Test 1
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Math 115

Name: _____KEY_____

Mathematicians have never been in full agreement on their science, though it is said to be the science of self-evident verities -- absolute, indisputable and definitive. They have always been in controversy over developing aspects of mathematics, and they have always considered their own age to be in a period of crisis.

No work = no credit

Henri Léon Lebesgue (1875 - 1941)
French mathematician

Warm-ups (1 pt each) $\dfrac{2}{0} = $ _undd_ $-3^2 = $ _-4_ $(-3)^2 = $ _9_

1.) (1 pt) According to the quote (see above), mathematicians have always considered their own age to be in a __period of crises__ (fill in the blank).

2.) (11 pts) Use the given graph of $f(x)$ to answer the following questions.

a.) $f(-3) = $ _~~0~~ 1_

b.) $f(-5) = $ _undefined_

c.) $f(4) = $ _undefined_.

d.) $f(x) = 3$: when _$x = -2$ OR $x = 0$_

e.) $f(x) = 4$: when _no solution_

f.) $f(f(-2)) = $ _$f(3) = 0$_
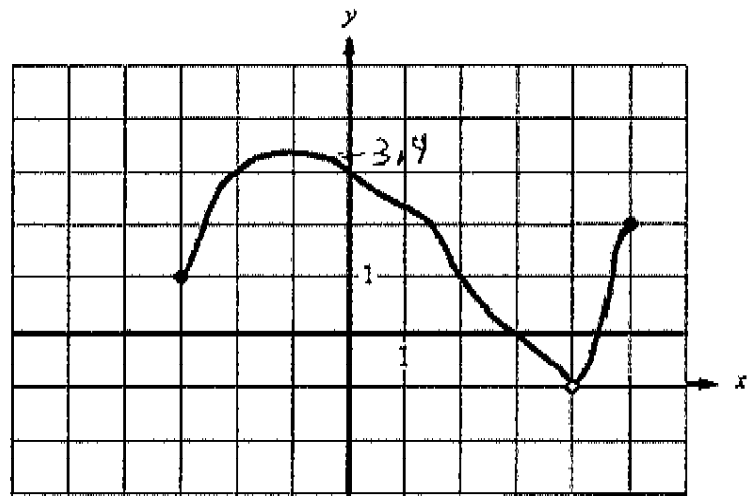
g.) The domain of f (use set notation)

$\{x | -3 \le x < 4 \text{ or } 4 < x \le 5\}$

h.) The range of f (use any notation)

$\{y | -1 < y \le 3, 4\}$

3.) (1 pt) Given some function $f(x)$, how would you determine algebraically whether f is an even function? (What is the test for even functions?)

$f(-x) = f(x)$

4.) (1 pt) Given the graph of some function $g(x)$, how would we determine if g is an odd function? symmetric about the origin

5.) (3 pts) Suppose that $h(x) = f(g(x))$. Complete the following table. If an item cannot be determined from the given information or is undefined, put an asterisk ("*") in the given space.

x	$f(x)$	$g(x)$	$h(x) = f(g(x))$
0	5	2	3
1	7	1	7
2	3	0	5

$h(1) = f(g(1)) = f(1) = 7$

$h(2) = f(g(2)) = f(0) = 5$

$f(2) = f(g(0)) = h(0) = 3$

6.) (4 pts) Express the statement, "The domain of f is the set of all x's such that x does not equal 2" in set notation.

Solution: $\{x \mid x \neq 2\}$

7.) (4 pts) Find the average rate of change of $f(x) = 3x^2 - 7$ on the interval $[a, a+h]$. Show all of your work. Hint, this requires evaluating the difference quotient.

$\text{Ave RoC} = \dfrac{f(a+h) - f(a)}{(a+h) - a}$

$= \dfrac{(3(a+h)^2 - 7) - (3a^2 - 7)}{h}$

$= \dfrac{3a^2 + 6ah + 3h^2 - 7 - 3a^2 + 7}{h}$

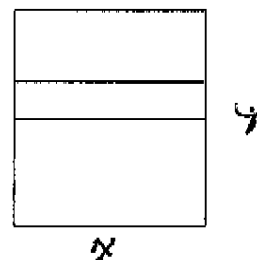$\Rightarrow = \dfrac{6ah + 3h^2}{h}$

Solution: $6a + 3h = \text{Ave Ra}$

8.) (4 pts) If 1200 feet of fence is to be used to enclose 3 adjacent rectangular pens, find the dimensions of the maximum total area that can be enclosed (see the picture)? Express your solution in a complete sentence.

