LESSON EII.F – ABSOLUTE VALUE

OVERVIEW

Here's what you'll learn in this lesson:

Solving Equations

a. Solving $|x| = a$

b. Solving $|Ax + B| = a$

c. Solving $|Ax + B| = |Cx + D|$

Solving Inequalities

a. Solving absolute value inequalities

Your favorite brand of candy is on sale, so you buy three bags. The labels on the bags say: "Contents: Approximately 25 pieces." When you count the pieces in each bag, you notice that every bag contains a different numbers of pieces, but the average number of pieces is 25.

To indicate how the number of pieces in each bag varies from the average, you can use absolute value.

In this lesson you will learn how to solve equations and inequalities involving absolute value.

EXPLAIN

SOLVING EQUATIONS

Summary

Absolute Value

You have learned that the absolute value of a number is the distance of that number from 0 on the number line. Since distance is always a nonnegative number, the absolute value of a number is always nonnegative.

For example, the absolute value of 7, denoted by $|7|$, is 7, since the number 7 is a distance of 7 from 0 on the number line. Similarly, the absolute value of –7, denoted by $|-7|$, is also 7, since the number –7 is also a distance of 7 from 0 on the number line.

Solving Equations of the Form $|x| = a$

You can use what you know about absolute value to solve an equation that can be written in the form $|x| = a$. Here are the steps:

1. Write the equation in the form $|x| = a$.
2. Find the solutions based on the following:
   - If $a > 0$, the equation has two solutions, $x = -a$ or $x = a$.
   - If $a < 0$, the equation has no solutions.
   - If $a = 0$, the equation has one solution, $x = 0$.

For example, solve the equation $|x| = 6$:

1. The equation is in the form $|x| = a$. $\quad |x| = 6$
2. Find the solutions. Since a is 6, $a > 0$, and the equation has two solutions. $\quad x = -6$ or $x = 6$

So the solutions of $|x| = 6$ are $x = -6$ or $x = 6$.

You can check these solutions by substituting them into the original equation.

Check $x = -6$:	Check $x = 6$:
Is $|-6| = 6$?	Is $|6| = 6$?
Is $6 = 6$? Yes.	Is $6 = 6$? Yes.

As another example, to solve the equation $2|x| + 5 = 5$:

1. Write the equation in the form $|x| = a$.

$$2|x| + 5 = 5$$
$$2|x| + 5 - 5 = 5 - 5$$
$$2|x| = 0$$
$$\frac{2|x|}{2} = \frac{0}{2}$$
$$|x| = 0$$

2. Find the solutions. Since $a = 0$, the equation has one solution. $x = 0$

So the solution of $2|x| + 5 = 5$ is $x = 0$.

You can check this solution by substituting it into the original equation.

Check $x = 0$:

Is $2|0| + 5 = 5$?

Is $2(0) + 5 = 5$?

Is $0 + 5 = 5$?

Is $5 = 5$? Yes.

Similarly, solve the equation $|x| = -21$:

1. The equation is in the form $|x| = a$. $|x| = -21$
2. Find the solutions. Since a is -21, $a < 0$, and the equation has no solutions. no solutions

So there are no solutions of the equation $|x| = -21$.

Solving Equations of the Form $|ax + b| = c$

Here are the steps for solving an equation that can be written in the form $|ax + b| = c$, where $c \geq 0$:

You can use any variable, not just z, as the value to substitute for ax + b.

1. Write the equation in the form $|ax + b| = c$.
2. Substitute z for $ax + b$.
3. Solve the equation $|z| = c$ to get $z = -c$ or $z = c$.
4. Replace z with $ax + b$.
5. Solve for x.

For example, solve the equation $|3x| = 15$:

1. The equation is in the form $\|ax + b\| = c$. (Here, b is 0.)	$\|3x\| = 15$		
2. Substitute z for $3x$.	$\|z\| = 15$		
3. Solve for z.	$z = -15$	or	$z = 15$
4. Replace z with $3x$.	$3x = -15$		$3x = 15$
5. Solve for x.	$\frac{3x}{3} = \frac{-15}{3}$		$\frac{3x}{3} = \frac{15}{3}$
	$x = -5$	or	$x = 5$

So the solutions of the equation $|3x| = 15$ are $x = -5$ or $x = 5$.

You can check these solutions by substituting them into the original equation.

Check $x = -5$:	Check $x = 5$:
Is $\lvert 3(-5) \rvert = 15$?	Is $\lvert 3(5) \rvert = 15$?
Is $\lvert -15 \rvert = 15$?	Is $\lvert 15 \rvert = 15$?
Is $15 = 15$? Yes.	Is $15 = 15$? Yes.

