LESSON F2.1 – FRACTIONS I

Overview

You have already studied whole numbers and their properties. Now you will study fractions.

In this lesson, you will learn to simplify fractions, determine whether two fractions are equivalent, and order fractions. You will also learn how to apply fractions to some everyday situations.

Before you begin, you may find it helpful to review the following mathematical ideas which will be used in this lesson. To help you review, you may want to work out each example.

To see these Review problems worked out, go to the Overview module of this lesson on the computer.

Review 1

Identify the factors of a whole number.

What are the factors of 20?

Answer: 1, 2, 4, 5, 10, and 20

Review 2

Determine whether a whole number is prime, composite, or neither.

Which of the following is a prime number?

1 2 4 44 100

Answer: 2

Review 3

Find the prime factorization of a whole number.

What is the prime factorization of 60?

Answer: $60 = 2 \times 2 \times 3 \times 5$

Review 4

Solve an equation of the form ax = b, where a, b, and x are whole numbers.

Find the value of x that makes this statement true: $5x = 105$

(Remember, 5x means 5 times x.)

Answer: $x = 21$

Review 5

Order whole numbers, using $<, \leq, >,$ or $\geq$.

Order the whole numbers 110 and 101 using $<, \leq, >,$ or $\geq$.

Answer: $101 < 110$ or $101 \leq 110$ or $110 > 101$ or $110 \geq 101$

Explain

In Concept 1: Equivalent Fractions, you will find a section on each of the following:

- **Finding a Fraction that is Equivalent to a Fraction with a Given Denominator**
- **Determining whether Two Fractions are Equivalent**
- **Finding the Greatest Common Factor (GCF) of Two or More Whole Numbers**
- **Simplifying a Fraction**

Concept 1: Equivalent Fractions

Finding Equivalent Fractions

Fractions can be used to represent many quantities. A fraction can represent

- part of 1 whole
  For example, 3 pieces of an 8 piece pizza can be represented by the fraction $\frac{3}{8}$.

- part of a collection
  For example, 2 pencils out of a collection of 7 pencils can be represented by the fraction $\frac{2}{7}$.

- one number divided by another number
  For example, $2 \div 15$ can be represented by the fraction $\frac{2}{15}$.

In a fraction, the bottom number is called the **denominator**. It is the number of parts that make up the whole.

The top number is called the **numerator**. It is the number of those parts that are selected.

Here are some special examples of fractions.

- Writing 1 as a Fraction
  When the numerator and denominator are the same, the fraction represents 1 whole. $\frac{5}{5} = 5 \div 5 = 1$

- Writing a Whole Number as a Fraction
  When the denominator of a fraction is 1, the fraction represents the same whole number as the numerator. $\frac{5}{1} = 5 \div 1 = 5$

- Writing 0 as a Fraction
  When the numerator of a fraction is zero, the fraction represents zero.
  The denominator can be any number except zero. $\frac{0}{5} = 0 \div 5 = 0$

Here is a caution about fractions:
The denominator of a fraction is never zero, since division by zero is not defined. $\require{cancel}\cancel{\frac{5}{0}}$

This may help you remember which part of a fraction is the numerator and which part is the denominator.

numerator ⟶ 2
denominator ⟶ 3

The ***d****enominator is* ***d****own under the fraction bar.*

Why can't you divide by zero?

A fraction represents one number divided by another number.

For example, $\frac{6}{2}$ *means 6 divided by 2.*

The quotient is 3 because $3 \times 2 = 6$.

$$2\overline{)6}^{\;3} \qquad 3 \times 2 = 6$$

Now try this with zero.

The fraction $\frac{6}{0}$ *seems to mean 6 divided by 0. But, this has no meaning because any number times 0 is 0, not 6.*

$$0\overline{)6}^{\;?} \qquad ? \times 0 \neq 6$$

(Here "?" is any real number.)

There are many fractions that represent the same number.

For example, $\frac{2}{1}$, $\frac{6}{3}$, and $\frac{18}{9}$ represent the number 2.

Two fractions are equivalent if they represent the same number.

Given a fraction, you can find an equivalent fraction with a particular denominator. To do so:
- Decide what to multiply (or divide) the "old" denominator by to get the "new" one.
- Multiply (or divide) the numerator of the "old" fraction by that number to get the numerator of the "new" fraction.

- *If the denominator of the "new" fraction is bigger than the denominator of the "old" fraction, you multiply.*
- *If the denominator of the "new" fraction is smaller than the denominator of the "old" fraction, you divide.*

Example 1

You may find these Examples useful while doing the homework for this section.

1. Find the missing number: $\frac{7}{8} = \frac{?}{48}$

Here, 48, the denominator of the "new" fraction, is bigger than 8, the denominator of the old fraction.

To find the missing number:

- *Decide what to multiply the "old" denominator, 8, by to get the "new" denominator, 48. Since 8 × 6 = 48, multiply by 6.* $\frac{7}{8} = \frac{?}{8 \times \mathbf{6}} = \frac{?}{48}$
- *Multiply the numerator, 7, of the "old" fraction by 6 to get the numerator of the "new" fraction.* $\frac{7}{8} = \frac{7 \times \mathbf{6}}{8 \times \mathbf{6}} = \frac{42}{48}$

So, $\frac{7}{8} = \frac{42}{48}$. *The missing number is 42.*

Example 2

2. What fraction with denominator 12 is equivalent to the fraction $\frac{2}{3}$?

Here, 12, the denominator of the "new" fraction, is bigger than 3, the denominator of the "old" fraction.

To find a fraction with denominator 12 that is equivalent to the fraction $\frac{2}{3}$:

- *Decide what to multiply the "old" denominator, 3, by to get the "new" denominator, 12. Since 3 × 4 = 12, multiply by 4.* $\frac{2}{3} = \frac{?}{3 \times \mathbf{4}} = \frac{?}{12}$
- *Multiply the numerator, 2, of the "old" fraction by 4 to get the numerator of the "new" fraction.* $\frac{2}{3} = \frac{2 \times \mathbf{4}}{3 \times \mathbf{4}} = \frac{8}{12}$

So, $\frac{2}{3} = \frac{8}{12}$.

Example 3

3. Find the missing number: $\frac{12}{15} = \frac{?}{5}$

Here, 5, the denominator of the "new" fraction, is smaller than 15, the denominator of the "old" fraction.

To find the missing number:

- *Decide what to divide the "old" denominator, 15, by to get the "new" denominator, 5. Since 15 ÷ 3 = 5, divide by 3.* $\frac{12}{15} = \frac{?}{15 \div \mathbf{3}} = \frac{?}{5}$
- *Divide the numerator, 12, of the "old" fraction by 3 to get the numerator of the "new" fraction.* $\frac{12}{15} = \frac{12 \div \mathbf{3}}{15 \div \mathbf{3}} = \frac{4}{5}$

So, $\frac{12}{15} = \frac{4}{5}$. *The missing number is 4.*

Example 4

4. What fraction with denominator 8 is equivalent to the fraction $\frac{10}{40}$?

Here, 8, the denominator of the "new" fraction, is smaller than 40, the denominator of the "old" fraction.

To find a fraction with denominator 8 that is equivalent to the fraction $\frac{10}{40}$:

- *Decide what to divide the "old" denominator, 40, by to get the "new" denominator, 8. Since 40 ÷ 5 = 8, divide by 5.*

$$\frac{10}{40} = \frac{?}{40 \div \mathbf{5}} = \frac{?}{8}$$

- *Divide the numerator, 10, of the "old" fraction by 5 to get the numerator of the "new" fraction.*

$$\frac{10}{40} = \frac{10 \div \mathbf{5}}{40 \div \mathbf{5}} = \frac{2}{8}$$

So, $\frac{10}{40} = \frac{2}{8}$.

Determining Whether Two Fractions are Equivalent

Given two fractions, you can check whether they are equivalent.
One method for doing so uses cross multiplication.

In cross multiplication, you multiply the numerator of one fraction by the denominator of another fraction. Thus, each pair of fractions has two cross products.

For example, the pair of fractions $\frac{2}{3}$ and $\frac{16}{24}$ have the cross products 16 × 3 and 2 × 24.

$$\frac{2}{3} \quad \frac{16}{24}$$

16 × 3 2 × 24

When the cross products of two fractions are equal, the two fractions are equivalent.

In the example above, the cross products, 16 × 3 and 2 × 24, both equal 48.

So, $\frac{2}{3}$ and $\frac{16}{24}$ are equivalent fractions.

When their cross products are unequal, two fractions are not equivalent.

You may find these Examples useful while doing the homework for this section.

Example 5

5. Are $\frac{5}{15}$ and $\frac{4}{12}$ equivalent fractions?

To answer the question, you can use cross multiplication:

- *To cross multiply, multiply the numerator of each fraction by the denominator of the other fraction.*
- *The cross products are equal, so the fractions*

 15

 are equivalent.

Is $\frac{5}{15} = \frac{4}{12}$?