$1200 = 4x + 2y \Rightarrow y = \dfrac{1200 - 4x}{2}$

Area $A = x \cdot y \Rightarrow A(x) = x(600 - 2x)$

zeros @ $x = 0$ & $x = 300$
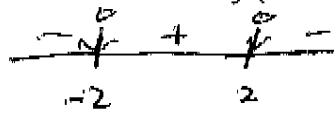
$\Rightarrow$ vertex @ $(150, A(150)) = (150, 45000)$

The rectangle should be 150 ft by 300 ft.

9.) (6 pts) Let $f(x) = \sqrt{4-x^2}$ and $g(x) = \sqrt{7-x}$. Find $f(g(x))$ its domain. Place all answers in the given spaces and use set notation when expressing the domain.

a.) (2 pts) Find the domain of f (the domain of g is given).

$$\frac{- + -}{\underset{-2}{} \quad \underset{2}{}}$$

$D_f =$ $\{x \mid -2 \leq x \leq 2\}$ $D_g = \{x \mid x \leq 7\}$

b.) (2 pts) Find $f(g(x))$.

$$f(g(x)) = f(\sqrt{7-x}) = \sqrt{4 - (\sqrt{7-x})^2}$$

$$= \sqrt{4 - (7-x)}$$

$$= \sqrt{x-3}$$

$f(g(x)) =$ $\sqrt{x-3}$

c.) (2 pts) What is the domain of $f \circ g$?   set notation

$X \leq 7$ AND $X \geq 3$

$D_{f \circ g} =$ $\{x \mid 3 \leq x \leq 7\}$

10.) (7 pts) Consider the graph of the piece-wise defined function h given. Use the graph to complete the following:
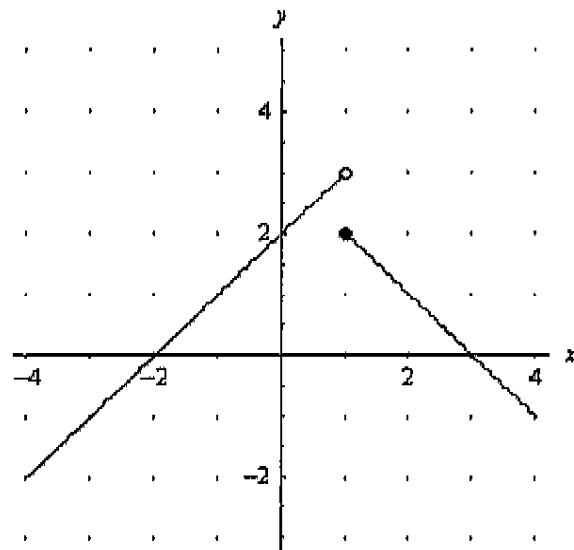
a.) $h(-2) =$ 0

b.) $h(1) =$ 2

c.) The range of h (set notation):

$\{y \mid y < 3\}$

d.) Complete the definition of h (fill in the blanks)

$$h(x) = \begin{cases} x+2 & , \ x < 1 \\ 3-x & , \ x \geq 2 \end{cases}$$

11.) (5 pts) <u>Algebraically</u>, find the intercepts and vertex of the parabola $q(x) = 4x^2 - 12x - 40$.
Circle your results.

$y-\text{int?} \quad q(0) = \underline{-40}$

$x-\text{int}: \quad 0 = 4x^2 - 12x - 40$

$\Rightarrow \quad 0 = x^2 - 3x - 10$

$\Rightarrow \quad 0 = (x-5)(x+2)$

$\Rightarrow \quad \underline{x = -2 \text{ or } x = 5}$

$\text{vertex}: q(x) = 4x^2 - 12x - 4$

$= 4(x^2 - 3x) - 40$

$= 4\left(x^2 - 3x + \dfrac{9}{4} - \dfrac{9}{4}\right) - 40$

$= 4\left(x - \dfrac{3}{2}\right)^2 - 49$

$\Rightarrow \quad \left(\dfrac{3}{2}, -49\right)$

12.) (4 pts) Given the graph of $h(x)$, perform the following:

Graph the following function on the given axes

a.) $h(-x) + 3$

Express the equation of the function graphed in terms of $h(x)$. For example, you might write "$h(x) + 1$."

b.) $\underline{\quad -h\left(\tfrac{1}{2}x\right) - 1 \quad}$