9:27
9:38 11m

Test 3
Dusty Wilson
Math 115

No work = no credit

Name: Key

Seeing there is nothing that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers ... I began therefore to consider in my mind by what certain and ready art I might remove those hindrances.

John Napier (1550 - 1670)
Scottish mathematician

Warm-ups (1 pt each) $\log_2(-1) =$ UND $\log_2(0) =$ UND $\log_2(1) =$ 0

1.) (1 pt) According to the quote why did John Napier invent the logarithm? Answer using a complete sentence.

To make math easier.

2.) (6 pts) The US population was 203 million in 1970 and 281 million in 2000. Assume that the population grows exponentially.

a.) Write an exponential model that describes that models the US population.

$m(t) = 203\left(\frac{281}{203}\right)^{t/30}$ in mil, $r = 0.01084$

t in yrs since 1970.

b.) Use your model to predict the population in 2010.

$m(40) \approx 313$ mil in 2010

c.) When does your model predict the population would reach 300 million?

$300 = 203\left(\frac{281}{203}\right)^{t/30} \Rightarrow t = \frac{30\ln(300/203)}{\ln(281/203)} \approx 36$ 1/2

the pop will reach 300 mil in 2006

3.) (2 pt) True or false, $e = \dfrac{128479085}{47264814}$? Explain your answer.

False – e is irrational

4.) (4 pts) Write the equation $7^3 = 343$ in logarithmic form.

Solution: $\log_7(343) = 3$

5.) (4 pts) Evaluate $\log_{1/2}(9)$ to 4 decimal places.

$\frac{\ln(9)}{\ln(1/2)}$

Solution: -3.1699

6.) (4 pts) Write $2\log(x-3)-\frac{1}{2}\log(x)+\log(x+6)$ as a single logarithm.

Solution: $\log\left[\frac{(x-3)^2(x+6)}{\sqrt{x}}\right]$

7.) (6 pts) Find the inverse function of $h(x)=2+(x-3)^2, x\le 3$ and graph $h^{-1}\left[h(x)\right]$.

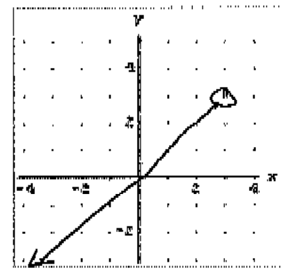
$y=2+(x-3)^2$

$\Rightarrow y-2=(x-3)^2$

$\Rightarrow \pm\sqrt{y-2}=x-3$

$\Rightarrow x=3\pm\sqrt{y-2}$

$\Rightarrow h^{-1}(x)=3-\sqrt{x-2}$



Solution: $h^{-1}(x)=3-\sqrt{x-2}$

8.) (4 pts) Solve $43=2\cdot 5^{-7x}$ exactly.

$\Rightarrow \frac{43}{2}=5^{-7x}$

$\Rightarrow \log_5(43/2)=-7x$

Solution: $x=-\frac{1}{7}\log_5\left(\frac{43}{2}\right)$

9.) (4 pts) Find the domain of $f(x)=\ln\left[(x+3)(4-x)\right]$. Express your answer in set notation.

$(x+3)(4-x)>0$

$-$ $+$ $-$ x

-3 4

Solution: $\{x \mid -3<x<4\}$

10.) (1 pt) Solve $2^x = \log_2(x)$. Hint: what do the graphs look like?

Solution: No solution

11.) (4 pts) Solve $\left(5^x\right)^2 - 2\cdot 5^x - 3 = 0$

$(5^x - 3)(5^x + 1) = 0$

$5^x = 3$ or $5^x = -1$

$x = \log_5 3$ No sol

Solution: $x = \log_5(3)$

12.) (1 pt) Circle the number that is closest to b if the graph of $y = b^x + c$ goes through the points $(1,3)$ and $(-1,7)$? Hint: You do not need to know b or c to answer the question.

-3 **0.5** **2** **Not enough info**

decay

13.) (4 pts) If $\log_b(u) = -6$ and $\log_b(v) = 4$, evaluate the following. Hint: your answer should not include logs.

a.) $\log_b\left(u^2 \cdot v\right)$.

$= 2\log_b u + \log_b v$

$2(-6)+4$

Solution: -8

b.) $\log_b\left(\dfrac{v}{b\cdot u}\right)$.

