Group Quiz 2
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Math 115

No Calculators

I think therefore I am.

Rene Descartes (1596 – 1650)
French philosopher and mathematician

1.) Find the intercepts and vertex of the function $k(x) = 2x(x-4)+7$ algebraically.

see attached

2.) If $f(x) = \frac{x^2}{x^2-5}$ and $g(x) = \sqrt{x+4}$, find $f[g(x)]$ and its domain $D_{f \circ g}$. Express the domain in set notation.

see attached

3.) A rectangle is inscribed in a right triangle whose height is twice the length of its base. Find the rectangle that maximizes the percentage of the triangle filled by the rectangle. Give your answer as a percent and use a complete sentence.

see attached.

Group Quiz 2 key

Find the intercepts & vertex of $h(x) = 2x(x-4)+7$.
Show all work algebraically.

$h(x) = 2x^2 - 8x + 7$

To find the x-intercepts, solve $0 = 2x^2 - 8x + 7$

$$\Rightarrow \quad x = \frac{8 \pm \sqrt{64 - 4(2)(7)}}{2(2)}$$

$$= \frac{8 \pm \sqrt{8}}{4}$$

$$= \frac{4 \pm \sqrt{2}}{2}$$

The y-intercept is @ $h(0) = 7$.

To find the vertex, complete the square on $h(x)$

$$h(x) = 2(x^2 - 4x) + 7$$

$$= 2(x^2 - 4x + 4 - 4) + 7$$

$$= 2(x^2 - 4x + 4) + 7 - 8$$

$$= 2(x-2)^2 - 1$$

$\Rightarrow$ the vertex is @ $(2, -1)$.

$$f(x) = \frac{x^2}{x^2 - 5} \quad \text{and} \quad g(x) = \sqrt{x+4}$$

a) find $f(g(x))$.

$$f(g(x)) = f(\sqrt{x+4})$$

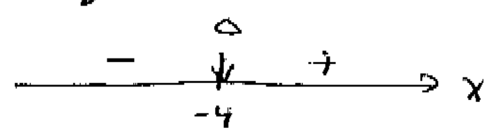
$$= \frac{(\sqrt{x+4})^2}{(\sqrt{x+4})^2 - 5}$$

$$= \frac{x+4}{(x+4) - 5}$$

$$\Rightarrow f(g(x)) = \frac{x+4}{x-1}$$

b) find the domain of $f \circ g$.

(i) $D_{AL} = \{x \mid x \neq 1\}$ (ii) $D_g = \{x \mid x \geq -4\}$

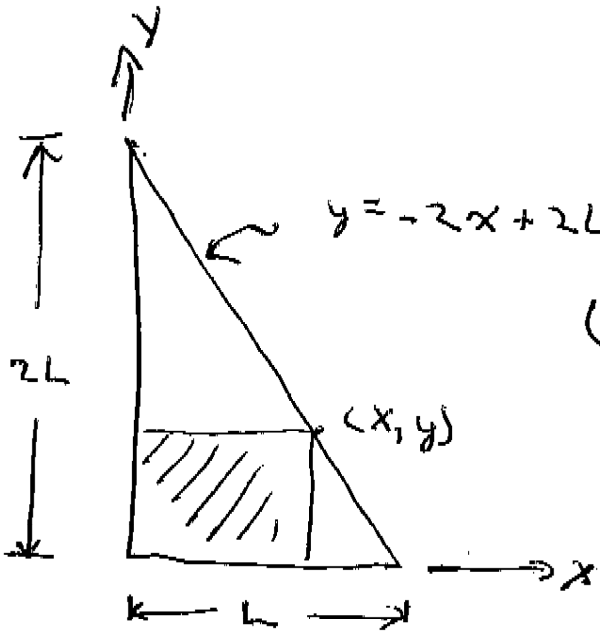


$$\Rightarrow D_{f \circ g} = D_{AL} \cap D_g = \{x \mid x \geq -4 \text{ AND } x \neq 1\}$$

$$= \{x \mid -4 \leq x < 1 \text{ OR } 1 < x\}$$

A rectangle is to be inscribed in a right triangle whose height is twice the length of the base. What is the greatest percentage of the triangle that the rectangle's area can fill?

(1)



(2) $A(x) = x(-2x + 2L)$

$= -2x(x \mp L)$

(3) max at $(x, A(\frac{L}{2}))$

$A(\frac{L}{2}) = -L(-\frac{L}{2}) = \frac{L^2}{2}$

(4) The triangle's total area is $A = \frac{1}{2} b \cdot h = L^2$.

so $\dfrac{\text{Area of rect}}{\text{Area of tri}} = \dfrac{L^2/2}{L^2} = \dfrac{1}{2}$

or the ~~t~~ rectangle's area is 50% that of the triangle