4 ×

5 × 12

60 = 60

So, $\frac{5}{15}$ and $\frac{4}{12}$ are equivalent fractions.

Example 6

6. Are $\frac{4}{9}$ and $\frac{2}{3}$ equivalent fractions?

To answer the question, you can use cross multiplication:

- *To cross multiply, multiply the numerator of each fraction by the denominator of the other fraction.*
- *The cross products are not equal, so the fractions are not equivalent.*

$$\text{Is } \frac{4}{9} = \frac{2}{3}\ ?$$

$$2 \times 9 \neq 4 \times 3$$

$$18 \neq 12$$

So, $\frac{4}{9}$ and $\frac{2}{3}$ are not equivalent fractions.

Simplifying Fractions

A fraction is simplified when it is written as an equivalent fraction with a smaller denominator. To simplify a fraction, divide its numerator and denominator by a common factor.

For example, $\frac{12}{15}$ can be simplified (or reduced) to $\frac{4}{5}$ by dividing both 12 and 15 by the common factor 3.

$$\frac{12}{15} = \frac{12 \div 3}{15 \div 3} = \frac{4}{5}$$

Since 3 divides 12 and 15 evenly, it is a factor of both of these numbers.

$12 = 3 \times 4 \qquad 15 = 3 \times 5$

Since 3 is a factor of both 12 and 15, you say 3 is a ***common*** *factor of 12 and 15.*

However, since 1 is the only whole number that divides both 4 and 5 evenly, the fraction $\frac{4}{5}$ can't be simplified further. The fraction $\frac{4}{5}$ is said to be in lowest terms.

A fraction is simplified to lowest terms if it can be written as an equivalent fraction in which the numerator and denominator have no common factors other than 1.

For example, the following three fractions are equivalent fractions. Look at the terms of each fraction.

$\frac{9}{12}$	=	$\frac{6}{8}$	=	$\frac{3}{4}$
The terms are 9 and 12.		The terms are 6 and 8.		The terms are 3 and 4.

$9 = 3 \times 3$, $12 = 3 \times 4$ — So 9 and 12 have a common factor 3.

$6 = 2 \times 3$, $8 = 2 \times 4$ — So 6 and 8 have a common factor 2.

Since 3 and 4 have no common factors other than 1, the fraction $\frac{3}{4}$ is in lowest terms.

The greatest factor that two or more numbers have in common is called their greatest common factor (GCF).

For example, here are the factors of 24 and 30:

24: 1, 2, 3, 4, 6, 8, 12, 24

30: 1, 2, 3, 5, 6, 10, 15, 30

You can see that the greatest factor these two numbers have in common is 6.

To simplify a fraction to lowest terms in one step, divide both the numerator and the denominator by their greatest common factor.

For example, to simplify $\frac{24}{30}$ to lowest terms in one step, divide both the numerator and the denominator by their greatest common factor, 6.

That is, $\frac{24}{30} = \frac{24 \div 6}{30 \div 6} = \frac{4}{5}$.

Here's another way to simplify a fraction to lowest terms:
- Write the numerator as a product of its prime factors.
- Write the denominator as a product of its prime factors.
- Cancel each pair of common factors.
- Multiply the remaining factors of the numerator and multiply the remaining factors of the denominator.

To see how to simplify a fraction to lowest terms using this method, go to Example 9.

You may find these Examples useful while doing the homework for this section.

Example 7

7. Simplify the fraction $\frac{20}{24}$ to lowest terms.

Here's one way to simplify the fraction $\frac{20}{24}$ to lowest terms.

- *Find a common factor of 20 and 24.*
  $20 = 4 \times 5$
  $24 = 4 \times 6$
  4 is a common factor of 20 and 24.
- *Divide the numerator and denominator of the fraction by the common factor.*
  $\frac{20}{24} = \frac{20 \div 4}{24 \div 4} = \frac{5}{6}$

Since the only common factor of 5 and 6 is 1, the fraction $\frac{5}{6}$ is in lowest terms.

So, $\frac{20}{24}$ simplified to lowest terms is $\frac{5}{6}$.

Example 8

8. Simplify the fraction $\frac{120}{150}$ to lowest terms.

Here's one way to simplify the fraction $\frac{120}{150}$ to lowest terms:

- *Find a common factor of 120 and 150.*
  $120 = 10 \times 12$
  $150 = 10 \times 15$
  10 is a common factor of 120 and 150.
- *Divide the numerator and denominator of the fraction by the common factor.*
  $\frac{120}{150} = \frac{120 \div 10}{150 \div 10} = \frac{12}{15}$
- *Since 12 and 15 have a common factor other than 1, find a common factor of 12 and 15.*
  $12 = 3 \times 4$
  $15 = 3 \times 5$
  3 is a common factor of 12 and 15.
- *Divide the numerator and denominator of $\frac{12}{15}$ by the common factor.*
  $\frac{12}{15} = \frac{12 \div 3}{15 \div 3} = \frac{4}{5}$

Since the only common factor of 4 and 5 is 1, the fraction $\frac{4}{5}$ is in lowest terms.

So, $\frac{120}{150}$ simplified to lowest terms is $\frac{4}{5}$.

Example 9

9. Simplify the fraction $\frac{24}{60}$ to lowest terms.

Here's a different way to simplify the fraction $\frac{24}{60}$ to lowest terms: $\frac{24}{60}$

- *Write the numerator as a product of its prime factors.* $= \frac{2 \times 2 \times 2 \times 3}{60}$
- *Write the denominator as a product of its prime factors.* $= \frac{2 \times 2 \times 2 \times 3}{2 \times 2 \times 3 \times 5}$
- *Cancel each pair of common factors.* $\frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times 2 \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}} \times 5}$
- *Multiply the remaining factors of the numerator and multiply the remaining factors of the denominator.* $\frac{2}{5}$

So, $\frac{24}{60}$ simplified to lowest terms is $\frac{2}{5}$.

Example 10

10. Simplify the fraction $\frac{36}{63}$ to lowest terms.

Here's one way to simplify the fraction $\frac{36}{63}$ to lowest terms.

- *Find the greatest common factor of 36 and 63.*
  The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.
  The factors of 63 are 1, 3, 7, 9, 21, and 63.
  The common factors of 36 and 63 are 1, 3, and 9.
  The greatest common factor of 36 and 63 is 9.
- *Divide the numerator and denominator of the fraction by this greatest common factor.* $\frac{36}{63} = \frac{36 \div 9}{63 \div 9} = \frac{4}{7}$

So, $\frac{36}{63}$ simplified to lowest terms is $\frac{4}{7}$.

Finding the Greatest Common Factor (GCF)

You have seen that you can simplify a fraction in one step, if you divide the numerator and denominator by their greatest common factor (GCF).

There are two ways to find the GCF of two numbers.

Method One: List the Factors
- List all the factors of each number.
- Find all of the common factors.
- Select the greatest of the common factors.

Method Two: Prime Factorization
- Find the prime factorization of each number.
- Use a diagram, if necessary, to identify the common prime factors.
- Multiply the common prime factors to get the GCF.

You may find these Examples useful while doing the homework for this section.

Example 11

11. Find the greatest common factor (GCF) of 75 and 125.

Here's one way to use Method One to find the GCF of 75 and 125.

- *List all the factors of each number.* 75: 1, 3, 5, 15, 25, 75
  125: 1, 5, 25, 125
- *Find all their common factors.* 1, 5, 25
- *Select the greatest of the common factors.* 25

So, the GCF of 75 and 125 is 25.

Example 12

12. Find the greatest common factor (GCF) of 75 and 125.

Here's a way to use Method Two to find the GCF of 75 and 125.

- *Find the prime factorization of each number.* $75 = 3 \times 5 \times 5$
  $125 = 5 \times 5 \times 5$
- *Identify the common prime factors.* $75 = 3 \times \textcircled{5} \times \boxed{5}$
  $125 = \textcircled{5} \times \boxed{5} \times 5$
- *Multiply the common prime factors to get the GCF.* $GCF = 5 \times 5 = 25$

So, the GCF of 75 and 125 is 25.

Example 13

13. Find the greatest common factor (GCF) of 50 and 63.

Here's one way to use Method One to find the GCF of 50 and 63.

- *List all the factors of each number.* 50: 1, 2, 5, 10, 25, 50
  63: 1, 3, 7, 9, 21, 63
- *Find all their common factors.* 1
- *Select the greatest of the common factors.* 1

So, the GCF of 50 and 63 is 1.

Example 14

14. Find the GCF of 30, 45, and 75.

Here's one way to use Method One to find the GCF of 30, 45, and 75.

- *List all the factors of each number.* 30: 1, 2, 3, 5, 6, 10, 15, 30
  45: 1, 3, 5, 9, 15, 45
  75: 1, 3, 5, 15, 25, 75
- *Find all their common factors.* 1, 3, 5, 15
- *Select the greatest of the common factors.* 15

So, the GCF of 30, 45, and 75 is 15.

15. Find the greatest common factor (GCF) of 18, 60, and 84.

Example 15

Here's one way to use Method Two to find the GCF of 18, 60, and 84.

