

2.7: Combining Functions

2.7
1/2

(I) Basic Operations

(II) Composition of Functions.

Given two sets f & g , we can combine them in various ways.

EX1: Basic Operations.

If $f(x) = \sqrt{1-x}$ & $g(x) = \sqrt{4+x}$, find the following.

a) Domains of f & g . (use sign diagrams)

b) $f+g$ & D_{f+g}

c) $f-g$ & D_{f-g}

d) fg & D_{fg}

e) $\frac{f}{g}$ & $D_{f/g}$

In general, if f & g are sets w/ domains ~~D_f & D_g~~ D_f & D_g respectively, then:

$$(f \pm g)(x) = f(x) \pm g(x)$$

$$D_{f \pm g} = D_f \cap D_g$$

$$(fg)(x) = f(x)g(x)$$

$$D_{fg} = D_f \cap D_g$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$D_{f/g} = \{x \in D_f \cap D_g \mid g(x) \neq 0\}$$

composition is like a multi-step process

$$\begin{array}{|c|} \hline 2.7 \\ \hline 2/2 \\ \hline \end{array}$$

ex: cutting wood.

NOTE: when are fcts equivalent?

ex: Evaluate $f(f(f(f(f(f(f(f(6))))))))$

$$\text{if } f(x) = \begin{cases} f(x-2), & x > 5 \\ x^2 - 3, & x = 5 \\ x^2 + 1, & x < 5 \end{cases}$$

Handout to follow.

Handout – Combining Functions

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Math 115

Example 1: Algebra of FunctionsSuppose the functions f and g are solely defined by the given table.

x	0	1	2	3	4
f	4	3	0	1	undefined
g	undefined	1	4	3	2

a.) What are the domain and range of f ?

$$D_f = \{0, 1, 2, 3\} \quad \& \quad R_f = \{0, 1, 3, 4\}$$

b.) What are the domain and range of g ?

$$D_g = \{1, 2, 3, 4\} \quad \& \quad R_g = \{1, 2, 3, 4\}$$

c.) Complete the table for the following:

x	0	1	2	3	4
$f+g$	*	4	4	4	*
$f-g$	*	2	-4	-2	*
f/g	*	3	0	3	*
f/g or g/f	*	1/3	*	3	*

d.) What are the domain and range of $f+g$?

$$D_{f+g} = \{1, 2, 3\} \quad \& \quad R_{f+g} = \{4\}$$

e.) What are the domain and range of f/g ?

$$D_{f/g} = \{1, 3\} \quad \& \quad R_{f/g} = \{1/3, 3\}$$

Example 2: Composition of Functionsa.) If f and g are as defined in the previous example, complete the table for the following:

x	0	1	2	3	4
$f \circ g$	$f(g(0)) = f(\text{undefined}) = *$	3	*	1	0
$g \circ f$	$g(f(0)) = g(4) = 2$	3	*	1	*
$f \circ f$	$f(f(0)) = *$	1	4	3	*
$g \circ g$	*	1	2	3	4

b.) Domain restrictions for composition can come from which two places?

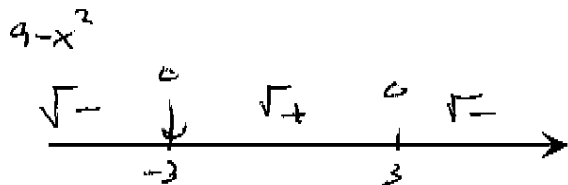
1) the inside function

2) outputs of the inside set are undefined in the outside set.

Example 3

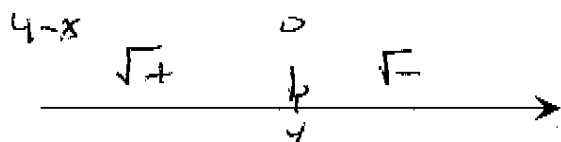
Let $f(x) = \sqrt{9-x^2}$ and $g(x) = \sqrt{4-x}$.

a.) Find the domain of f .