As a second example, solve $\lvert 2x - 3 \rvert = 9$:

1. The equation is in the form $\lvert ax + b \rvert = c$. $\quad \lvert 2x - 3 \rvert = 9$
2. Substitute z for $2x - 3$. $\quad \lvert z \rvert = 9$
3. Solve for z. $\quad z = -9 \quad$ or $\quad z = 9$
4. Replace z with $2x - 3$. $\quad 2x - 3 = -9 \quad$ or $\quad 2x - 3 = 9$
5. Solve for x. $\quad 2x - 3 + 3 = -9 + 3 \quad$ or $\quad 2x - 3 + 3 = 9 + 3$

$$2x = -6 \qquad\qquad 2x = 12$$

$$\frac{2x}{2} = \frac{-6}{2} \qquad\qquad \frac{2x}{2} = \frac{12}{2}$$

$$x = -3 \quad \text{or} \quad x = 6$$

So the solutions of the equation $\lvert 2x - 3 \rvert = 9$ are $x = -3$ or $x = 6$.

You can check these solutions by substituting them into the original equation.

Check $x = -3$:	Check $x = 6$:
Is $\lvert 2(-3) - 3 \rvert = 9$?	Is $\lvert 2(6) - 3 \rvert = 9$?
Is $\lvert -6 - 3 \rvert = 9$?	Is $\lvert 12 - 3 \rvert = 9$?
Is $\lvert -9 \rvert = 9$?	Is $\lvert 9 \rvert = 9$?
Is $9 = 9$? Yes.	Is $9 = 9$? Yes.

Here's another example. To solve the equation $7\lvert 2x + 5 \rvert - 3 = 18$:

1. Write the equation in the form $\lvert ax + b \rvert = c$.

$$7\lvert 2x + 5 \rvert - 3 = 18$$
$$7\lvert 2x + 5 \rvert - 3 + 3 = 18 + 3$$
$$7\lvert 2x + 5 \rvert = 21$$
$$\frac{7\lvert 2x + 5 \rvert}{7} = \frac{21}{7}$$
$$\lvert 2x + 5 \rvert = 3$$

2. Substitute z for $2x + 5$. $\quad \lvert z \rvert = 3$
3. Solve for z. $\quad z = -3 \quad$ or $\quad z = 3$

4. Replace z with $2x+5$. $\quad 2x+5=-3 \quad$ or $\quad 2x+5=3$

5. Solve for x. $\quad 2x+5-5=-3-5 \quad$ or $\quad 2x+5-5=3-5$

$$2x=-8 \qquad\qquad 2x=-2$$

$$\frac{2x}{2}=\frac{-8}{2} \qquad\qquad \frac{2x}{2}=\frac{-2}{2}$$

$$x=-4 \quad \text{or} \quad x=-1$$

So the solutions of the equation $7|2x+5|-3=18$ are $x=-4$ or $x=-1$.

You can check these solutions by substituting them into the original equation.

Check $x=-4$:	Check $x=-1$:
Is $7\|2(-4)+5\|-3=18$?	Is $7\|2(-1)+5\|-3=18$?
Is $7\|-8+5\|-3=18$?	Is $7\|-2+5\|-3=18$?
Is $7\|-3\|-3=18$?	Is $7\|3\|-3=18$?
Is $7(3)-3=18$?	Is $7(3)-3=18$?
Is $21-3=18$?	Is $21-3=18$?
Is $18=18$? Yes.	Is $18=18$? Yes.

Solving Equations of the Form $|ax+b|=|cx+d|$

Here are the steps for solving an equation that can be written in the form $|ax+b|=|cx+d|$:

Again, you can use any variables you like…you don't have to use z and w.

1. Write the equation in the form $|ax+b|=|cx+d|$.
2. Substitute z for $ax+b$ and w for $cx+d$.
3. Solve the equation $|z|=|w|$ to get $z=w$ or $z=-w$.
4. Replace z with $ax+b$ and w with $cx+d$.
5. Solve for x.

For example, solve the equation $|3x-4|=|x+8|$:

1. The equation is in the form $|ax+b|=|cx+d|$. $\quad |3x-4|=|x+8|$
2. Substitute z for $3x-4$ and w for $x+8$. $\quad |z|=|w|$
3. Solve for z. $\quad z=w \quad$ or $\quad z=-w$
4. Replace z with $3x-4$ and w with $x+8$. $\quad 3x-4=x+8 \quad$ or $\quad 3x-4=-(x+8)$

5. Solve for x: $3x - 4 - x = x + 8 - x$ or $3x - 4 = -x - 8$

$$2x - 4 = 8 \qquad 3x - 4 + x = -x - 8 + x$$

$$2x - 4 + 4 = 8 + 4 \qquad 4x - 4 = -8$$

$$2x = 12 \qquad 4x - 4 + 4 = -8 + 4$$

$$\frac{2x}{2} = \frac{12}{2} \qquad 4x = -4$$

$$\frac{4x}{4} = \frac{-4}{4}$$

$$x = 6 \quad \text{or} \quad x = -1$$

So the solutions of the equation $|3x - 4| = |x + 8|$ are $x = 6$ or $x = -1$.

You can check these solutions by substituting them into the original equation.

