Test 2
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Math 115

No work = no credit

Name: Key

Of all things, good sense is the most fairly distributed: everyone thinks he is so well supplied with it that even those who are the hardest to satisfy in every other respect never desire more of it than they already have.

Rene Descartes (1596 – 1650)
French philosopher and mathematician

3:11
3:24

Warm-ups (1 pt each) $\frac{1}{0}$ = und $i^2 = -1$ $\frac{0}{1} = 0$

1.) (1 pt) According to the quote (see above), in what way is everyone satisfied? Answer using a complete English sentence.

Everyone is satisfied w/his allotment of good sense,

2.) (2 pts) If $P(x) = 3x^6 - 7x^5 + 4x^4 - x^3 - 4x^2 - 7$, find $P(2)$ using synthetic division (the Remainder Theorem).

(P(2) = -3 if they missed an x)

	3	-7	4	-1	-4	0	-7
2	3	-1	2	3	2	4	1

Solution: P(2) = 1

3.) (2 pts) State the Fundamental Theorem of Algebra.

Every poly P w/ deg n > 0 Ex complex coef. has @ least one complex zero.

4.) (2 pts) List all numbers that could potentially be rational zeros of $P(x) = 3x^3 - x^2 - 8x - 14$ according the rational zeros (root) theorem.

1 3 1, 2, 7, 14

Solution: $\pm 1, \pm 2, \pm 7, \pm 14, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{7}{3}, \pm \frac{14}{3}$

5.) (4 pts) Factor the function $P(x) = 3x^3 - x^2 - 8x - 14$ into the product of linear factors. (< (4.))

	3	-1	-8	-14
$\frac{3}{3}$	3			
$\frac{7}{3}$	3	6	6	0

set to 0 & solve.

$P(x) = (3x+7)(x^2 + 2x + 2)$

$x = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2}$

$= \frac{-2 \pm \sqrt{-4}}{2}$

$\Rightarrow x = -1 \pm i$

Solution: $P(x) = (3x+7)(x-(-1+i))(x-(-1-i))$

6.) (4 pts) Let $f(x)=\sqrt{9-x^2}$ and $g(x)=\sqrt{5-x}$. Find $f(g(x))$.

$f(\sqrt{5-x}) = \sqrt{9-(5-x)}$

$= \sqrt{9-5+x}$

$f(g(x)) =$ $\sqrt{4+x}$

7.) (1 pt) $f(x)$ is a polynomial of degree 4 and $g(x)$ is a polynomial of degree 3. What is the most times that the graphs of f and g can intersect?

Solution: 4

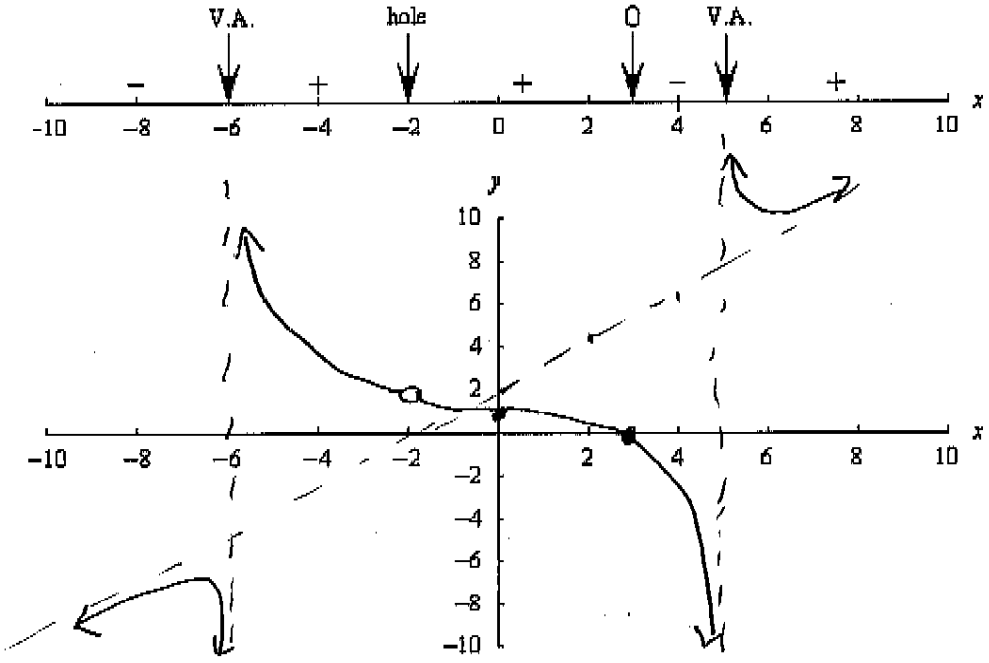
8.) (4 pts) The rational function $g(x)$ has the following characteristics and sign diagram. Use this information to sketch a graph of g.

