this system because the shaded regions do not overlap.

Like systems of linear equations, it is possible for systems of linear inequalities to have no solution only when the corresponding equations are parallel lines (as in the previous example).

However, unlike systems of linear equations, it is possible for a system of linear inequalities to have a solution even when the corresponding lines are parallel. For example, this system of two parallel linear inequalities has a solution:

$$y \leq -\frac{2}{3}x + 2$$
$$y < -\frac{2}{3}x - 3$$

The solution of this system is shown to the left.



Sample Problems

1. Graph the system below to find its solution:

$$x - 4y \geq 2$$
$$2x - y > -3$$

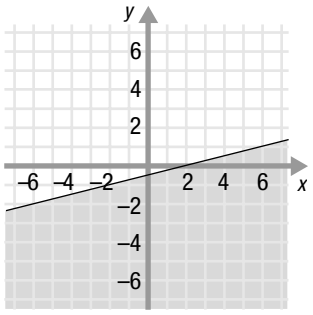
☑ a. Graph the line $x - 4y = 2$.

☑ b. Shade the region whose points satisfy the inequality.

☐ c. Graph the line $2x - y = -3$.

☐ d. Shade the region whose points satisfy the inequality.

☐ e. Indicate the region whose points are the solution of the system.



2. Graph the system below to find its solution:

$$y > x + 3$$
$$y \geq x - 4$$

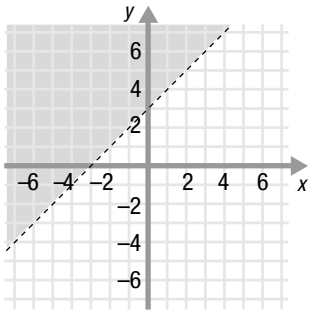
☑ a. Graph the line $y = x + 3$.

☑ b. Shade the region whose points satisfy the inequality.

☐ c. Graph the line $y = x - 4$.

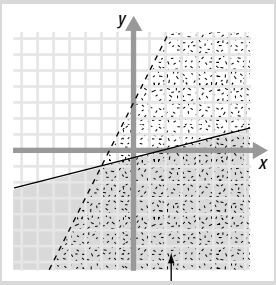
☐ d. Shade the region whose points satisfy the inequality.

☐ e. Indicate the region whose points are the solution of the system.

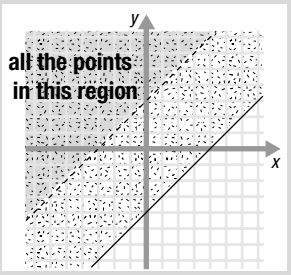


Answers to Sample Problems

c., d., e.

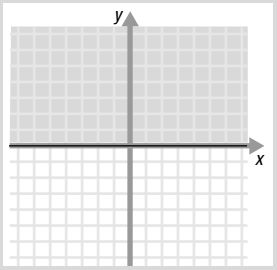


c., d., e.

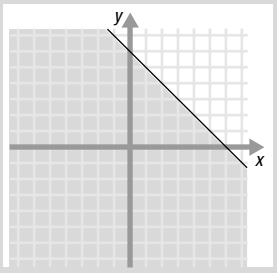


Answers to Sample Problems

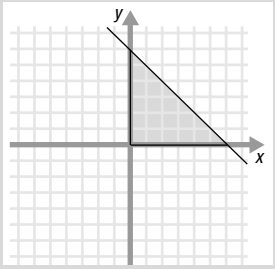
b.



c.



a., b., c.

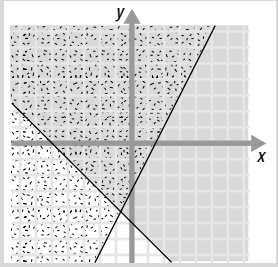


e. (6, 0)

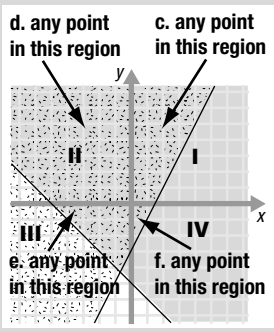
f. (0, 0)

g. 6, 6
18

a., b.



c., d., e., f.