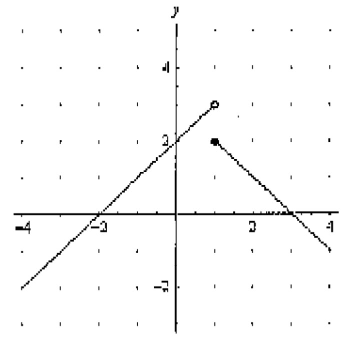
$\log_b v - (\log_b b + \log_b u)$

$4 - (1 + (-6))$

Solution: ~~1~~ 9

14.) (3 pts) Use the graph of the piece-wise defined function h to complete the definition of h (fill in the blanks).

$$h(x)=\begin{cases} x+2, & x<1 \\ 3-x, & x\geq 1 \end{cases}$$



15.) (4 pts) Solve $\log(3-x)+\log(7-x)=\log(32)$ analytically (using algebraic methods).

$\Rightarrow \log[(3-x)(7-x)]=\log(32)$

$\Rightarrow (3-x)(7-x)=32$

$\Rightarrow 21-10x+x^2=32$

$\Rightarrow x^2-10x-11=0$

$\Rightarrow (x-11)(x+1)=0$

$\Rightarrow x=11$ or $x=-1$

extraneous

Solution: $x=-1$

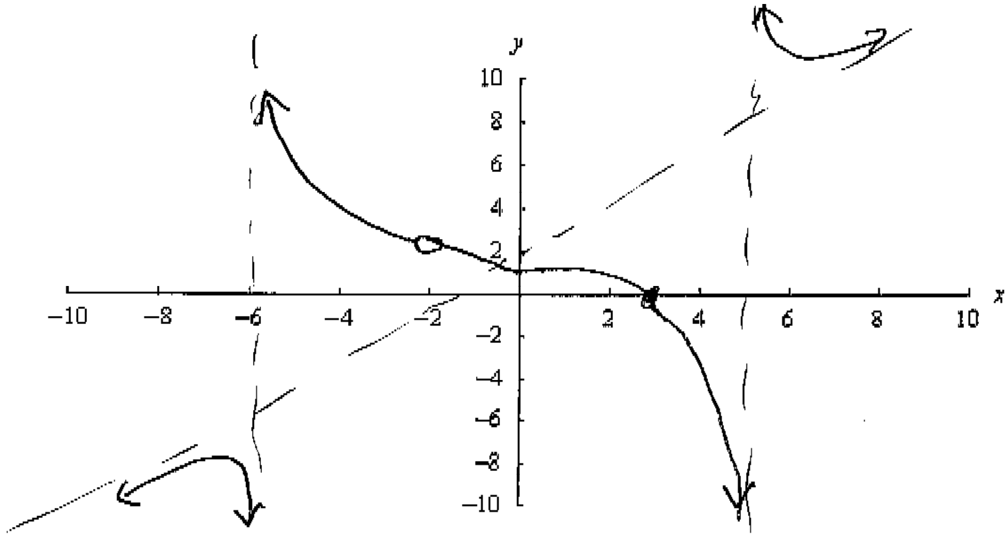
16.) (2 pts) The rational function $g(x)$ has the following characteristics and sign diagram. Use this information to sketch a graph of g.

i.) Vertical asymptotes at $x=-6$ and $x=5$

iii.) An oblique asymptote at $y=x+2$

ii.) A hole at $x=-2$

iv.) A y-intercept at $y=1$



3 min

Extra Credit (Credit will only be given for one bonus question).

17.) (4 pts) Prove that $\log_b(A \cdot B) = \log_b(A) + \log_b(B)$ for $A, B > 0$, $b > 0$, and $b \neq 1$

Let $\log_b A = u$ & $\log_b B = v$

$\Leftrightarrow$ $b^u = A$ AND $b^v = B$

So $\log_b(AB) = \log_b(b^u \cdot b^v)$

$= \log_b(b^{u+v})$

$= u + v$

$= \log_b(A) + \log_b(B)$

18.) (4 pts) Verify that the inverse of $\sinh(x) = \frac{e^x - e^{-x}}{2}$ is $\sinh^{-1}(x) = \ln\left(x + \sqrt{x^2+1}\right)$

$$2y = e^x - \frac{1}{e^x}$$

$$\Rightarrow 0 = (e^x)^2 - 2y(e^x) - 1$$

$$\Rightarrow e^x = \frac{2y \pm \sqrt{4y^2 - 4(1)(-1)}}{2(1)}$$

$$\Rightarrow e^x = \frac{2y \pm \sqrt{4y^2+4}}{2}$$

$$\Rightarrow e^x = y \pm \sqrt{y^2+1}$$

$$\Rightarrow x = \ln\left(y \pm \sqrt{y^2+1}\right)$$

AND $y - \sqrt{y^2+1} < 0$ for $y \in \mathbb{R}$,

so $\sinh^{-1}(x) = \ln\left(x + \sqrt{x^2+1}\right)$.