i.) Vertical asymptotes at $x=-6$ and $x=5$

ii.) A hole at $x=-2$

iii.) An oblique asymptote at $y=x+2$

iv.) A y-intercept at $y=1$

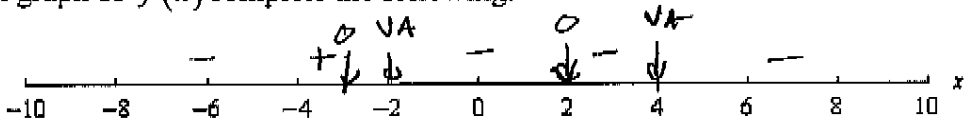


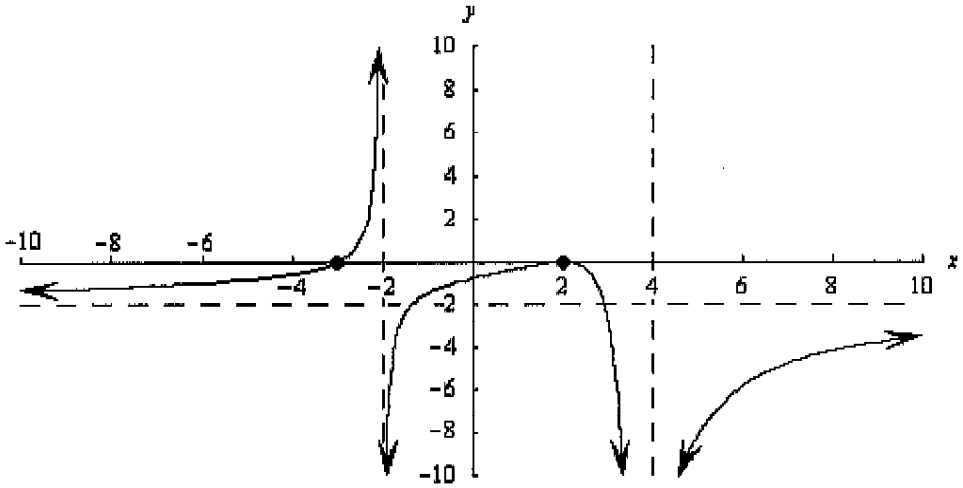
9.) (2 pts) Can a function cross its asymptote(s)? Explain your answer.

A fct can cross a non vertical asymptote.

10.) (6 pts) Use the given graph of $f(x)$ complete the following:

a.) A sign diagram of f





b.) An equation for f that shares the zeros, asymptotes, and has appropriate sign.

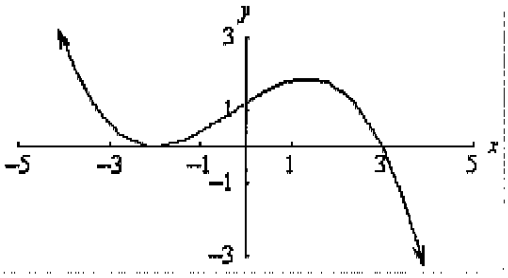
$$f(x) = \frac{-2(x+3)(x-2)^2}{(x+2)(x-4)^2}$$

11.) (4 pts) The following three graphs are of polynomials. Using the graphs, determine whether the degree is even or odd, and whether the leading coefficient is positive or negative. Circle your answer

a.) Graph (a.)