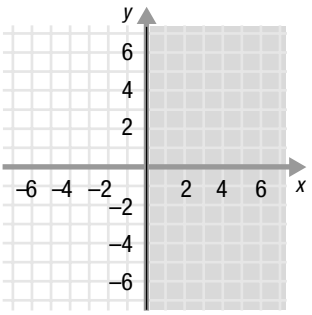
EXPLORE

Sample Problems

On the computer you used the Grapher to solve and analyze systems of linear inequalities. Below are some additional exploration problems.

1. Graph the system of inequalities below. Find the points of intersection of each pair of lines and the area of the region that is the solution.

$$x \geq 0$$
$$y \geq 0$$
$$x + y \leq 6$$



☑ a. Graph the inequality $x \geq 0$.

☐ b. Graph the inequality $y \geq 0$.

☐ c. Graph the inequality $x + y \leq 6$.

☑ d. Find the intersection of $x = 0$ and $x + y = 6$.

$$\begin{aligned} x + y &= 6 \\ 0 + y &= 6 \\ y &= 6 \end{aligned}$$

intersection: (0, 6)

.

☐ e. Find the intersection of $y = 0$ and $x + y = 6$. intersection: (___, ___)

☐ f. Find the intersection of $x = 0$ and $y = 0$. intersection: (___, ___)

☐ g. Find the area of the triangle.

$$\text{area} = \frac{1}{2} \cdot \text{base} \cdot \text{height}$$
$$= \frac{1}{2} \cdot \_\_\_\_ \cdot \_\_\_\_$$
$$= \_\_\_\_ \text{ square units}$$

2. Find the coordinates of four points—one in each quadrant—that satisfy both inequalities in the system below.

$$y \geq 2x - 3$$
$$x + y \geq -5$$

☐ a. Graph the inequality $y \geq 2x - 3$.

☐ b. Graph the inequality $x + y \geq -5$.

☐ c. Find a point in Quadrant I that satisfies both inequalities.

☐ d. Find a point in Quadrant II that satisfies both inequalities.

☐ e. Find a point in Quadrant III that satisfies both inequalities.

☐ f. Find a point in Quadrant IV that satisfies both inequalities.

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain
Solving Linear Systems

1. Graph the system of inequalities below. Use the grid in Figure 5.3.1. Then write the coordinates of the plotted points that are solutions of this system.

$$x + 2y < 3$$
$$2x - 3y \geq 12$$

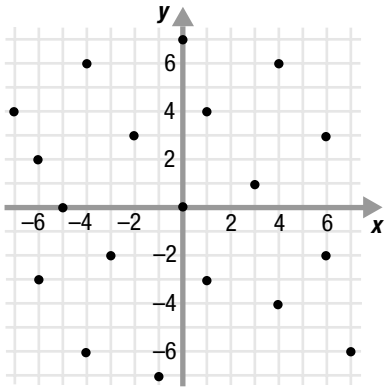


Figure 5.3.1

2. Graph the system of inequalities below to find its solution.

$$y \geq x - 4$$
$$x + 2y < 6$$

3. Graph the system of inequalities below to find its solution.

$$x \leq 5$$
$$y > \frac{2}{3}x - 1$$

4. Graph the system of inequalities below to find its solution.

$$y > 3x + 4$$
$$2y < 3x + 8$$

5. Graph the system of inequalities below to find its solution.

$$y \geq \frac{3}{4}x + 5$$
$$y \geq \frac{3}{4}x - 1$$

6. Graph the system of inequalities below to find its solution.

$$4x + y < -11$$
$$3x - 8y \leq 2$$

7. Graph the system of inequalities below to find its solution.

$$y < 6$$
$$x > -\frac{7}{2}$$

8. Graph the system of inequalities below to find its solution.

$$y \geq -\frac{5}{3}x + 1$$
$$y \geq -\frac{5}{3}x - 4$$

9. Conchita is moving out of her rental house and needs to clean it up. Because she is so busy, she won't have more than 10 hours to spend cleaning. She wants to spend at least as much time cleaning the inside of the house as she spends picking up the yard. Let x equal the number of hours Conchita spends picking up the yard; let y equal the number of hours she spends cleaning the house. Write a system of two linear inequalities that represents Conchita's cleaning options. Then graph the system.