- *Find the prime factorization of each number.* $18 = 2 \times 3 \times 3$

  $60 = 2 \times 2 \times 3 \times 5$

  $84 = 2 \times 2 \times 3 \times 7$

- *Identify the common prime factors. Use a diagram as in Figure 1.*

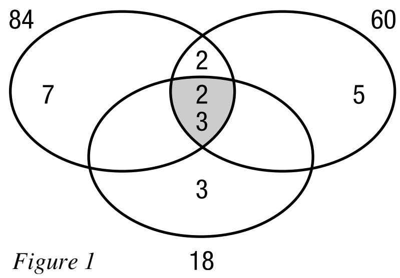

Figure 1

- *Multiply the common prime factors to get the GCF.* $GCF = 2 \times 3 = 6$

So the GCF of 18, 60, and 84 is 6.

Explain

In Concept 2: Multiplying and Dividing, you will find a section on each of the following:

- **Writing a Mixed Numeral as an Improper Fraction**
- **Writing an Improper Fraction as a Mixed Numeral**
- **Multiplying Fractions**
- **Finding the Reciprocal of a Fraction**
- **Dividing Fractions**
- **Finding z in Some Equations that Contain Fractions**

CONCEPT 2: MULTIPLYING AND DIVIDING

Writing a Mixed Numeral as an Improper Fraction

Numbers like $2\frac{1}{2}$ may be written as the sum of a whole number and a fraction less than 1.

For example, $2\frac{1}{2}$ may be written as the sum of 2, a whole number, and $\frac{1}{2}$, a fraction less than 1.

$$2 + \frac{1}{2} = 2\frac{1}{2}$$

These numbers are called mixed numbers.

A fraction that is greater than or equal to 1 is called an improper fraction. In an improper fraction, the numerator is greater than or equal to the denominator.

Examples of improper fractions are $\frac{4}{3}$, $\frac{13}{5}$, 1, 5, and $\frac{18}{3}$.

(A whole number, such as 5, is an improper fraction since 5 can be written as $\frac{5}{1}$.)

To convert a mixed numeral to an improper fraction:
- Multiply the whole number by the denominator of the fraction.
- Add the numerator of the fraction to that product.
- Put this numerator over the same denominator as the original denominator in the mixed numeral.

The result is an improper fraction with the same denominator as the fraction in the mixed numeral.

You may find these Examples useful while doing the homework for this section.

Example 16

16. Write $3\frac{2}{5}$ as an improper fraction.

To write $3\frac{2}{5}$ as an improper fraction:

- *Multiply the whole number by the denominator of the fraction.* $3 \times 5 = 15$
- *Add the numerator of the fraction to 15.* $15 + 2 = 17$
- *Put this numerator over the same denominator as the original denominator in the mixed numeral.* $\frac{17}{5}$

So, $3\frac{2}{5}$ written as an improper fraction is $\frac{17}{5}$.

Example 17

17. Write $4\frac{7}{8}$ as an improper fraction.

To write $4\frac{7}{8}$ as an improper fraction:

- *Multiply the whole number by the denominator of the fraction.* $4 \times 8 = 32$
- *Add the numerator of the fraction to 32.* $32 + 7 = 39$
- *Put this numerator over the same denominator as the original denominator in the mixed numeral.* $\frac{39}{8}$

So, $4\frac{7}{8}$ written as an improper fraction is $\frac{39}{8}$.

Writing an Improper Fraction as a Mixed Numeral

To write an improper fraction as a mixed numeral:

- Divide the numerator by the denominator.
- Use the division to write the mixed numeral.

You may find these Examples useful while doing the homework for this section.

Example 18

18. Write $\frac{9}{2}$ as a mixed numeral.

To write $\frac{9}{2}$ as a mixed numeral:

- *Divide the numerator by the denominator.* $\quad 2\overline{)9}$ quotient **4**, $\underline{8}$, remainder **1**
- *Use the division to write the mixed numeral.* $\quad 4\frac{1}{2}$

So, $\frac{9}{2}$ written as a mixed number is $4\frac{1}{2}$.

The whole number part of the mixed numeral is the whole number part of the quotient. The fraction part of the mixed numeral is given by $\frac{\text{remainder}}{\text{denominator}}$.

Example 19

19. Write $\frac{13}{5}$ as a mixed numeral.

To write $\frac{13}{5}$ as a mixed numeral:

- *Divide the numerator by the denominator.* $\quad 5\overline{)13}$ quotient **2**, $\underline{10}$, remainder **3**
- *Use the division to write the mixed numeral.* $\quad 2\frac{3}{5}$

So, $\frac{13}{5}$ written as a mixed number is $2\frac{3}{5}$.

Multiplying Fractions

To multiply fractions:

- Multiply their numerators.
- Multiply their denominators.

In general: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

Remember, the denominators b and d cannot be zero.

To multiply mixed numerals:

- First, write the mixed numerals as improper fractions.
- Then, multiply as above.

When you multiply fractions or mixed numerals, you may need to simplify the product.

For example, to find this product: $\quad \frac{1}{2} \times \frac{4}{5}$

- Multiply the numerators.
- Multiply the denominators.

$= \frac{1 \times 4}{2 \times 5}$

$= \frac{4}{10}$

- Simplify by factoring and canceling the common factor 2 from the numerator and the denominator. $\quad \frac{\overset{1}{\cancel{2}} \times 2}{\underset{1}{\cancel{2}} \times 5}$
- Finish multiplying. $\quad \frac{1 \times 2}{1 \times 5} = \frac{2}{5}$

So, $\frac{1}{2} \times \frac{4}{5} = \frac{2}{5}$.

In the previous example, you could have simplified before you finished multiplying. Here are the steps to use:

- Multiply the numerators.
- Multiply the denominators.
- Factor the numerator and denominator into primes.
- Cancel factors common to the numerator and denominator.
- Finish multiplying.

For an example of multiplying using these steps, go to Example 23.

You may find these Examples useful while doing the homework for this section.

Example 20

20. Find $\frac{1}{3} \times \frac{1}{4}$.

To find this product: $\frac{1}{3} \times \frac{1}{4}$

- *Multiply the numerators.*
- *Multiply the denominators.*

$= \frac{1 \times 1}{3 \times 4}$

- *The result is this fraction.* $= \frac{1}{12}$

So, $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$.

Example 21

21. Find $\frac{1}{2}$ of $\frac{5}{6}$.

Remember, in mathematics, the word "of" often means multiply. So you're looking for this product: $\frac{1}{2} \times \frac{5}{6}$

- *Multiply the numerators.*
- *Multiply the denominators.*

$= \frac{1 \times 5}{2 \times 6}$

- *The result is this fraction.* $= \frac{5}{12}$

So, $\frac{1}{2}$ *of* $\frac{5}{6}$ *is* $\frac{5}{12}$.

Here's how to use a rectangle to picture $\frac{1}{2}$ *of* $\frac{5}{6}$.

Cut a rectangle into sixths.
Shade 5 of the 6 strips to show $\frac{5}{6}$. *See Figure 2.*

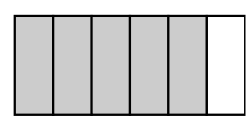

Figure 2

Now cut this rectangle in half horizontally. See Figure 3.
Shade 1 of the 2 horizontal strips to show $\frac{1}{2}$.

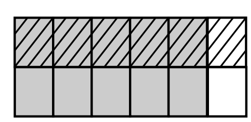

Figure 3

The rectangle is divided into 12 equal parts. 5 of these 12 parts have this shading: ▨

So, $\frac{1}{2}$ *of* $\frac{5}{6}$ *is* $\frac{5}{12}$.

Example 22

22. Find: $\frac{5}{6} \times \frac{3}{4}$.

Here's one way to find this product: $\frac{5}{6} \times \frac{3}{4}$

- *Multiply the numerators.*
- *Multiply the denominators.*

$= \frac{5 \times 3}{6 \times 4}$

- *The result is this fraction.* $= \frac{15}{24}$

Simplify the result.

- *Factor the numerator and denominator into prime factors.* $\frac{15}{24} = \frac{3 \times 5}{2 \times 2 \times 2 \times 3}$
- *Divide the numerator and denominator by each common prime factor.* $\frac{15}{24} = \frac{\overset{1}{\cancel{3}} \times 5}{2 \times 2 \times 2 \times \underset{1}{\cancel{3}}}$
- *Multiply.* $\frac{15}{24} = \frac{5}{8}$

So, $\frac{5}{6} \times \frac{3}{4} = \frac{5}{8}$.