Domain of f : $D_f = \{x \mid -3 \leq x \leq 3\}$

b.) Find the domain of g .



Domain of g : $D_g = x \leq 4$

c.) Create a table of values.

x	-6	-5	-4	3	4	5
$g(x)$	$\sqrt{10}$	3	$\sqrt{8}$	$\sqrt{1}$	0	*
$f(g(x))$	*	0	1	$\sqrt{8}$	3	*

d.) Find $(f \circ g)(x) = f(g(x))$

$$\begin{aligned} f(\sqrt{4-x}) &= \sqrt{9-(\sqrt{4-x})^2} \\ &= \sqrt{9-(4-x)} \\ &= \sqrt{5+x} \end{aligned}$$

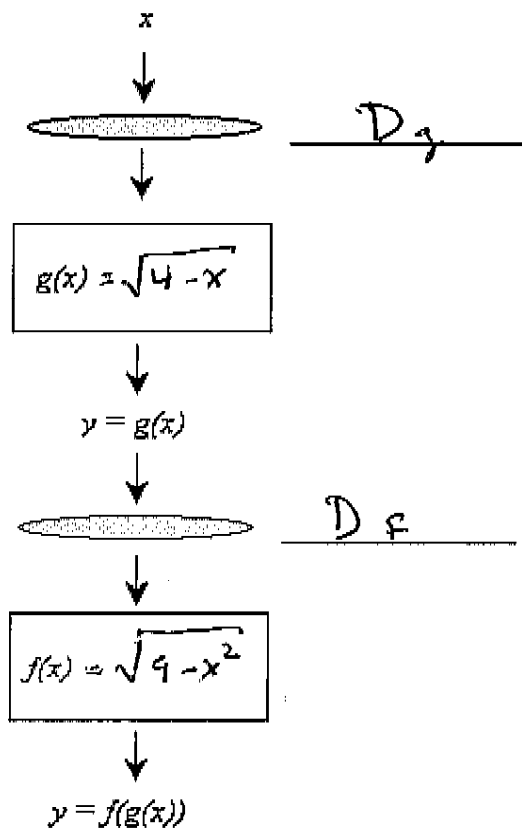
$f(g(x)) = \sqrt{5+x}$

e.) If we call the expression found in (d.) the algebraic composition (AC), find D_{AC} .

Domain of AC : $D_{AC} = \{x \mid x \geq -5\}$

f.) The domain of $f(g(x))$ is $D_g \cap D_{AC}$

g.) Domain of $f(g(x))$: $D_{f \circ g} = \{x \mid -5 \leq x \leq 4\}$



Example 4

Let $f(x) = \frac{x}{x+4}$ and $g(x) = 2-x$. Find $f(g(x))$ and $g(f(x))$ and the domains of each composition.

a.) Find the domain of f and g .

$$D_f = \underline{\{x \mid x \neq -4\}}$$

$$D_g = \underline{\{x \mid x \in \mathbb{R}\}}$$

b.) Find $f(g(x))$.

$$\begin{aligned} f(2-x) &= \frac{2-x}{(2-x)+4} \\ &= \frac{2-x}{6-x} \end{aligned}$$

$$f(g(x)) = \underline{\frac{2-x}{6-x}}$$

c.) What is the domain of the algebraic composition?

$$D_{AC} = \underline{\{x \mid x \neq 6\}}$$

d.) What is $D_{f \circ g}$?

$$D_{f \circ g} = \underline{\{x \mid x \neq 6\}}$$

e.) Find $g(f(x))$.

$$\begin{aligned} g\left(\frac{x}{x+4}\right) &= 2 - \frac{x}{x+4} \\ &= \frac{2x+8-x}{x+4} \end{aligned}$$

$$g(f(x)) = \underline{2 - \frac{x}{x+4} = \frac{x+8}{x+4}}$$

f.) What is the domain of the algebraic composition?