Check $x = 6$:	Check $x = -1$:
Is $|3(6) - 4| = |6 + 8|$?	Is $|3(-1) - 4| = |-1 + 8|$?
Is $|18 - 4| = |14|$?	Is $|-3 - 4| = |7|$?
Is $|14| = 14$?	Is $|-7| = 7$?
Is $14 = 14$? Yes.	Is $7 = 7$? Yes.

Sample Problems

1. Solve this equation: $|x| - 5 = 23 + 5$

✔ a. Write the equation in the form $|x| = a$.

$$|x| - 5 = 23 + 5$$
$$|x| - 5 = 28$$
$$|x| - 5 + 5 = 28 + 5$$
$$|x| = 33$$

☐ b. Find the solutions. Since a is 33, $a > 0$, and the equation has two solutions. $x =$ ______ or $x =$ ______

☐ c. Check the solutions in the original equation.

2. Solve this equation: $3|x + 9| - 7 = 24 - 28$

✔ a. Write the equation in the form $|ax + b| = c$.

$$3|x + 9| - 7 = 24 - 28$$
$$3|x + 9| - 7 = -4$$
$$3|x + 9| - 7 + 7 = -4 + 7$$
$$3|x + 9| = 3$$
$$\frac{3|x + 9|}{3} = \frac{3}{3}$$
$$|x + 9| = 1$$

Answers to Sample Problems

b. –33, 33

c. Here's one way to check.

Check $x = -33$:

Is $|-33| - 5 = 23 + 5$?

Is $33 - 5 = 23 + 5$?

Is $28 = 28$? Yes.

Check $x = 33$:

Is $|33| - 5 = 23 + 5$?

Is $33 - 5 = 23 + 5$?

Is $28 = 28$? Yes.

Answers to Sample Problems

b. 1

c. −1, 1

d. −1, 1

e. −10, −8

f. *Here's one way to check.*

Check $x = -10$:

Is $3|-10+9| - 7 = 24 - 28$?

Is $3|-1| - 7 = -4$?

Is $3(1) - 7 = -4$?

Is $3 - 7 = -4$?

Is $-4 = -4$? Yes.

Check $x = -8$:

Is $3|-8+9| - 7 = 24 - 28$?

Is $3|1| - 7 = -4$?

Is $3(1) - 7 = -4$?

Is $3 - 7 = -4$?

Is $-4 = -4$? Yes.

c. $w, -w$

d. $2x - 5$

f. *Here's one way to check.*

Check $x = 0$:

Is $|2(0) + 5| = |2(0) - 5|$?

Is $|0 + 5| = |0 - 5|$?

Is $|5| = |-5|$?

Is $5 = 5$? Yes.

☐ b. Substitute z for $x + 9$. $|z| = $ ____

☐ c. Solve for z. $z = $ ____ or $z = $ ____

☐ d. Replace z with $x + 9$. $x + 9 = $ ____ or $x + 9 = $ ____

☐ e. Solve for x. $x = $ _____ or $x = $ _____

☐ f. Check the solutions in the original equation.

3. Solve this equation: $|2x + 5| = |2x - 5|$

✔ a. The equation is in the form $|ax + b| = |cx + d|$. $|2x + 5| = |2x - 5|$

✔ b. Substitute z for $2x + 5$ and w for $2x - 5$. $|z| = |w|$

☐ c. Solve for z. $z = $ ____ or $z = $ ____

☐ d. Replace z with $2x + 5$ and w with $2x - 5$. $2x + 5 = 2x - 5$ or $2x + 5 = -($_____$)$

✔ e. Solve for x.

$$2x + 5 = 2x - 5 \quad \text{or} \quad 2x + 5 = -(2x - 5)$$
$$2x + 5 - 2x = 2x - 5 - 2x \qquad 2x + 5 = -2x + 5$$
$$5 = -5 \qquad 2x + 5 - 5 = -2x + 5 - 5$$
$$2x = -2x$$
$$2x + 2x = -2x + 2x$$
$$4x = 0$$
$$\frac{4x}{4} = \frac{0}{4}$$
$$x = 0$$

Since $5 \neq -5$, there is only one solution, $x = 0$.

☐ f. Check the solution in the original equation.

SOLVING INEQUALITIES

Summary

Solving Inequalities of the Form $|x| < a$ or $|x| \le a$

Recall that the absolute value of x, denoted by $|x|$, is the distance of x from 0 on the number line. You can use this fact to solve inequalities of the form $|x| < a$ (where $a > 0$) or $|x| \le a$ (where $a \ge 0$).

To solve an inequality that can be written in the form $|x| < a$ or $|x| \le a$.

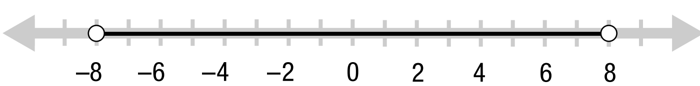
1. Write the inequality of the form $|x| < a$ or $|x| \le a$.
2. Find the solution based on the following:
   - If $|x| < a$, then the solution is all x such that $-a < x < a$.
   - If $|x| \le a$, then the solution is all x such that $-a \le x \le a$.