You could have also divided the numerators and denominators by common factors before you multiplied. Here's how:

$$\frac{5}{6} \times \frac{3}{4} = \frac{5}{\underset{2}{\cancel{6}}} \times \frac{\overset{1}{\cancel{3}}}{4} = \frac{5 \times 1}{2 \times 4} = \frac{5}{8}$$

Example 23

23. Find $\frac{5}{6}$ of $\frac{9}{10}$.

Since "of" means multiply here, you're looking for this product: $\frac{5}{6} \times \frac{9}{10}$

- *Multiply the numerators.*
- *Multiply the denominators*

$= \frac{5 \times 9}{6 \times 10}$

- *Factor the numerator and denominator into prime factors.* $= \frac{5 \times 3 \times 3}{2 \times 3 \times 2 \times 5}$
- *Divide the numerator and denominator by by each common prime factor.* $= \frac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{3}} \times 3}{2 \times \underset{1}{\cancel{3}} \times 2 \times \underset{1}{\cancel{5}}}$
- *Finish the multiplication.* $= \frac{3}{4}$

So, $\frac{5}{6}$ *of* $\frac{9}{10}$ *is* $\frac{3}{4}$.

You could have also divided the numerators and denominators by common factors before you multiplied. Here's how:

$$\frac{5}{6} \times \frac{9}{10} = \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}} = \frac{1 \times 3}{2 \times 2} = \frac{3}{4}$$

Example 24

24 . Find: $2\frac{1}{10} \times \frac{5}{6}$.

Here's one way to find this product: $2\frac{1}{10} \times \frac{5}{6}$

• *Write the mixed numeral as an improper fraction.* $= \frac{21}{10} \times \frac{5}{6}$

• *Multiply.* $= \frac{21 \times 5}{10 \times 6}$

$= \frac{105}{60}$

• *Simplify* $\frac{105}{60} = \frac{105 \div 15}{60 \div 15}$

$= \frac{7}{4}$

So, $2\frac{1}{10} \times \frac{5}{6} = \frac{7}{4}$.

Finding the Reciprocal of a Fraction

The reciprocal of a fraction is the fraction with the numerator and denominator interchanged.

For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

To find the reciprocal of a fraction, just switch its numerator and denominator.

To find the reciprocal of a whole number, first write the whole number as a fraction with denominator 1. Then switch the numerator and denominator.

For example, the reciprocal of 5 is the reciprocal of $\frac{5}{1}$, which is $\frac{1}{5}$.

The number 0 does not have a reciprocal. That's because you cannot divide by zero.

Reciprocals are important because they are used when you divide fractions and when you solve certain equations that contain fractions.

Example 25

25. What is the reciprocal of $\frac{2}{7}$?

To find the reciprocal of $\frac{2}{7}$*, switch its numerator and denominator.* $\frac{2}{7}$

$\frac{7}{2}$

So the reciprocal of $\frac{2}{7}$ *is* $\frac{7}{2}$.

Example 26

26. What is the reciprocal of $\frac{1}{9}$?

To find the reciprocal of $\frac{1}{9}$*, switch its numerator and denominator.* $\frac{1}{9}$

$\frac{9}{1}$

So the reciprocal of $\frac{1}{9}$ *is* $\frac{9}{1}$*. That is 9.*

27. What is the reciprocal of 8? **Example 27**

To find the reciprocal of 8:

- *Write 8 as a fraction.* $\frac{8}{1}$
- *Switch its numerator and denominator.* $\frac{1}{8}$

So the reciprocal of 8 is $\frac{1}{8}$.

28. What is the reciprocal of $\frac{0}{5}$? **Example 28**

To find the reciprocal of $\frac{0}{5}$*, switch its numerator and denominator.* $\frac{0}{5}$

$\frac{5}{0}$

But, division by 0 is undefined.

So the reciprocal of $\frac{0}{5}$ *is undefined.*

Dividing Fractions

To divide one fraction by a second fraction:

- Find the reciprocal of the second fraction.
- Multiply the first fraction by the reciprocal of the second fraction.

To divide mixed numerals:

- First, write the mixed numerals as improper fractions.
- Then divide as above.

Why is multiplying by the reciprocal the same as dividing?

Here's one way to think about it.

Remember that a division problem can be written as a fraction. Here's an example:

$$\frac{2}{3} \div \frac{1}{6} = \frac{\frac{2}{3}}{\frac{1}{6}}$$

To simplify, make the denominator 1.

To do this, multiply the bottom and top by $\frac{6}{1}$, the reciprocal of $\frac{1}{6}$.

$$= \frac{\frac{2}{3} \times \frac{6}{1}}{\frac{1}{6} \times \frac{6}{1}} \qquad \frac{1}{6} \times \frac{6}{1} = \frac{6}{6} = 1$$

$$= \frac{\frac{2}{3} \times \frac{6}{1}}{1}$$

After you divide by 1, you are left with this multiplication problem: $\frac{2}{3} \times \frac{6}{1}$

You may find these Examples useful while doing the homework for this section.

Example 29

29. What is $\frac{3}{5} \div \frac{3}{4}$?

Here's one way to find this quotient: $\frac{3}{5} \div \frac{3}{4}$

- *Find the reciprocal of the second fraction.*
  It's $\frac{4}{3}$.
- *Multiply the first fraction by the reciprocal of the second fraction.*

$$= \frac{3}{5} \times \frac{4}{3}$$
$$= \frac{3 \times 4}{5 \times 3}$$
$$= \frac{12}{15}$$

- *Simplify the answer.* $= \frac{4}{5}$

  So, $\frac{3}{5} \div \frac{3}{4} = \frac{4}{5}$.

Example 30

30. What is $\frac{4}{5} \div \frac{1}{10}$?

Here's one way to find this quotient: $\frac{4}{5} \div \frac{1}{10}$

- *Find the reciprocal of the second fraction.*
  It's $\frac{10}{1}$.
- *Multiply the first fraction by the reciprocal of the second fraction.*

$$= \frac{4}{5} \times \frac{10}{1}$$
$$= \frac{4 \times 10}{5 \times 1}$$
$$= \frac{40}{5}$$

- *Simplify the answer.* $= \frac{8}{1}$

$$= 8$$

So, $\frac{4}{5} \div \frac{1}{10} = 8$.

Rectangles can be used to illustrate this division.

To find $\frac{4}{5} \div \frac{1}{10}$:

- *Draw a rectangle.*

  Cut the rectangle into fifths.

  Shade 4 of the 5 strips to show $\frac{4}{5}$.

  See Figure 4.

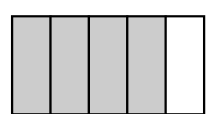

Figure 4

- *Cut the rectangle into tenths.*

  See Figure 5.

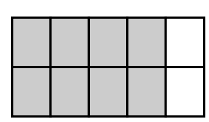

Figure 5

- *Now, count the tenths that are in* $\frac{4}{5}$.

  There are 8.

So, $\frac{4}{5} \div \frac{1}{10} = 8$.

Example 31

31. Find: $4\frac{2}{5} \div 3\frac{3}{10}$.

Here's one way to find this quotient: $4\frac{2}{5} \div 3\frac{3}{10}$

• *Write the mixed numerals as improper fractions.* $= \frac{22}{5} \div \frac{33}{10}$

• *Divide by multiplying the first fraction by the reciprocal of the second fraction.* $= \frac{22}{5} \times \frac{10}{33}$

$= \frac{22 \times 10}{5 \times 33}$

$= \frac{220}{165}$

• *Simplify.* $\frac{220}{165} = \frac{220 \div 55}{165 \div 55}$

$= \frac{4}{3}$

So, $4\frac{2}{5} \div 3\frac{3}{10} = \frac{4}{3}$.

Finding z in Some Equations that Contain Fractions

Sometimes the letter "*z*" is used to represent an unknown quantity, and you may be asked to figure out the value of *z*.

To find the value of *z* in an equation of the form $az = b$, get *z* by itself on one side of the equation:

- Multiply both sides of the equation by the reciprocal of *a*.
- Simplify both sides of the equation.

Another way to get *z* by itself on one side of the equation is to divide both sides of the equation by *a*.

Here, a is not zero.

Example 32

You may find these Examples useful while doing the homework for this section.

32. Find z: $\frac{4}{5} \times z = 12$

Here's a way to find z: $\frac{4}{5} \times z = 12$

• *Multiply both sides of the equation by* $\frac{5}{4}$*, the reciprocal of* $\frac{4}{5}$. $\frac{5}{4} \times \frac{4}{5} \times z = \frac{5}{4} \times 12$

• *Simplify both sides of the equation.*

A fraction times its reciprocal is 1. $1 \times z = \frac{5}{4} \times 12$

1 times z is z. Now z is by itself. $z = \frac{5}{4} \times 12$

To finish finding z, multiply. $z = \frac{5}{4} \times \frac{12}{1}$

$z = \frac{5 \times 12}{4 \times 1}$

Simplify. $z = \frac{60}{4}$

$z = 15$

So, $z = 15$.

You can check that if $z = 15$*, then* $\frac{4}{5} \times z = 12$.