$$D_{AC} = \underline{\{x \mid x \neq -4\}}$$

g.) What is $D_{g \circ f}$?

$$D_{g \circ f} = \underline{\{x \mid x \neq -4\}}$$

Example 5

Let $f(x) = \frac{2x+1}{x-2}$ and $g(x) = \frac{x+2}{x-3}$. Find $f(g(x))$ and $g(f(x))$ and the domains of each composition.

a.) Find the domain of f and g .

$$D_f = \underline{\{x \mid x \neq 2\}}$$

$$D_g = \underline{\{x \mid x \neq 3\}}$$

b.) Find $f(g(x))$.

$$f(g(x)) = f\left(\frac{x+2}{x-3}\right)$$

$$= \frac{2\left(\frac{x+2}{x-3}\right) + 1}{\left(\frac{x+2}{x-3}\right) - 2}$$

$$= \frac{2(x+2) + 1(x-3)}{(x+2) - 2(x-3)}$$

$$f(g(x)) = \underline{\frac{3x+1}{8-x}}$$

c.) What is the domain of the algebraic composition?

$$D_{AC} = \underline{\{x \mid x \neq 8\}}$$

d.) What is $D_{f \circ g}$?

$$D_{f \circ g} = \underline{\{x \mid x \neq 3 \text{ and } x \neq 8\}}$$

e.) Find $g(f(x))$.

$$g(f(x)) = g\left(\frac{2x+1}{x-2}\right)$$

$$= \frac{\left(\frac{2x+1}{x-2}\right) + 2}{\left(\frac{2x+1}{x-2}\right) - 3}$$

$$= \frac{2x+1 + 2(x-2)}{2x+1 - 3(x-2)}$$

$$= \frac{(2x+1) + 2(x-2)}{(2x+1) - 3(x-2)}$$

$$g(f(x)) = \underline{\frac{4x-3}{7-x}}$$

f.) What is the domain of the algebraic composition?

$$D_{AC} = \underline{\{x \mid x \neq 7\}}$$

g.) What is $D_{g \circ f}$?

$$D_{g \circ f} = \underline{\{x \mid x \neq 2 \text{ and } x \neq 7\}}$$

Example 6

Let $f(x) = \sqrt{x^2 + 8}$ and $g(x) = \sqrt{x^2 - 9}$. Find $f(g(x))$ and $g(f(x))$ and the domains of each composition.

a.) Find the domain of f and g .

NOTE: D_f must be written in two parts → why

$D_f = \underline{\mathbb{R}}$

$D_g = \underline{\{x \mid x \leq -3 \text{ OR } x \geq 3\}}$

b.) Find $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(\sqrt{x^2 - 9}) \\ &= \sqrt{(\sqrt{x^2 - 9})^2 + 8} \\ &= \sqrt{x^2 - 9 + 8} \end{aligned}$$

$f(g(x)) = \underline{\sqrt{x^2 - 1}}$

c.) What is the domain of the algebraic composition?

$D_{AC} = \underline{\{x \mid x \leq -1 \text{ OR } x \geq 1\}}$

d.) What is $D_{f \circ g}$?

$D_{f \circ g} = \underline{\{x \mid x \leq -3 \text{ OR } x \geq 3\}}$

e.) Find $g(f(x))$.

$$\begin{aligned} g(f(x)) &= g(\sqrt{x^2 + 8}) \\ &= \sqrt{(\sqrt{x^2 + 8})^2 - 9} \end{aligned}$$

$g(f(x)) = \underline{\sqrt{x^2 - 1}}$

f.) What is the domain of the algebraic composition?

$D_{AC} = \underline{\{x \mid x \leq -1 \text{ OR } x \geq 1\}}$

NOTE: $f(g(x)) \neq g(f(x))$. why?

g.) What is $D_{g \circ f}$?

$D_{g \circ f} = \underline{\{x \mid x \leq -1 \text{ OR } x \geq 1\}}$