When you write $-3 < x < 3$, read this $-3 < x$ ***and*** *$x < 3$.*

For example, solve the inequality $|x| < 8$:

1. The inequality is in the form $|x| < a$. $\quad |x| < 8$
2. Find the solution. Here, $a = 8$. $\quad -8 < x < 8$

So the solution of $|x| < 8$ is all x such that $-8 < x < 8$. This solution consists of all numbers whose distance from 0 is less than 8 on the number line.



Remember, the open circles on this number line are used to indicate that the numbers -8 and 8 are ***not*** *included in the solution.*

Many numbers are part of the solution of this inequality. Here, two of these numbers have been checked.

Check $x = -3.5$:	Check $x = 5$:
Is $|-3.5| < 8$?	Is $|5| < 8$?
Is $3.5 < 8$? Yes.	Is $5 < 8$? Yes.

Solving Inequalities of the Form $|x| > a$ or $|x| \ge a$

Here are the steps to solve inequalities that can be written in the form $|x| > a$ or $|x| \ge a$, where $a \ge 0$:

1. Write the inequality in the form $|x| > a$ or $|x| \ge a$.
2. Find the solution based on the following:
   - If $|x| > a$, then the solution is all x such that $x < -a$ or $x > a$.
   - If $|x| \ge a$, then the solution is all x such that $x \le -a$ or $x \ge a$.

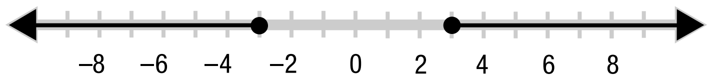
For example, solve the inequality $|x| \ge 3$:

1. The inequality is in the form $|x| \ge a$. $|x| \ge 3$
2. Find the solution. Here, $a = 3$. $x \le -3$ or $x \ge 3$

*You use **or** instead of **and** because x can't be less than –3 **and** greater than 3 at the same time.*

So the solution of $|x| \ge 3$ is all x such that $x \le -3$ or $x \ge 3$. This solution consists of all numbers whose distance from 0 is greater than 3 on the number line.

Remember, the closed circles on this number line are used to indicate that the numbers 3 and –3 are included in the solution.



Many numbers are part of the solution of this inequality. Here, two of these numbers have been checked.

Check $x = -3$:	Check $x = 7$:
Is $|-3| \ge 3$?	Is $|7| \ge 3$?
Is $3 \ge 3$? Yes.	Is $7 \ge 3$? Yes.

Solving Inequalities of the Form $|ax + b| < c$ or $|ax + b| \le c$

Just as you do when you solve equations involving absolute value, you can use substitution to solve inequalities involving absolute value.

To solve an inequality that can be written in the form $|ax + b| < c$ (where $c > 0$) or $|ax + b| \le c$ (where $c \ge 0$):

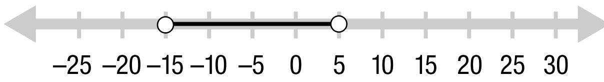
1. Write the inequality in the form $|ax + b| < c$ or $|ax + b| \le c$.
2. Substitute w for $ax + b$.
3. Solve the inequality $|w| < c$ or solve the inequality $|w| \le c$.
4. Replace w with $ax + b$.
5. Solve for x.

For example, solve the inequality $|x + 5| < 10$:

1. The inequality is in the form $|ax + b| < c$. $|x + 5| < 10$
2. Substitute w for $x + 5$. $|w| < 10$
3. Solve the inequality $|w| < 10$. $-10 < w < 10$
4. Replace w with $x + 5$. $-10 < x + 5 < 10$
5. Solve for x. $-10 - 5 < x + 5 - 5 < 10 - 5$

   $-15 < x < 5$

So the solution of the inequality $|x + 5| < 10$ is all x such that $-15 < x < 5$. This solution can be graphed on a number line:



Many numbers are part of the solution of this inequality. Here, two of these numbers have been checked.

Check $x = 2.5$:	Check $x = -7.5$:
Is $\lvert 2.5 + 5 \rvert < 10$?	Is $\lvert -7.5 + 5 \rvert < 10$?
Is $\lvert 7.5 \rvert < 10$?	Is $\lvert -2.5 \rvert < 10$?
Is $7.5 < 10$? Yes.	Is $2.5 < 10$? Yes.

Solving Inequalities of the Form $|ax + b| > c$ or $|ax + b| \geq c$

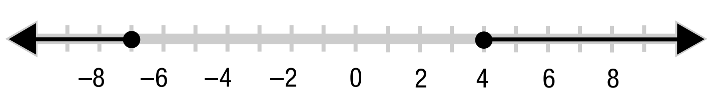
To solve an inequality that can be written in the form $|ax + b| > c$ or $|ax + b| \geq c$, where $c \geq 0$:

1. Write the inequality in the form $|ax + b| > c$ or $|ax + b| \geq c$.
2. Substitute w for $ax + b$.
3. Solve the inequality $|w| > c$ or solve the inequality $|w| \geq c$.
4. Replace w with $ax + b$.
5. Solve for x.

