3.5 1/2

3.5: FToA

FToA: Every poly P w/ deg $n>0$ & complex coef has at least one complex zero.

Factor Thm: Every poly w/deg $n>0$ can be written as $P(x) = a\cdot(x-c_1)(x-c_2)\cdots(x-c_n)$ for some $a, c_1, \dots, c_n \in \mathbb{C}$

I) proof
(somewhat inductive using FToA).

ex1: Factor $P(x) = x^3 + 2x^2 + 4x + 8$

ex2: Factor $P(x) = x^3 - x^2 - 2x - 12$

multiplicity.

ex3: Factor $P(x) = 2x^5 + 36x^3 + 162x$

ex4: make up a poly w/ given zeros, etc.

ex5: Factor $P(x) = 8x^4 - 20x^3 - 2x^2 + 23x - 3$
zeros: $x = \frac{3}{2}, -1, \frac{2 \pm \sqrt{3}}{2}$

ex6: Factor $P(x) = 2x^4 - 9x^3 + 15x^2 + 5x - 21$
zeros: $x = \frac{3}{2}, -1, 2 \pm i\sqrt{3}$

$\boxed{\begin{matrix}3.5\\ 2/2\end{matrix}}$

<u>complex conjugate zeros</u>: If the poly P has real coef & if $z = a+bi$ is a zero of P, then $\bar{z} = a-bi$ is also a zero of P.

□ <u>proof</u>.

$P(\bar{z}) = \ldots$

the proof requires that you notice that the coefficients $a_i = \bar{a_i}$ since they are real.

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<u>ex7</u>: Find a poly w/ int. coef & zeros @ $x = -3/2$ & $x = 2-i$

<u>NOTE</u>: Every poly w/ real coef. can be factored into a product of linear & irreducible quad factors.

□ <u>proof</u>.

<u>1st</u>: show that $(x-c)(x-\bar{c})$ has real coef.

<u>2nd</u>: refer to the factor thm.

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<u>ex8</u>: Factor $x^4 - 6x^2 - 16$