10. Aki is trying to decide how to divide her exercise time between running and swimming. She wants to spend at least three times as much time swimming as she spends running, but she doesn't want to spend more than 9 hours per week swimming. Let x equal the number of hours per week Aki spends running; let y equal the number of hours per week she spends swimming. Write a system of two linear inequalities that represents Aki's exercise options. Then graph the system.

11. Graph the system of inequalities below to find its solution.

$$x \leq 1$$
$$x \geq -3$$

12. Graph the system of inequalities below to find its solution.

$$y \leq x + 4$$
$$x + 5y \geq -16$$
$$2x + y \leq 4$$

Explore

13. Graph the system of inequalities below. Find the points of intersection of each pair of lines.

$$\begin{aligned} x &\geq 0 \\ 3y &\geq 5x - 15 \\ 5x + 3y &\leq 15 \end{aligned}$$

14. Graph the system of inequalities below. Then find the coordinates of four points that satisfy both inequalities in the system.

$$\begin{aligned} y &\leq 7 \\ y &\geq -2 \end{aligned}$$

15. Graph the system of inequalities below. Find the points of intersection of each pair of lines and the area of the region whose points are the solution of the system.

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x &\leq 3 \\ y &\leq 5 \end{aligned}$$

16. Graph the system of inequalities below. If the direction of both inequality signs are reversed, what region contains the points that are the solution of the new system?

$$\begin{aligned} 2y &< x + 4 \\ 2x + y &< 7 \end{aligned}$$

17. Graph the system of inequalities below. Then find the coordinates of four points that satisfy both inequalities in the system.

$$\begin{aligned} 2x + y &\geq -6 \\ y &\leq x + 4 \end{aligned}$$

18. Graph the system of inequalities below. Find the points of intersection of each pair of lines and the area of the region whose points are the solution of the system.

$$\begin{aligned} y &\geq x - 4 \\ x + y &\geq -4 \\ 2y &\leq x + 10 \\ x + 2y &\leq 10 \end{aligned}$$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Solving Linear Systems