For example, solve the inequality $|2x + 3| \geq 11$:

1. The inequality is in the form $\lvert ax + b \rvert \geq c$.	$\lvert 2x + 3 \rvert \geq 11$		
2. Substitute w for $2x + 3$.	$\lvert w \rvert \geq 11$		
3. Solve the inequality $\lvert w \rvert \geq 11$.	$w \leq -11$	or	$w \geq 11$
4. Replace w with $2x + 3$.	$2x + 3 \leq -11$	or	$2x + 3 \geq 11$
5. Solve for x.	$2x + 3 - 3 \leq -11 - 3$	or	$2x + 3 - 3 \geq 11 - 3$
	$2x \leq -14$		$2x \geq 8$
	$\frac{2x}{2} \leq \frac{-14}{2}$		$\frac{2x}{2} \geq \frac{8}{2}$
	$x \leq -7$	or	$x \geq 4$

So the solution of the inequality $|2x + 3| \geq 11$ is all x such that $x \leq -7$ or $x \geq 4$. This solution can be graphed on a number line:



Many numbers are part of the solution of this inequality. Here, two of these numbers have been checked.

Check $x = -8.3$:

Is $|2(-8.3) + 3| \geq 11$?

Is $|-16.6 + 3| \geq 11$?

Is $|-13.6| \geq 11$?

Is $13.6 \geq 11$? Yes.

Check $x = 5$

Is $|2(5) + 3| \geq 11$?

Is $|10 + 3| \geq 11$?

Is $|13| \geq 11$?

Is $13 \geq 11$? Yes.

Sample Problems

1. Solve this inequality: $|x| \leq 2.5$

   ✔ a. The inequality is in the form $|x| \leq a$. $|x| \leq 2.5$

   ✔ b. Find the solution. Here, $a = 2.5$. The solution is all numbers x whose distance from 0 is less than or equal to 2.5. $-2.5 \leq x \leq 2.5$

   ☐ c. Check two of the numbers that are part of the solution, $x = -1$ and $x = 2.5$.

2. Solve this inequality: $2|x| - 5 > 3$

   ☐ a. Write the inequality in the form $|x| > a$.

   $$2|x| - 5 > 3$$
   $$2|x| - 5 + 5 > 3 + 5$$
   $$2|x| > 8$$
   $$\frac{2|x|}{2} > \frac{8}{2}$$
   $$|x| > \_\_\_\_\_\_$$

   ☐ b. Find the solutions. ________________

   ✔ c. Check two of the numbers that are part of the solution, $x = -5$ and $x = 7$.

   Check $x = -5$:
   Is $2|-5| - 5 > 3$?
   Is $2(5) - 5 > 3$?
   Is $10 - 5 > 3$?
   Is $5 > 3$? Yes.

   Check $x = 7$:
   Is $2|7| - 5 > 3$?
   Is $2(7) - 5 > 3$?
   Is $14 - 5 > 3$?
   Is $9 > 3$? Yes.

3. Solve this inequality: $|7 - 3x| \leq 28$

   ✔ a. The inequality is in the form $|ax + b| \leq c$. $|7 - 3x| \leq 28$

Answers to Sample Problems

c. Here's one way to check.
Check $x = -1$:
Is $|-1| \leq 2.5$?
Is $1 \leq 2.5$? Yes.

Check $x = 2.5$:
Is $|2.5| \leq 2.5$?
Is $2.5 \leq 2.5$? Yes.

a. 4

b. $x < -4$ or $x > 4$

☑ b. Substitute w for $7 - 3x$. $|w| \le 28$

☐ c. Solve the inequality $|w| \le 28$. ____________________

☐ d. Replace w with $7 - 3x$. ____________________

☐ e. Solve for x. (Remember to reverse the direction of the inequality sign when you divide both sidesof an inequality by a negative number.) ____________________

☑ f. Check two of the numbers that are part of the solution, $x = -2$ and $x = 10$.

Check $x = -2$:
Is $|7 - 3(-2)| \le 28$?
Is $|7 + 6| \le 28$?
Is $|13| \le 28$?
Is $13 \le 28$? Yes.

Check $x = -10$:
Is $|7 - 3(10)| \le 28$?
Is $|7 - 30| \le 28$?
Is $|-23| \le 28$?
Is $23 \le 28$? Yes.

4. Solve this inequality: $2|3x + 7| - 10 \ge 74$

☑ a. Write the inequality in the form $|ax + b| \ge c$.

$$2|3x + 7| - 10 \ge 74$$
$$2|3x + 7| - 10 + 10 \ge 74 + 10$$
$$2|3x + 7| \ge 84$$
$$\frac{2|3x + 7|}{2} \ge \frac{84}{2}$$
$$|3x + 7| \ge 42$$

☑ b. Substitute w for $3x + 7$. $|w| \ge 42$

☐ c. Solve the inequality $|w| \ge 42$. ____________________

☐ d. Replace w with $3x + 7$. ____________________

☐ e. Solve for x.