Example 33

33. Find z: $\frac{2}{3} \times z = 6$

Here's a way to find z: $\frac{2}{3} \times z = 6$

- *Divide both sides of the equation by* $\frac{2}{3}$. $\frac{\frac{2}{3} \times z}{\frac{2}{3}} = \frac{6}{\frac{2}{3}}$
- *Simplify both sides of the equation.*

Cancel the $\frac{2}{3}$*'s on the left side.* $\frac{\frac{\not{2}^{1}}{\not{3}} \times z}{\frac{\not{2}}{\not{3}}_{1}} = \frac{6}{\frac{2}{3}}$

Use ÷ on the right side. $z = 6 \div \frac{2}{3}$

To finish finding z, divide. $z = \frac{6}{1} \times \frac{3}{2}$

$z = \frac{6 \times 3}{1 \times 2}$

Simplify. $z = \frac{18}{2}$

$z = 9$

So, $z = 9$.

You can check that if $z = 9$*, then* $\frac{2}{3} \times z = 6$.

Example 34

34. Find z: $\frac{7}{4} \times z = 21$

Here's a way to find z: $\frac{7}{4} \times z = 21$

- *Multiply both sides of the equation by* $\frac{4}{7}$*, the reciprocal of* $\frac{7}{4}$. $\frac{4}{7} \times \frac{7}{4} \times z = \frac{4}{7} \times 21$
- *Simplify both sides of the equation.*

A fraction times its reciprocal is 1. $1 \times z = \frac{4}{7} \times 21$

1 times z is z. Now z is by itself. $z = \frac{4}{7} \times 21$

To finish finding z, multiply. $z = \frac{4}{7} \times \frac{21}{1}$

$z = \frac{4 \times 21}{7 \times 1}$

Simplify. $z = \frac{84}{7}$

$= 12$

So, $z = 12$.

You can check that if $z = 12$*, then* $\frac{7}{4} \times z = 21$.

Explore

This Explore contains two investigations.

- ***Data Analysis***
- ***Fractions and Triangles***

You have been introduced to these investigations in the Explore module of this lesson on the computer. You can complete them using the information given here.

Investigation 1: Data Analysis

1. Keep track of the time you spend eating, sleeping, attending class, studying, and completing miscellaneous tasks (like showering, cleaning, or shopping) in one 24 hour period. Record your data in the table below.

Activity	Hours Spent on Activity	Fraction of Day (24 hours) Spent on Activity	Fraction in Lowest Terms
eating		$\frac{\quad}{24}$	
sleeping		$\frac{\quad}{24}$	
attending class		$\frac{\quad}{24}$	
miscellaneous tasks		$\frac{\quad}{24}$	

2. The line below is marked in 24 parts. Illustrate the results of your data collection by starting at zero, counting the number of hours you spent eating, and marking a mark at that point. Now, start at this point, count the number of hours you spent sleeping, and make a mark at that point. Continue this procedure for the other activities. Your last mark should be at 24.

3. Now, join the ends of the line to make a circle. Connect the center of the circle with each mark you made on the line. Your picture should look like a pie. The whole pie represents represents your 24 hour day. Each piece of pie represents the fraction of a day you spent on the corresponding activity.

Investigation 2: Fractions and Triangles

You will need graph paper for this investigation.

1. Draw a triangle on a piece of graph paper so that the shortest side is 1 unit long and one of the other sides is 2 units long. See Figure 6.

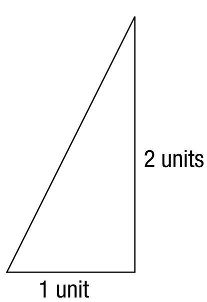

Figure 6

2. Write the fraction that expresses the relationship between the two sides.

$$\frac{\text{length of long side}}{\text{length of short side}} = \underline{\qquad\qquad}$$

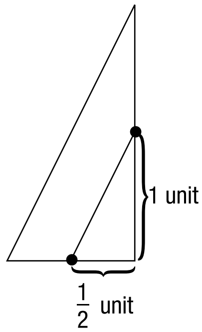

Figure 7

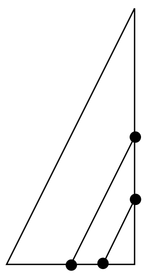
Figure 8

3. Now, draw a new triangle by joining the midpoints (halfway points) of the two sides. That is, draw a triangle so that the shortest side is $\frac{1}{2}$ unit and the other side is 1 unit. See Figure 7.
4. Write the fraction that expresses the relationship between the two sides of the new triangle.

   $\frac{\text{length of long side}}{\text{length of short side}}$ = ____________

5. Simplify the fraction in question 4.
6. Now draw a third triangle by joining the midpoints (halfway points) of the two sides of the second triangle. See Figure 8.
7. Write the fraction that expresses the relationship between the two sides of the new triangle.

   $\frac{\text{length of long side}}{\text{length of short side}}$ = ____________

8. Simplify the fraction in question 7.
9. If you were to keep drawing triangles in this way, what do you think the relationship between sides would be for the 6th triangle? 10th triangle? 100th triangle? You may want to draw a few more triangles to help you see the pattern.

   6th triangle: _______

   10th triangle: _______

   100th triangle: _______

10. Record the lengths of the shortest side in each triangle you drew in the space provided below.

    Length of shortest side:

    triangle 1: _______

    triangle 2: _______

    triangle 3: _______

    triangle 4: _______

11. Predict the length of the shortest side in the seventh triangle.
12. If the length of the shortest side of the twelfth triangle is $\frac{1}{2048}$, what is the length of the shortest side of the thirteenth triangle?

Homework

CONCEPT 1: EQUIVALENT FRACTIONS

Finding Equivalent Fractions

For help working these types of problems, go back to Examples 1–4 in the Explain section of this lesson.

1. Find the missing number: $\frac{4}{5} = \frac{?}{25}$
2. Find the missing number: $\frac{3}{5} = \frac{?}{35}$
3. Find the missing number: $\frac{3}{7} = \frac{?}{42}$
4. Find the missing number: $\frac{5}{6} = \frac{?}{42}$
5. Find the missing number: $\frac{14}{18} = \frac{?}{9}$
6. Find the missing number: $\frac{12}{18} = \frac{?}{6}$
7. Find the missing number: $\frac{27}{81} = \frac{?}{3}$
8. Find the missing number: $\frac{9}{81} = \frac{?}{27}$
9. What fraction with denominator 48 is equivalent to the fraction $\frac{2}{3}$?
10. What fraction with denominator 33 is equivalent to the fraction $\frac{1}{3}$?
11. What fraction with denominator 48 is equivalent to the fraction $\frac{5}{8}$?
12. What fraction with denominator 56 is equivalent to the fraction $\frac{3}{8}$?
13. What fraction with denominator 12 is equivalent to the fraction $\frac{25}{60}$?
14. What fraction with denominator 15 is equivalent to the fraction $\frac{24}{60}$?
15. What fraction with denominator 13 is equivalent to the fraction $\frac{36}{52}$?
16. What fraction with denominator 4 is equivalent to the fraction $\frac{39}{52}$?
17. Shelly had a party and invited 20 people. Only 15 people came to the party. The fraction of people who came to the party is $\frac{15}{20}$. What fraction with denominator 4 is equivalent to the fraction $\frac{15}{20}$?
18. Ron and Melissa are getting married. They've invited 200 people, but only expect 125 people to attend. The fraction of people expected to attend the wedding is $\frac{125}{200}$. What fraction with denominator 8 is equivalent to the fraction $\frac{125}{200}$?
19. In a prealgebra class of 18 students there are 2 men and 16 women. The fraction of men in the class is $\frac{2}{18}$. What fraction with denominator 9 is equivalent to the fraction $\frac{2}{18}$?

20. In a prealgebra class of 18 students there are 2 men and 16 women. The fraction of women in the class is $\frac{16}{18}$. What fraction with denominator 9 is equivalent to the fraction $\frac{16}{18}$?

21. Kelsey has a closet full of clothes. If 2 out of every 3 outfits in the closet are blue, how many outfits are blue if there are a total of 24 outfits? (Hint: To answer the question, find the numerator of the fraction with denominator 24 that is equivalent to the fraction $\frac{2}{3}$.)

22. Leslie has a closet full of clothes. If 1 out of every 5 outfits in the closet are red, how many outfits are red if there are a total of 30 outfits? (Hint: To answer the question, find the numerator of the fraction with denominator 30 that is equivalent to the fraction $\frac{1}{5}$.)

23. Casey has planted a small orchard of peach and apricot trees. For every peach tree, he planted three apricot trees. The fraction of trees he planted that are apricot trees is $\frac{3}{4}$. How many apricot trees did he plant if he planted a total of 40 trees? (Hint: To answer the question, find the numerator of the fraction with denominator 40 that is equivalent to the fraction $\frac{3}{4}$.)

24. Casey has planted a small orchard of peach and apricot trees. For every peach tree, he planted three apricot trees. The fraction of trees he planted that are peach trees is $\frac{1}{4}$. How many peach trees did he plant if he planted a total of 40 trees? (Hint: To answer the question, find the numerator of the fraction with denominator 40 that is equivalent to the fraction $\frac{1}{4}$.)