1. Graph the system of inequalities below.

   $y > x + 3$

   $y < -x - 2$

2. Graph the system of inequalities below.

   $y \geq x + 1$

   $y \leq -x + 4$

3. Graph the system of inequalities below.

   $y > x - 2$

   $y < -2x + 3$

4. Graph the system of inequalities below.

   $y < x - 2$

   $y < -2x + 3$

5. Graph the system of inequalities below.

   $y \leq 3x - 1$

   $y \leq -x + 3$

6. Graph the system of inequalities below.

   $y < x - 1$

   $y < -3x + 2$

7. Graph the system of inequalities below.

   $x + y \geq 4$

   $x - y < -2$

8. Graph the system of inequalities below.

   $x - y > -4$

   $x + y \leq -5$

9. Graph the system of inequalities below.

   $x + y \leq 6$

   $x - y > 2$

10. Graph the system of inequalities below.

    $y > x + 5$

    $y \leq -3x + 2$

11. Graph the system of inequalities below.

    $y \leq -x - 4$

    $y < 2x + 5$

12. Graph the system of inequalities below.

    $y \geq x - 3$

    $y < -2x - 3$

13. Graph the system of inequalities below.

    $y > \frac{3}{2}x + 3$

    $y < \frac{1}{2}x - 2$

14. Graph the system of inequalities below.

    $y \geq \frac{1}{4}x + 4$

    $y \leq \frac{1}{3}x - 3$

15. Graph the system of inequalities below.

    $y \leq -\frac{4}{3}x + 2$

    $y > \frac{1}{3}x + 5$

16. Graph the system of inequalities below.

    $x + y > 3$

    $2x - y < 2$

17. Graph the system of inequalities below.

$3x + y \geq 4$

$x - y < 1$

18. Graph the system of inequalities below.

$x - y \geq 3$

$2x + y \geq 1$

19. Graph the system of inequalities below.

$y \geq \frac{3}{4}x + 1$

$y \leq -\frac{1}{3}x - 2$

20. Graph the system of inequalities below.

$y > \frac{1}{4}x + 5$

$y \leq \frac{3}{2}x + 1$

21. Graph the system of inequalities below.

$y > \frac{3}{5}x + 2$

$y < -\frac{2}{3}x - 1$

22. Graph the system of inequalities below.

$y > 3$

$y < 3x - 4$

23. Graph the system of inequalities below.

$x \geq -1$

$y \geq 2x + 5$

24. Graph the system of inequalities below.

$y < -2$

$y \leq -2x - 1$

25. Graph the system of inequalities below.

$y > x$

$y \leq 3$

26. Graph the system of inequalities below.

$y < -x$

$x > -2$

27. Graph the system of inequalities below.

$y > 3x + 1$

$y < 3x - 4$

28. Graph the system of inequalities below.

$y \geq -2x - 3$

$y \leq -2x + 4$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Graph the system of inequalities below. Use the grid in Figure 5.3.2. Then write the coordinates of the plotted points that are solutions of this system.

$$y < \frac{2}{3}x + 1$$
$$x + y \geq -6$$

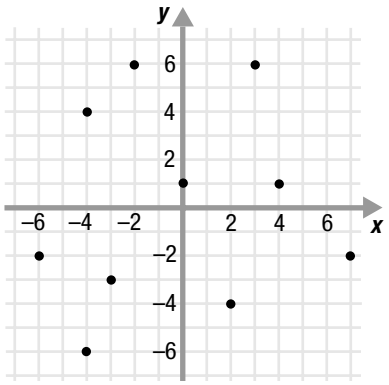


Figure 5.3.2

2. Graph the system of inequalities below to find its solution.

$$x + 2y \leq 4$$
$$y > 3$$

3. Graph the system of inequalities below. Use the grid in Figure 5.3.3. Then write the coordinates of the plotted points that are **not** solutions of either inequality in this system.

$$2x - 5y < 15$$
$$3x + 4y \leq -5$$

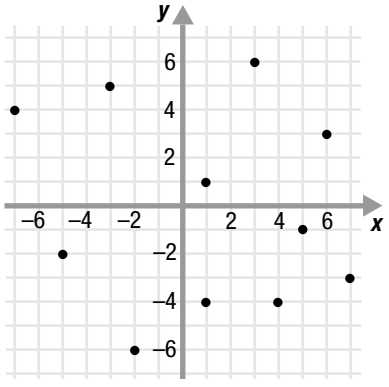


Figure 5.3.3

4. Graph the system of inequalities below to find its solution.

$$y \leq x + 3$$
$$y \geq x - 4$$

5. Graph the system of inequalities below to find its solution.

$$x \leq 2$$
$$y \geq -3$$
$$2y \leq 3x + 2$$

6. Graph the system of inequalities below. Use the grid in Figure 5.3.4. Then write the coordinates of the plotted points that are solutions of this system.

$$y \geq 0$$
$$x \geq -6$$
$$y \leq -\frac{1}{2}x + 2$$

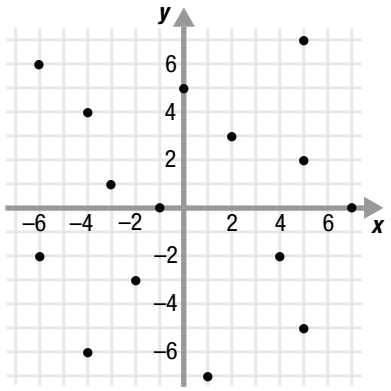


Figure 5.3.4

7. The system of inequalities below is graphed in Figure 5.3.5. If the direction of the first inequality sign is reversed, outline the region(s) that contains the points that are the solution of the new system.

$$2x + y < 4$$
$$x + 2y > -4$$

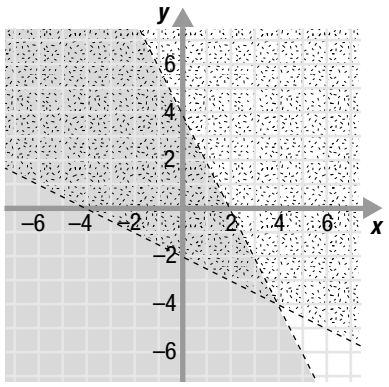


Figure 5.3.5

8. Graph the system of inequalities below. Then write the coordinates of a point that satisfies the first two inequalities but not the third inequality.

$$x \geq 0$$
$$2x - y \leq 6$$
$$y \leq x$$

TOPIC 5 CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic, or you may wish to do these problems to review for a test.

