3.3: Real zeros of polys

ex1: For observation

$$p(x) = (x+2)(x-3)(x+3)$$
$$= x^3 + 2x^2 - 9x - 18$$

Notice that the zeros: $-2, \pm 3$ are all factors of the constant -18.

ex2: If the poly $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + \dots$ has integer coefficients, then every rational zero has the form

$$x = \frac{p}{q} = \frac{\text{factor of the const } a_0}{\text{factor of the l.c. } a_n}$$

□ proof.

If p/q is a $\mathbb{Q}$ zero, in lowest terms, of poly P, then we have:

$$a_n\left(\frac{p}{q}\right)^n + a_{n-1}\left(\frac{p}{q}\right)^{n-1} + \dots + a_1\left(\frac{p}{q}\right) + a_0 = 0$$

$$\Rightarrow a_n p^n + a_{n-1} p^{n-1} q + \dots + a_1 p q^{n-1} + a_0 q^n = 0$$

$$\Rightarrow q\left(a_{n-1} p^{n-1} + \dots + a_1 p q^{n-2} + a_0 q^{n-1}\right) = -a_n p^n$$

$\Rightarrow$ q is a factor of the LHS. & so must be a factor of the RHS.

$\Rightarrow$ since q & p are relatively prime, q must be a factor of a_n.

The book shows that p is a factor of a_0

QED. ■

$\boxed{\begin{matrix} 3,3 \\ 2/3 \end{matrix}}$

ex2: list all possible rat zeros of $P(x) = 12x^5 + 6x^3 - 2x - 8$

ex3: Find all rat zeros of $P(x) = \cancel{x^4 - 5x^3 + 4}$
$= x^4 - 2x^3 - 3x^2 + 8x - 4$

ex4: Find all zeros of $P(x) = 4x^5 - 18x^4 - 6x^3 + 91x^2 - 60x + $

Note: variation in signs

Descartes' Rule of Signs If P is a poly w/ real coef.

(1) The number of pos. real zeros of P is either equal to the number of variations in sign in $P(x)$ or less by an even whole number.

(2) same as (1), but for neg. zeros & uses the variation in $P(-x)$.

Reflect back on ex 3 & ex 4 in light of Descartes.

Upper & Lower Bounds Thm: P a poly w/ real coef.

(1) Divide P by $(x-b)$, $b > 0$, using syn div. If the quotient row is all non-neg, then b is an U. B. on the real zeros of P.

$\boxed{\begin{array}{c}3,3 \\ 3/3\end{array}}$

(2) same as (1). Divide by $(x-a)$, $a<0$.
If signs alternate (0 is wild), then
a LB of the real zeros.

Reflect on ex3 & ex4 in the light of UB & LBs.

ex5: Find a zeros of $P(x) = 3x^6 - 5x^5 + 2x^4 + 3x^2 - 5x + 2$

ex6: Find the zeros of $P(x) = x^5 + 2x^4 + 0.96x^3 + 5x^2 + 10x + 4.8$