Determining Whether Two Fractions are Equivalent

For help working these types of problems, go back a few pages to Examples 5–6 in the Explain section of this lesson.

25. Are the fractions $\frac{6}{10}$ and $\frac{21}{35}$ equivalent fractions?
26. Are the fractions $\frac{4}{10}$ and $\frac{6}{15}$ equivalent fractions?
27. Are the fractions $\frac{28}{49}$ and $\frac{12}{21}$ equivalent fractions?
28. Are the fractions $\frac{3}{5}$ and $\frac{7}{12}$ equivalent fractions?
29. Are the fractions $\frac{16}{17}$ and $\frac{9}{10}$ equivalent fractions?
30. Are the fractions $\frac{9}{12}$ and $\frac{3}{4}$ equivalent fractions?
31. Are the fractions $\frac{40}{64}$ and $\frac{35}{56}$ equivalent fractions?
32. Are the fractions $\frac{21}{24}$ and $\frac{49}{56}$ equivalent fractions?
33. Are the fractions $\frac{20}{36}$ and $\frac{11}{16}$ equivalent fractions?
34. Are the fractions $\frac{20}{36}$ and $\frac{25}{45}$ equivalent fractions?
35. Are the fractions $\frac{18}{27}$ and $\frac{22}{33}$ equivalent fractions?
36. Are the fractions $\frac{27}{36}$ and $\frac{21}{33}$ equivalent fractions?
37. Are the fractions $\frac{5}{17}$ and $\frac{15}{34}$ equivalent fractions?
38. Are the fractions $\frac{11}{15}$ and $\frac{30}{34}$ equivalent fractions?
39. Are the fractions $\frac{100}{124}$ and $\frac{124}{136}$ equivalent fractions?
40. Are the fractions $\frac{90}{100}$ and $\frac{18}{20}$ equivalent fractions?

41. Katie and Ali each had the same size chocolate bar. After Katie had eaten $\frac{2}{6}$ of her bar and Ali had eaten $\frac{4}{12}$ of her bar, Katie claimed that they each had the same amount of chocolate left. Is she right? To answer the question, determine whether the fractions $\frac{2}{6}$ and $\frac{4}{12}$ are equivalent fractions.

42. George and Arthur each had a bag of chocolates containing the exact same amount of chocolates. George ate $\frac{13}{39}$ of his bag of chocolates and Arthur ate $\frac{12}{38}$ of his bag of chocolates. Did they eat the same amount of chocolates? To answer the question, determine whether the fractions $\frac{13}{39}$ and $\frac{12}{38}$ are equivalent fractions.

43. Jose and Delilah worked at a gift wrapping station in the mall. They each had the same number of packages to wrap. At the end of an hour, Jose had wrapped $\frac{3}{15}$ of his packages and Dalia had wrapped $\frac{2}{10}$ of hers. Were they wrapping packages at the same rate? To answer the question, determine whether the fractions $\frac{3}{15}$ and $\frac{2}{10}$ are equivalent fractions.

44. Jenny and Carmen worked at a gift wrapping station in the mall. They each had the same number of packages to wrap. At the end of an hour, Jenny had wrapped $\frac{12}{15}$ of her packages and Carmen had wrapped $\frac{8}{10}$ of her packages. Were they wrapping packages at the same rate? To answer the question, determine whether the fractions $\frac{12}{15}$ and $\frac{8}{10}$ are equivalent fractions.

45. During the Fall Semester, 4 out of 20 students dropped a class by the first drop date. During the spring semester, 3 out of 19 students dropped. Was the drop rate the same for each semester? To answer the question, determine whether the fractions $\frac{4}{20}$ and $\frac{3}{19}$ are equivalent fractions.

46. During the Fall Semester, 16 out of 54 students dropped a class by the first drop date. During the spring semester, 12 out of 48 students dropped. Was the drop rate the same for each semester? To answer the question, determine whether the fractions $\frac{16}{54}$ and $\frac{12}{48}$ are equivalent fractions.

47. Jacqueline and Julian's pool needs to have some water added. Jacqueline claims that the pool is $\frac{6}{15}$ full while Julian says that it is $\frac{8}{20}$ full. Are these fractions equivalent?

48. Marina and Cecil are painting their living room. After an hour of painting, Marina claims that the room is $\frac{4}{10}$ complete while Cecil says that it is $\frac{1}{2}$ complete. Are these fractions equivalent?

Simplifying Fractions

For help working these types of problems, go back to Examples 7–10 in the Explain section of this lesson.

49. Simplify the fraction $\frac{45}{60}$ to lowest terms.

50. Simplify the fraction $\frac{15}{36}$ to lowest terms.

51. Simplify the fraction $\frac{114}{126}$ to lowest terms.

52. Simplify the fraction $\frac{27}{54}$ to lowest terms.

53. Simplify the fraction $\frac{44}{80}$ to lowest terms.

54. Simplify the fraction $\frac{18}{42}$ to lowest terms.

55. Simplify the fraction $\frac{9}{72}$ to lowest terms.

56. Simplify the fraction $\frac{28}{84}$ to lowest terms.

57. Simplify the fraction $\frac{65}{115}$ to lowest terms.

58. Simplify the fraction $\frac{126}{144}$ to lowest terms.

59. Simplify the fraction $\frac{32}{80}$ to lowest terms.

60. Simplify the fraction $\frac{1024}{1048}$ to lowest terms.

61. Simplify the fraction $\frac{6}{213}$ to lowest terms.

62. Simplify the fraction $\frac{90}{105}$ to lowest terms.

63. Simplify the fraction $\frac{12}{112}$ to lowest terms.

64. Simplify the fraction $\frac{84}{108}$ to lowest terms.

65. Shauna surveyed 500 people and found that 175 of them go to the movies at least once a month. What fraction of people go to the movies at least once a month? Write the fraction in lowest terms.

66. Jeremy answered 35 out of 40 questions correctly on his history exam. What fraction of questions did he answer correctly? Write the fraction in lowest terms.

67. In one season, Larry made 120 free throws. What fraction of his free throws did he make if he attempted 150 free throws? Write the fraction in lowest terms.

68. In one season, Chuck missed 6 of his point-after-touchdown attempts. What fraction of his point-after-touchdown attempts did he miss if he attempted 48 point-after-touchdowns? Write the fraction in lowest terms.

69. In a class of 36 students, 30 are twenty years of age or older. What fraction of students are twenty years of age or older? Write the fraction in lowest terms.

70. An ice-cream shop offers 36 different flavors. If 24 of the flavors contain chocolate, what fraction of the flavors contain chocolate? Write the fraction in lowest terms.

71. Janelle has a monthly budget of $1500. If she spends $600 on her house payment, what fraction of her budget is designated for housing? Write the fraction in lowest terms.

72. Janelle has a monthly budget of $1500. If she spends $200 on groceries, what fraction of her budget is designated for groceries? Write the fraction in lowest terms.

Finding the Greatest Common Factor (GCF)

For help working these types of problems, go back to Examples 11–15 in the Explain section of this lesson.

73. Find the GCF of 26 and 54.

74. Find the GCF of 42 and 56.

75. Find the GCF of 28 and 42.

76. Find the GCF of 27 and 66.

77. Find the GCF of 75 and 105.

78. Find the GCF of 52 and 78.

79. Find the GCF of 120 and 75.

80. Find the GCF of 132 and 77.

81. Find the GCF of 36, 54, and 108.

82. Find the GCF of 24, 64, and 104.

83. Find the GCF of 48, 63, and 81.

84. Find the GCF of 35, 70, and 100.

85. Find the GCF of 66, 132, and 231.

86. Find the GCF of 48, 72, and 108.

87. Find the GCF of 15, 49, and 95.

88. Find the GCF of 16, 81, and 100.

89. Kyle is working for the state highway department putting up mileage markers. The stretch of road he is working on is straight and has three towns located on it. The second town is 18 miles from the first and the third town is 63 miles from the second. See Figure 9. Kyle wants to keep the distance between each mileage marker the same, wants to place consecutive markers as far apart as possible, and wants to place a mileage marker at each town. How far apart along the road should Kyle place the mileage markers? (Hint: To answer this question, find the GCF of 18 and 63.)

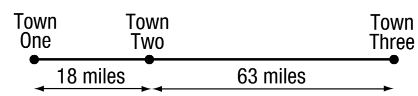

Figure 9

90. Sarah is working for the state highway department putting up mileage markers. The stretch of road she is working on is straight and has three towns located on it. The second town is 36 miles from the first and the third town is 54 miles from the second. See Figure 10. Sarah wants to keep the distance between each mileage marker the same, wants to place consecutive markers as far apart as possible, and wants to place a mileage marker at each town. How far apart along the road should Sarah place the mileage markers? (Hint: To answer this question, find the GCF of 36 and 54.)

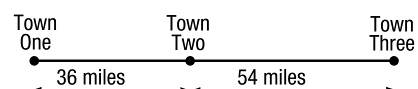

Figure 10

91. Marvin needs to cut as many pieces of rope of equal length as possible from two strands of rope that are 36 feet and 48 feet long. If the pieces need to be as long as possible and no rope can be wasted, how long should each piece be? (Hint: To answer this question, find the GCF of 36 and 48.)