1. Evaluate the expression $3x^3 - 7x^2y + 8y^2 + 2$ when $x = 3$ and $y = 6$.

2a. Find the GCF of 75, 84, and 180.

b. Find the LCM of 75, 84, and 180.

3. The sum of two numbers is 53. The difference of these numbers is 15. What are the two numbers?

4. Write the equation of the line through the point (4, 1) with slope 1:

   a. in point-slope form.

   b. in slope-intercept form.

   c. in standard form.

5. Solve this system:

$$x + y = 4$$
$$x - y = 7$$

6. Solve $-4 \leq 3 + 4x < 5$ for x, then graph its solution on the number line below.

7. Find the slope of the line through the points (0, 5) and (8, –3).

8. Solve $5x = 3(x - 2) - x$ for x.

9. Graph the system of inequalities below to find its solution.

$$y \leq 2x + 3$$
$$y \geq 2x - 4$$

10. Reduce to lowest terms: $\frac{3080}{5390}$

11. Solve this system:

$$x + 2y = -2$$
$$3x - 7y = 33$$

12. The point (–2, –6) lies on a line with slope 3. Graph this line by finding another point that lies on the line.

13. Find the slope of the line that is perpendicular to the line $4x - 9y = 5$.

14. Graph the system of inequalities below to find its solution.

$$y \geq 2x - 4$$
$$y \leq x + 2$$

15a. Find the GCF of 168, 231, and 315.

b. Find the LCM of 168, 231, and 315.

16. Write the equation of the line through the point (2, –5) with slope -3:

   a. in point-slope form.

   b. in slope-intercept form.

   c. in standard form.

17. Solve: $3y + 5 = 9 - 2y$ for y.

18. Last year Keith split $4,500 between his savings account, which paid 4% in interest, and his checking account, which paid 1.5% in interest. If he earned a total of $143.25 in interest, how was his money divided between the two accounts?

19. Solve this system:

$$4x + 5y = 12$$
$$5x + 4y = -3$$

20. Graph the system of inequalities below to find its solution.

$$3y < 2x + 12$$
$$x - 5y \geq -6$$

21. Evaluate the expression $11a^2 - b + 6ab$ when $a = 4$ and $b = 12$.

Use Figure 5.1 to answer questions 22 through 25.

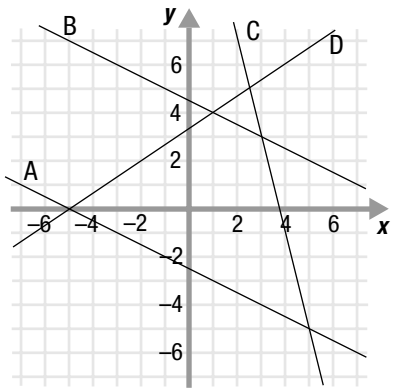


Figure 5.1

22. Which two lines form a system that has a solution of (5, –5)?

23. Which two lines form a system that has a solution of (1, 4)?

24. Which two lines form a system that has a solution of (–5, 0)?

25. Which two lines form a system that has no solution?

26. Find the slope of the line through the points (–19, 12) and (23, –11).

27. Find: $\frac{6}{7} - \frac{5}{6}$

28. Solve $\frac{2}{13}(\frac{9}{2}x + 5) = \frac{7}{13}(\frac{1}{2} + x) + 1$ for x.

29. Solve this system:

$$11x - 7y = -19$$
$$17x + 14y = -23$$

30. Graph this system of inequalities to find its solution.

$$x < -1$$
$$y \geq -3$$

31. Solve $2 < 10 - 9y < 4$ for y, then graph its solution on the number line below.

32. Write the equation of the line through the point (–8, –7) with slope $\frac{6}{5}$:

    a. in point-slope form.

    b. in slope-intercept form.

    c. in standard form.

33. Brad emptied the money from a newspaper vending machine that accepted only dimes and quarters and got 181 coins worth a total of $31.30. How many dimes did he collect?

34. Find: $\frac{6}{35} \div \frac{8}{21}$

35. Solve $8y + 7 = 1 - 3y$ for y.

36. Solve this system:

$$9x + y = 58$$
$$\frac{1}{2}x + 8y = 35$$

37. Find the slope of the line $6x - 5y = 8$.