Degree: EVEN (ODD)

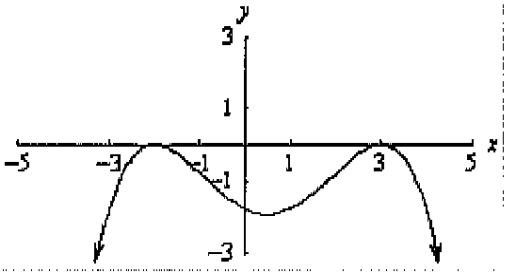
Leading Coefficient: POSITIVE (NEGATIVE)



b.) Graph (b.)

Degree: (EVEN) ODD

Leading Coefficient: POSITIVE (NEGATIVE)



12.) (2 pts) Circle the function(s) that has an oblique (slant) asymptote. Then find the equation of the oblique (slant) asymptote.

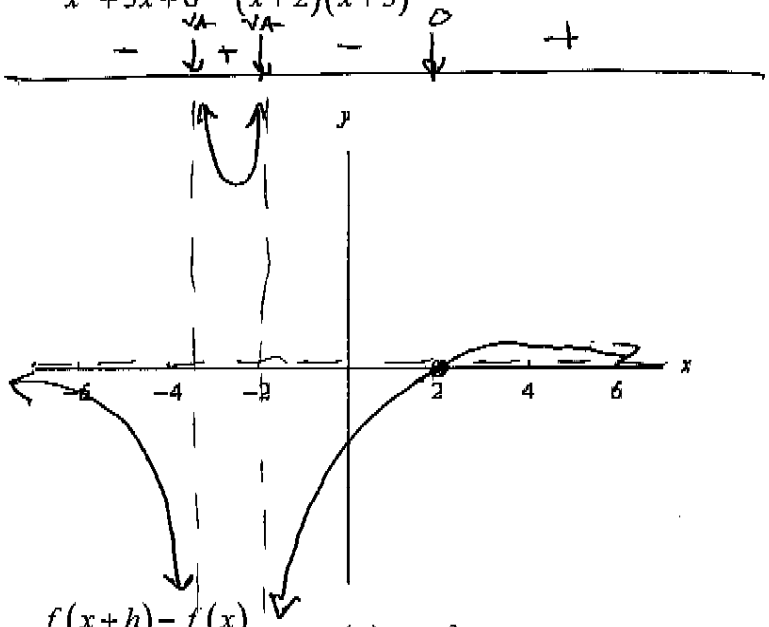
$f(x)=\dfrac{2x^2+6x+8}{3x^2-x-2}$ $\quad$ $g(x)=\dfrac{4x^2-10x-1}{2x-4}$ $\quad$ $h(x)=\dfrac{7x^2}{(2x-3)^2}$

$$2x-4\overline{)\,4x^2-10x-1}\qquad 2x-1$$
$$-(4x^2-8x)$$
$$-2x$$

Solution: $y=2x-1$

13.) (4 pts) Carefully sketch a graph of $q(x)=\dfrac{x-2}{x^2+5x+6}=\dfrac{x-2}{(x+2)(x+3)}$.

NJA @ $y=0$



14.) (4 pts) Calculate the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for $f(x)=4x^2-3$

$$DQ=\frac{4(x+h)^2-3-(4x^2-3)}{h}$$
$$=\frac{4x^2+8xh+4h^2-3-4x^2+3}{h}$$
$$=\frac{8xh+4h^2}{h}$$

Solution: $DQ=8x+4h$

15.) (3 pts) $P(x)$ is a polynomial of odd degree n that has a leading coefficient of -3. Name three unique characteristics of the graph of P (or a fourth for extra credit).

1.) At most n x-int.	3.) At most, $(n-1)$ turning points.
2.) At least 1 x-int.	4.) $\lim_{x\to\infty} P(x) = -\infty$