92. Myrtle needs to cut as many pieces of rope of equal length as possible from two strands of rope that are 42 feet and 70 feet long. If the pieces need to be as long as possible and no rope can be wasted, how long should each piece be? (Hint: To answer this question, find the GCF of 42 and 70.

93. Paula is a landscape architect who is installing a sprinkler system on the edge of a rectangular yard that has dimensions 16 feet by 24 feet. She wants to use the least number of sprinkler heads possible. If she wants to place a sprinkler head in each corner of the yard, and if she wants to place each sprinkler head so it is the same distance away from the two sprinkler heads on either side of it, how far apart should she place each sprinkler head? See Figure 11. (Hint: To answer this question, find the GCF of 16 and 24.)

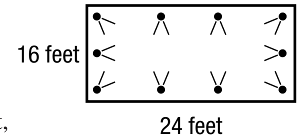

Figure 11

94. Peter is a landscape architect who is installing a sprinkler system on the edge of a rectangular yard that has dimensions 18 feet by 30 feet. He wants to use the least number of sprinkler heads possible. If he wants to place a sprinkler head in each corner of the yard, and if he wants to place each sprinkler head so it is the same distance away from the two sprinkler heads on either side of it, how far apart should he place each sprinkler head? See Figure 12. (Hint: To answer this question, find the GCF of 18 and 30.)

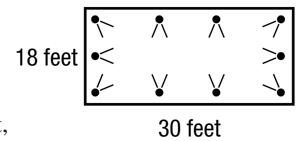

Figure 12

95. Penny works for three people who entrust her with their charity accounts. Once a month, it is Penny's job to decide how much money to give from each charity account to different charities. Penny has strict guidelines to follow. She needs to give the same amount from each account to each charity, and no money can be left in the accounts. If the accounts have \$480, \$520, and \$740 in them, what is the greatest amount she can give to each charity? (Hint: To answer this question, find the GCF of 480, 520, and 740.)

96. Quincy works for three people who entrust him with their charity accounts. Once a month, it is Quincy's job to decide how much money to give from each charity account to different charities. Quincy has strict guidelines to follow. He needs to give the same amount from each account to each charity, and no money can be left in the accounts. If the accounts have \$475, \$525, and \$625 in them, what is the greatest amount he can give to each charity? (Hint: To answer this question, find the GCF of 475, 525, and 625.)

CONCEPT 2: MULTIPLYING AND DIVIDING

Writing a Mixed Numeral as an Improper Fraction

For help working these types of problems, go back to Examples 16–17 in the Explain section of this lesson.

97. Write $4\frac{3}{7}$ as an improper fraction.

98. Write $7\frac{3}{5}$ as an improper fraction.

99. Write $1\frac{13}{15}$ as an improper fraction.

100. Write $1\frac{12}{13}$ as an improper fraction.

101. Write $6\frac{2}{3}$ as an improper fraction.

102. Write $8\frac{2}{5}$ as an improper fraction.

103. Write $21\frac{3}{7}$ as an improper fraction.

104. Write $32\frac{5}{9}$ as an improper fraction.

105. Write $7\frac{11}{35}$ as an improper fraction.

106. Write $8\frac{21}{26}$ as an improper fraction.

107. Write $13\frac{4}{5}$ as an improper fraction.

108. Write $12\frac{3}{8}$ as an improper fraction.

109. Write $9\frac{7}{10}$ as an improper fraction.

110. Write $7\frac{3}{10}$ as an improper fraction.

111. Write $44\frac{5}{8}$ as an improper fraction.

112. Write $75\frac{9}{11}$ as an improper fraction.

113. A recipe calls for $2\frac{1}{4}$ cups of flour. Write $2\frac{1}{4}$ as an improper fraction.

114. A recipe calls for $1\frac{1}{2}$ cups of sugar. Write $1\frac{1}{2}$ as an improper fraction.

115. Jo lives $3\frac{4}{5}$ miles from school. Write $3\frac{4}{5}$ as an improper fraction.

116. Andy lives $11\frac{1}{2}$ miles from the store. Write $11\frac{1}{2}$ as an improper fraction.

117. A certain car requires $4\frac{2}{3}$ quarts of oil. Write $4\frac{2}{3}$ as an improper fraction.

118. A truck uses $1\frac{1}{4}$ gallons of antifreeze. Write $1\frac{1}{4}$ as an improper fraction.

119. The Rostenberger family drinks $3\frac{7}{8}$ gallons of milk in a week. Write $3\frac{7}{8}$ as an improper fraction.

120. The Mathematics Club hosts an annual picnic. They need $15\frac{3}{4}$ gallons of lemonade. Write $15\frac{3}{4}$ as an improper fraction.

Writing an Improper Fraction as a Mixed Numeral

For help working these types of problems, go back to Examples 18–19 in the Explain section of this lesson.

121. Write $\frac{25}{7}$ as a mixed numeral.

122. Write $\frac{37}{8}$ as a mixed numeral.

123. Write $\frac{21}{5}$ as a mixed numeral.

124. Write $\frac{14}{3}$ as a mixed numeral.

125. Write $\frac{19}{4}$ as a mixed numeral.

126. Write $\frac{47}{18}$ as a mixed numeral.

127. Write $\frac{45}{12}$ as a mixed numeral.

128. Write $\frac{65}{15}$ as a mixed numeral.

129. Write $\frac{129}{6}$ as a mixed numeral.

130. Write $\frac{246}{9}$ as a mixed numeral.

131. Write $\frac{369}{23}$ as a mixed numeral.

132. Write $\frac{545}{12}$ as a mixed numeral.

133. Write $\frac{10}{9}$ as a mixed numeral.

134. Write $\frac{17}{16}$ as a mixed numeral.

135. Write $\frac{123}{10}$ as a mixed numeral.

136. Write $\frac{447}{10}$ as a mixed numeral.

137. When Benji works overtime, he is paid $\frac{3}{2}$ of his normal pay. Write $\frac{3}{2}$ as a mixed numeral.

138. When Lisa works overtime, she is paid $\frac{8}{5}$ of her normal pay. Write $\frac{8}{5}$ as a mixed numeral.

139. A sewing pattern calls for a $\frac{5}{4}$ inch hem. Write $\frac{5}{4}$ as a mixed numeral.

140. A sewing pattern calls for a $\frac{11}{8}$ inch hem. Write $\frac{11}{8}$ as a mixed numeral.

141. A trip takes $\frac{15}{4}$ hours. Write $\frac{15}{4}$ as a mixed numeral.

142. A trip takes $\frac{16}{5}$ hours. Write $\frac{16}{5}$ as a mixed numeral.

143. Sophie is $\frac{11}{4}$ feet tall. Write $\frac{11}{4}$ as a mixed numeral.

144. The height of a building is $\frac{75}{4}$ feet. Write $\frac{75}{4}$ as a mixed numeral.

Multiplying Fractions

For help working these types of problems, go back to Examples 20–24 in the Explain section of this lesson.

145. Find: $\frac{3}{5} \times \frac{6}{7}$

146. Find: $\frac{2}{3} \times \frac{8}{9}$

147. Find: $\frac{7}{15} \times \frac{4}{5}$

148. Find: $\frac{9}{10} \times \frac{11}{16}$

149. Find: $\frac{6}{14} \times \frac{21}{32}$

150. Find: $\frac{12}{25} \times \frac{15}{26}$

151. Find: $\frac{24}{49} \times \frac{7}{8}$

152. Find: $\frac{4}{55} \times \frac{105}{8}$

153. Find $2\frac{1}{9}$ of $\frac{18}{95}$

154. Find $3\frac{3}{7}$ of $\frac{21}{84}$

155. Find $5\frac{5}{6}$ of $4\frac{4}{7}$

156. Find $3\frac{3}{8}$ of $11\frac{1}{9}$

157. Find $2\frac{1}{2}$ of $4\frac{4}{5}$

158. Find $2\frac{2}{5}$ of $6\frac{2}{3}$

159. Find $\frac{65}{144}$ of $\frac{24}{45}$

160. Find $\frac{88}{129}$ of $\frac{387}{84}$

161. Helen bought $\frac{2}{3}$ pounds of cheese at \$3 per pound. How much did she pay for the cheese?

162. Art bought $2\frac{1}{4}$ gallons of milk at \$2 per gallon. How much did he pay for the milk?

163. Sasha bought $\frac{3}{8}$ yards of fabric at \$6 a yard. How much did she pay for the fabric?

164. Jose bought $3\frac{3}{4}$ yards of ribbon at \$2 a yard. How much did he pay for the ribbon?

165. A certain stock sells for $15\frac{3}{4}$ dollars per share. How much would 56 shares cost?

166. A certain stock sells for $16\frac{2}{5}$ dollars per share. How much would 50 shares cost?

167. A rectangular yard is $16\frac{1}{2}$ feet wide and $25\frac{1}{3}$ feet long. What is the area of the yard?
(Hint: To find the area of a rectangle, multiply the width by the length.)