____

☐ f. Check two of the numbers that are part of the solution, $x = -17$ and $x = 12$. ____________ or ____________

Answers to Sample Problems

3.

c. $-28 \le w \le 28$

d. $-28 \le 7 - 3x \le 28$

e. Here's a way to solve for x:

$$-28 \le 7 - 3x \le 28$$
$$-28 - 7 \le 7 - 3x - 7 \le 28 - 7$$
$$-35 \le -3x \le 21$$
$$\frac{-35}{-3} \ge \frac{-3x}{-3} \ge \frac{21}{-3}$$
$$\frac{35}{3} \ge x \ge -7$$

4.

c. $w \le -42$ *or* $w \ge 42$

d. $3x + 7 \le -42$ *or* $3x + 7 \ge 42$

e. Here's a way to solve for x:

$$3x + 7 \le -42 \quad \text{or} \quad 3x + 7 \ge 42$$
$$3x + 7 - 7 \le -42 - 7 \quad 3x + 7 - 7 \ge 42 - 7$$
$$3x \le -49 \quad 3x \ge 35$$
$$\frac{3x}{3} \le \frac{49}{3} \quad \frac{3x}{3} \ge \frac{35}{3}$$
$$x \le -\frac{49}{3} \quad \text{or} \quad x \ge \frac{35}{3}$$

f. Here's one way to check.

Check $x = -17$:
Is $2|3(-17) + 7| - 10 \ge 74$?
Is $2|-51 + 7| - 10 \ge 74$?
Is $2|-44| - 10 \ge 74$?
Is $2(44) - 10 \ge 74$?
Is $88 - 10 \ge 74$?
Is $78 \ge 74$? *Yes.*

Check $x = 12$:
Is $2|3(12) + 7| - 10 \ge 74$?
Is $2|36 + 7| - 10 \ge 74$?
Is $2|43| - 10 \ge 74$?
Is $2(43) - 10 \ge 74$?
Is $86 - 10 \ge 74$?
Is $76 \ge 74$? *Yes.*

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Solving Equations

Solve the equations in problems (1) – (8) for x:

1. $|x| = 100$
2. $3|x| = 51$
3. $|x| + 10 = 7$
4. $|x| - 23 = 5 - 28$
5. $|x - 5| = 27$
6. $2|3x - 7| = 42$
7. $|5x - 6| + 71 = 72$
8. $4|3x + 8| - 18 = 14$
9. To win a prize in a contest, a contestant must guess how many jelly beans are in a jar. The jar contains 457 jelly beans. If a contestant's guess is within 15 of the actual number of jelly beans, the contestant wins a prize. Write an absolute value equation to represent the highest and lowest guesses that will receive a prize. Solve your equation to find the highest and lowest possible guesses.
10. The following formula is used to calculate percent error in a scientific experiment: $E = \frac{|a - e|}{e}$. Use this formula to find a if e is 0.156 and E is 0.1.
11. Solve for x: $|x - 8| = |x|$
12. Solve for x: $|2x + 7| = |3x - 5|$

Solving Inequalities

Solve the inequalities in problems (13) – (20) for x:

13. $|x| \geq 24$
14. $|x| < 8.27$
15. $|x| - 5 \leq -4$
16. $2|x| > 15$
17. $|x - 7.2| < 18.7$
18. $|2x + 5| \leq 21$
19. $3|x - 8| \geq 48$
20. $|5x + 9| \leq 100.5$
21. The absolute value of 5 less than 3 times a number is greater than 23. Find all possible numbers that satisfy this statement.
22. A sawmill produces 8 ft. long wall studs. The maximum desirable percent error for the lengths of the studs is 0.005. What range of lengths of studs is allowable.

    To answer this question, use the formula $E \geq \frac{|a - e|}{e}$. Let $E = 0.005$ and $e = 8$. Solve the inequality for a.
23. $3|4x + 2.4| - 9 < 23 - 26$
24. $|7 - 6x| > 42$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Solving Equations

1. Solve for y: $|y| = 128$
2. Solve for x: $|x| = -4$
3. Solve for x: $|x| = 250$
4. Solve for y: $4|y| = 56$
5. Solve for x: $-3|x| = -27$
6. Solve for x: $3|x| = 33$
7. Solve for x: $|x| + 5 = 26$
8. Solve for y: $|y| + 21 = 20$
9. Solve for x: $2|x| - 16 = 14$
10. Solve for y: $|y + 7| = 34$
11. Solve for x: $|3x + 6| = 21$
12. Solve for x: $|x - 6| = 48$
13. Solve for y: $3|2y - 5| = 39$
14. Solve for x: $4|3x - 3| = 60$
15. Solve for y: $-2|5y + 5| = 50$
16. Solve for y: $5|3y - 9| + 8 = 53$
17. Solve for x: $-3|4x + 8| + 38 = 2$
18. Solve for x: $4|2x + 5| - 6 = 22$
19. Solve for y: $\frac{1}{3}|3y - 3| + 13 = 25$
20. Solve for x: $\frac{2}{5}|5x + 15| - 24 = 36$
21. Solve for x: $\frac{1}{2}|2x - 4| - 19 = 5$
22. Solve for x: $|x - 8| = |x|$
23. Solve for y: $|y| = |12 - y|$
24. Solve for y: $|3y - 2| = |4y + 9|$
25. Solve for x: $|5x + 1| = |7x - 1|$
26. Solve for x: $|6x - 1| = |9 - 4x|$
27. Solve for y: $5|4y - 4| = 120$
28. Solve for x: $4|2x + 3| = 60$

