

Group Quiz 5
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 Math 115

Key

Calculators Allowed

Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate.

Leonhard Euler (1707 – 1783)
 Swiss mathematician

1.) If $g(x) = 3 - \sqrt{x+2}$, find and graph $g^{-1}(x)$.

a.) Find $g^{-1}(x)$.

$$y = 3 - \sqrt{x+2}$$

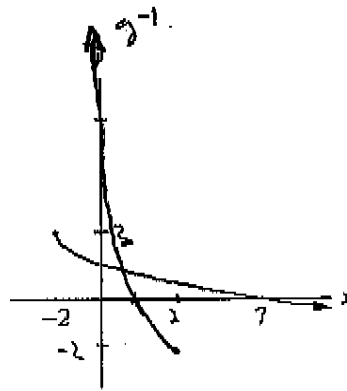
$$\Rightarrow y - 3 = -\sqrt{x+2}$$

$$\Rightarrow (y - 3)^2 = x + 2, \quad y \leq 3 \quad (\text{why?})$$

$$\Rightarrow x = -2 + (y - 3)^2, \quad y \leq 3$$

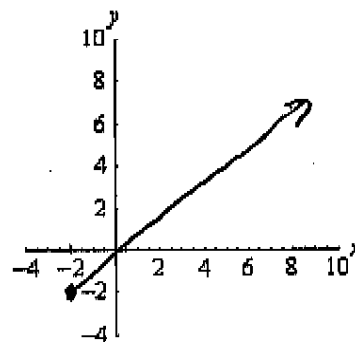
$$\text{AND } g^{-1}(x) = (x - 3)^2 - 2, \quad x \leq 3$$

b.) Carefully graph $g^{-1}(x)$ on the given axes.



c.) Carefully graph $g^{-1}(g(x))$ on the given axes.

$$x > -2$$



2.) The population of aardvarks on an island is limited by the food supply available. The population behaves according to the logistic growth model $n(t) = \frac{500}{1+9e^{-0.1t}}$ where t is measured in months since the introduction of aardvarks to the island. A graph of $n(t)$ is given.

a.) Find and interpret $n(0)$.

The initial aardvark population was fifty.

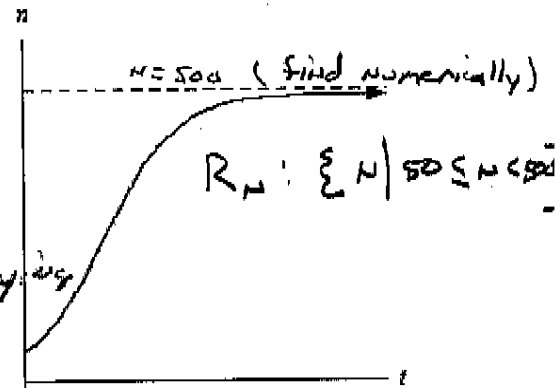
b.) Find and interpret the horizontal asymptote.

The aardvark population grows to 500 and stabilizes @ 500 (island carrying capacity)

c.) State a realistic domain and range for $n(t)$.

$$D_n : \{t \mid 0 \leq t\}$$

$$R_n : \{N \mid 50 \leq N < 500\}$$



d.) The graph shows that in the years immediately after introduction, the aardvark population grew slowly. Use your calculator to estimate when the population grew most rapidly.

The pop grows fastest after about 22 months

e.) Find $n^{-1}(t)$.

$$N = \frac{500}{1+9e^{-0.1t}}$$

$$\Rightarrow \frac{500}{N} = 1 + 9e^{-0.1t}$$

$$\Rightarrow \frac{500}{N} - 1 = 9e^{-0.1t}$$

$$\Rightarrow \frac{1}{9} \left(\frac{500}{N} - 1 \right) = e^{-0.1t}$$

$$\ln \left[\frac{1}{9} \left(\frac{500}{N} - 1 \right) \right] = -0.1t$$

$$\Rightarrow t = -\frac{1}{0.1} \ln \left[\frac{1}{9} \left(\frac{500}{N} - 1 \right) \right]$$

$$\Rightarrow n^{-1}(t) = -10 \ln \left[\frac{1}{9} \left(\frac{500}{N} - 1 \right) \right]$$

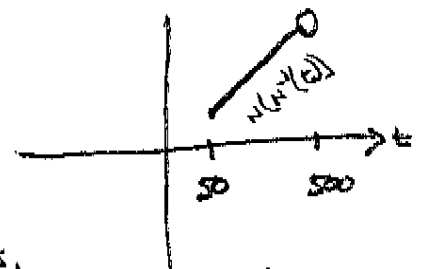
$$D_{n^{-1}} : \{t \mid 50 \leq t < 500\}$$

f.) What is the physical meaning of t in n^{-1} ?

t represents the number of aardvarks on the island

g.) Find and interpret $n^{-1}(491)$.

The pop is 491 after 61.9 months.



h.) (Extra credit) Sketch a graph of $n(n^{-1}(t))$. Draw your own axes and label all important points.

Attach an additional sheet if it is necessary.