168. A desktop is $2\frac{1}{2}$ feet wide by $6\frac{3}{4}$ feet long. What is the area of the desktop?
(Hint: To find the area of the desktop, multiply the width by the length.)

Finding the Reciprocal of a Fraction

For help working these types of problems, go back to Examples 25–28 in the Explain section of this lesson.

169. What is the reciprocal of $\frac{1}{8}$?

170. What is the reciprocal of $\frac{1}{7}$?

171. What is the reciprocal of $\frac{1}{37}$?

172. What is the reciprocal of $\frac{1}{112}$?

173. What is the reciprocal of $\frac{2}{7}$?

174. What is the reciprocal of $\frac{3}{8}$?

175. What is the reciprocal of $\frac{15}{29}$?

176. What is the reciprocal of $\frac{29}{81}$?

177. What is the reciprocal of 6?

178. What is the reciprocal of 3?

179. What is the reciprocal of $\frac{0}{17}$?

180. What is the reciprocal of $\frac{0}{9}$?

181. What is the reciprocal of $\frac{21}{8}$?

182. What is the reciprocal of $\frac{37}{6}$?

183. What is the reciprocal of $\frac{115}{9}$?

184. What is the reciprocal of $\frac{14}{3}$?

185. A 6 foot long board is to be divided into smaller boards that are $\frac{1}{3}$ of a foot long. What is the reciprocal of $\frac{1}{3}$?

186. A 12 foot long board is to be divided into smaller boards that are $\frac{1}{4}$ of a foot long. What is the reciprocal of $\frac{1}{4}$?

187. A $\frac{1}{2}$ gallon container of juice is to be poured into $\frac{1}{8}$ gallon glasses. What is the reciprocal of $\frac{1}{8}$?

188. A 6 gallon container is to be filled using a $\frac{3}{4}$ gallon container. What is the reciprocal of $\frac{3}{4}$?

189. A family is taking a 4 hour trip and stops every $\frac{7}{4}$ hour. What is the reciprocal of $\frac{7}{4}$?

190. A family is taking a 10 hour trip and stops every $\frac{5}{3}$ hour. What is the reciprocal of $\frac{5}{3}$?

191. Knots are tied in a 30 foot piece of rope every $\frac{2}{3}$ feet. What is the reciprocal of $\frac{2}{3}$?

192. Knots are tied in a 48 foot piece of rope every $\frac{12}{5}$ feet. What is the reciprocal of $\frac{12}{5}$?

Dividing Fractions

For help working these types of problems, go back to Examples 29–31 in the Explain section of this lesson.

193. Find: $\frac{3}{5} \div \frac{9}{20}$

194. Find: $\frac{4}{7} \div \frac{16}{35}$

195. Find: $\frac{1}{2} \div \frac{5}{7}$

196. Find: $\frac{3}{4} \div \frac{2}{3}$

197. Find: $\frac{15}{16} \div \frac{45}{96}$

198. Find: $\frac{18}{25} \div \frac{9}{20}$

199. Find: $\frac{3}{7} \div 12$

200. Find: $\frac{4}{9} \div 20$

201. Find: $36 \div \frac{9}{20}$

202. Find: $84 \div \frac{12}{25}$

203. Find: $1\frac{3}{4} \div 2\frac{1}{2}$

204. Find: $6\frac{3}{22} \div 7\frac{1}{11}$

205. Find: $24 \div 1\frac{3}{5}$

206. Find: $75 \div 4\frac{1}{6}$

207. Find: $\frac{4}{5} \div 12$

208. Find: $\frac{7}{8} \div 49$

209. How many $\frac{1}{4}$ foot long pieces of board can be cut from a 12 foot long board? That is, what is $12 \div \frac{1}{4}$?

210. How many $\frac{3}{4}$ foot long pieces of board can be cut from a 15 foot long board? That is, what is $15 \div \frac{3}{4}$?

211. How many $\frac{1}{8}$ gallon glasses can be filled from a $\frac{1}{2}$ gallon container of juice? That is, what is $\frac{1}{2} \div \frac{1}{8}$?

212. A 6 gallon container is to be filled using a $\frac{3}{4}$ gallon container. How many times will the smaller container have to be filled to complete the task? That is, what is $6 \div \frac{3}{4}$?

213. A family is taking a 14 hour trip and stops every $\frac{7}{4}$ hour. How many stops will the family make before reaching its destination? That is, what is $14 \div \frac{7}{4}$?

214. A family is taking a 10 hour trip and stops every $\frac{5}{3}$ hour. How many stops will the family make before reaching its destination? That is, what is $10 \div \frac{5}{3}$?

215. If knots are tied in a rope every $\frac{2}{3}$ feet, how many knots are tied in a 30 foot piece of rope? That is, what is $30 \div \frac{2}{3}$?

216. If knots are tied in a rope every $\frac{12}{5}$ feet, how many knots are tied in a 48 foot piece of rope? That is, what is $48 \div \frac{12}{5}$?

Finding z in Some Equations that Contain Fractions

For help working these types of problems, go back to Examples 32–34 in the Explain section of this lesson.

217. Find z: $\frac{2}{3} \times z = 16$

218. Find z: $\frac{5}{4} \times z = 35$

219. Find z: $\frac{7}{8} \times z = 49$

220. Find z: $\frac{8}{11} \times z = 24$

221. Find z: $\frac{7}{5} \times z = 14$

222. Find z: $\frac{9}{4} \times z = 36$

223. Find z: $\frac{15}{16} \times z = 45$

224. Find z: $\frac{11}{12} \times z = 77$

225. Find z: $\frac{1}{3} \times z = 21$

226. Find z: $\frac{1}{2} \times z = 35$

227. Find z: $\frac{5}{8} \times z = \frac{15}{16}$

228. Find z: $\frac{2}{3} \times z = \frac{8}{27}$

229. Find z: $1\frac{3}{4} \times z = 21$

230. Find z: $2\frac{3}{5} \times z = 39$

231. Find z: $6\frac{2}{5} \times z = 7\frac{1}{9}$

232. Find z: $2\frac{1}{5} \times z = 3\frac{3}{10}$

233. When a number is multiplied by $\frac{6}{15}$ the result is 12. Find the number. (Hint: Let the number be z and find z if $\frac{6}{15} \times z = 12$.)

234. $\frac{4}{3}$ of what number is sixteen? (Hint: Let the number be z and find z if $\frac{4}{3} \times z = 16$.)

235. The area of a rectangle is 45 square feet. Find the length of the rectangle if the width is $5\frac{1}{4}$ feet. (Hint: Area = length × width. So let z be the length and find z if $45 = z \times 5\frac{1}{4}$.)

236. The area of a rectangle is 145 square inches. Find the width of the rectangle if the length is $17\frac{2}{5}$ inches. (Hint: Area = length × width. So let z be the width and find z if $145 = 17\frac{2}{5} \times z$.)

237. Kelvin spends $\frac{1}{5}$ of his monthly budget on food. How much money is in his budget if he spends $200 per month on food? (Hint: Let z be the amount of money in Kelvin's budget and find z if $\frac{1}{5} \times z = 200$.)

238. Kim spends $\frac{2}{5}$ of her monthly budget on her rent. How much money is in her budget if her rent is $400 per month? (Hint: Let z be the amount of money in Kim's budget and find z if $\frac{2}{5} \times z = 400$.)

239. Leslie spends $\frac{2}{3}$ of her free time each week quilting. How much free time does Leslie have in a week if she spends 16 hours quilting? (Hint: Let z be the amount of free time Leslie has and find z if $\frac{2}{3} \times z = 16$.)

240. Jayne spends $\frac{1}{4}$ of her study time each week doing mathematics. How much time does Jayne spend studying in a week if she spends 8 hours doing mathematics? (Hint: Let z be the amount of time Jayne spends studying and find z if $\frac{1}{4} \times z = 8$.)

Evaluate

Take this Practice Test to prepare for the final quiz in the Evaluate module of this lesson on the computer.

Practice Test

1. Fill in the missing numerator that makes the fractions equivalent.

   $\frac{5}{8} = \frac{?}{32}$

2. Choose the fraction that is equivalent to $\frac{2}{7}$.

   $\frac{4}{14}$ $\frac{5}{16}$ $\frac{3}{8}$

3. Find the greatest common factor (GCF) of 42 and 30.

4. Simplify to lowest terms: $\frac{30}{75}$

5. A soup recipe calls for $3\frac{3}{4}$ cups of broth. Write the mixed numeral $3\frac{3}{4}$ as an improper fraction.

6. Do the multiplication below. Write the answer in lowest terms.

   $\frac{1}{2} \times \frac{14}{25}$

7. Do the division below. Write the answer as an improper fraction in lowest terms.

   $4\frac{1}{3} \div \frac{5}{6}$

8. Find the value of z in this equation: $\frac{2}{7} \times z = 12$