Solving Inequalities

29. Solve for y: $|y| < 7$
30. Solve for x: $|x| \leq 3$
31. Solve for x: $|3x| < 12$
32. Solve for y: $|y| \geq 23$
33. Solve for x: $|x| > 1$
34. Solve for x: $|5x| > 95$
35. Solve for y: $4|y| \leq 36$
36. Solve for x: $3|x| \geq 45$
37. Solve for x: $5|3x| > 45$
38. Circle the inequalities below which have no solution.

    $|y| - 18 < -18$

    $|y + 7| \geq -4$

    $8|y| - 36 \leq 27$

    $-3|y| - 10 < 8$

39. Circle the inequalities below which have no solution.

    $|x| + 9 < 9$

    $|x + 2| < -3$

    $7|x| < 99$

    $-|2x + 9| > 0$

40. Solve for y: $5 - 4|2y - 3| \le -39$

41. Solve for x: $4 + 3\,|4x - 2| > 22$

42. Solve for x: $-7 + 3|5x - 10| > 38$

43. Solve for y: $12 + 3|y - 6| \le 48$

44. Solve for x: $-3 + 2\,|x + 4| \le 7$

45. Solve for x: $-8 + 4|3x + 6| < 28$

46. Solve for y: $|12 - 6y| \le 48$

47. Solve for x: $|8 - 4x| < 16$

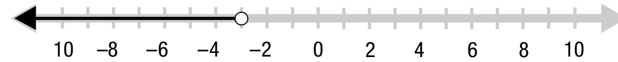
48. Solve for x: $2|14 - 7x| < 56$

49. Solve for y: $|8 - y| > 8$

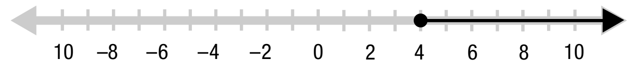
50. Solve for y: $|5 - 2y| \ge 9$

51. Solve for x: $5|18 - 6x| > 30$

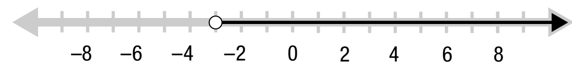
52. Find the inequality whose solution is graphed below.



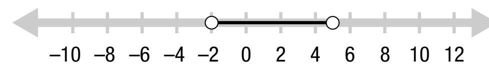
53. Find the inequality whose solution is graphed below.



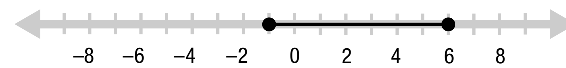
54. Find the inequality whose solution is graphed below.



55. Find the inequality whose solution is graphed below.



56. Find the inequality whose solution is graphed below.



EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Find the solutions of the following equations.

   a. $|x - 5| = 9$ b. $|8x| = 24$

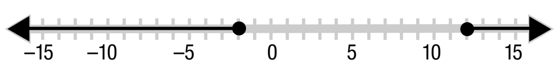
2. Solve for x: $|x + 3| - 8 = 19$

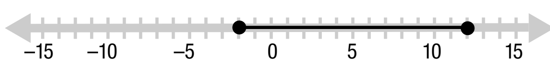
3. Solve for x: $|2x + 5| = |x + 7|$

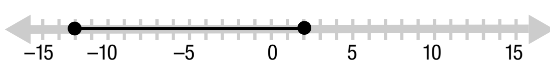
4. Circle the solution of this equation: $5|4x - 7| + 12 = 7$

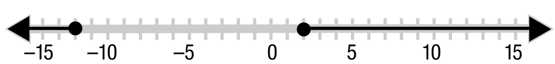
   $x = 2$ or $x = 3$
   $x = -2$ or $x = -3$
   $x = 2$ or $x = -3$
   $x = -2$ or $x = 3$
   the equation has no solution

5. Circle the graph that represents the solution of this inequality: $|x - 5| \leq 7$



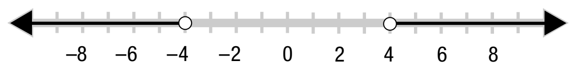






6. Solve for x: $|4x - 6| > 18$

7. Circle the inequality whose solution is graphed below.



   $|x| \geq 4$

   $|x| \leq 4$

   $|x| > 4$

   $|x| \leq 4$

8. Solve for x: $|-4x - 4| < 16$

TOPIC EII CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic. Or you may wish to do these problems to review for a test.

1. Which of the following is **not** a real number:

   $-\frac{3}{8}$, 0, $\sqrt{(-3)\cdot 5}$, 42, $\sqrt{6}$
2. Factor: $3x^2 + 21xy - x - 7y$
3. Find: $\frac{5x^2y^8}{2z} \cdot \frac{12x^2z}{y^6}$
4. Solve for x: $\frac{x-7}{x-2} - \frac{x}{x+7} = \frac{x+12}{x^2+5x-14}$
5. Solve for x: $\frac{4+3x}{2} > \frac{3}{4}(x - \frac{1}{3}) > \frac{5}{12} + \frac{3}{2}x$
6. Solve for x: $|3x + 7| = 12$
7. Find: $3 + 3\,[2^2 - 2(8 + 1)]$
8. Find the equation of the line through the point (–3, –3) that is perpendicular to the line $y = 4x + 12$. Write your answer in slope-intercept form.
9. Find the reciprocal of $3 \cdot \left(\frac{x+y}{2x^2+4y^3}\right)$.
10. Find the slope m of the line through the points (11, –4) and (–3, 6).
11. Find the coordinates of the points in Figure EII.1.

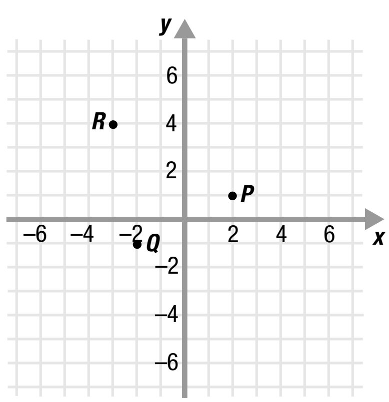


Figure EII.1

12. Solve for x: $\frac{1}{x^2+x-12} = \frac{3}{3x^2+12x}$
13. Solve for x: $\frac{x}{5} = \frac{x}{25} - \frac{1}{30}$
14. Solve for x: $|x| < 11$
15. Find the degree of this polynomial:

    $17x^6y^2 + 3y^4 - 8x^2y + 11x^3y^6 - 4x + 1$
16. Find the point-slope form of the equation of the line through the point (–1, 2) with slope 3.
17. Simplify using the properties of exponents: $\left[\frac{(2x)^4}{(3y)^2}\right]^3 \cdot x^5y^6$
18. Reduce to lowest terms and write using positive exponents: $\frac{45x^{-3}y^{-3}z^{-4}}{9x^5y^{-2}z}$
19. Factor: $y^4 - 49$
20. Find: $\frac{48x^9 - 3x^4}{12x^3}$
21. Solve for x: $y = mx + b$
22. Find: $\frac{x^3+8}{32xyz^2} + \frac{x^2-4}{32xyz^2}$
23. Solve for x: $|x + 2| = |11x - 4|$
24. Factor: $28x^5y^2 - 12x^5y^4 + 20x^4y^3$
25. Solve for z: $-5(4z - 3) > 24 - 9z$
26. Simplify using the properties of exponent: $\frac{x^{17}}{x^{15}}$
27. Find: $\frac{6x^3}{2y^2} + \frac{13y^9}{x^2}$
28. Solve for x: $3|x - 2| \geq 12$
29. Reduce to lowest terms: $\frac{6xy^2 + 4y - 15xy - 10}{3xy^2 + 2y + 24xy + 16}$

30. Find:
$(6a^2 + 7ab + 6ab^2 + b^2 - 3) - (5b^2 - 17 - 4a^2b + 5ab)$

31. Solve for z: $-4 < 2(z - 5) \leq 10$

32. Find: $\dfrac{x^2 + 2x - 15}{x^2 - 4} \div \dfrac{4x + 20}{3x - 6}$

33. Factor: $8x^2 + 2x - 21$

34. Solve for y: $\dfrac{2}{3}(y + 2) = \dfrac{1}{8}y + \dfrac{5}{6}$

35. Find the distance d between the points (1, 3) and (–4, 7).

36. Graph the solution to the inequality $-2|3x - 3| > -6$ on a number line.

37. Simplify: $\dfrac{\frac{x^4}{10}}{\frac{x^2}{5}}$

38. Evaluate when $y = -3$: $-2y^2 - 9y - 9$

39. Rewrite using the distributive property: $7 \cdot (12 + 33)$

40. Factor: $90x^3 - 25x^2 - 35x$

41. Simplify: $\dfrac{-3 + \frac{1}{x}}{-2 - \frac{1}{2x}}$

42. Simplify: $\dfrac{8b}{2a^2 + 5a - 3} - \dfrac{4}{a + 3} + \dfrac{9}{2a - 1}$

43. Find the distance d between the points (1, 15) and (1, –8).

44. Graph the line $4x + y = 12$.

45. Solve for x: $\dfrac{2x}{5} + 7 = \dfrac{8}{3}$

46. Factor: $16a^2 - b^8$

47. Find the equations of the vertical line and the horizontal line that pass through the point (7, 1).

48. Solve for x: $|x| = 99$

49. Find: $(6x - 9)(3x + 5)$

50. Find the x- and y-intercepts of the line $y - 2 = 5(x + 1)$.