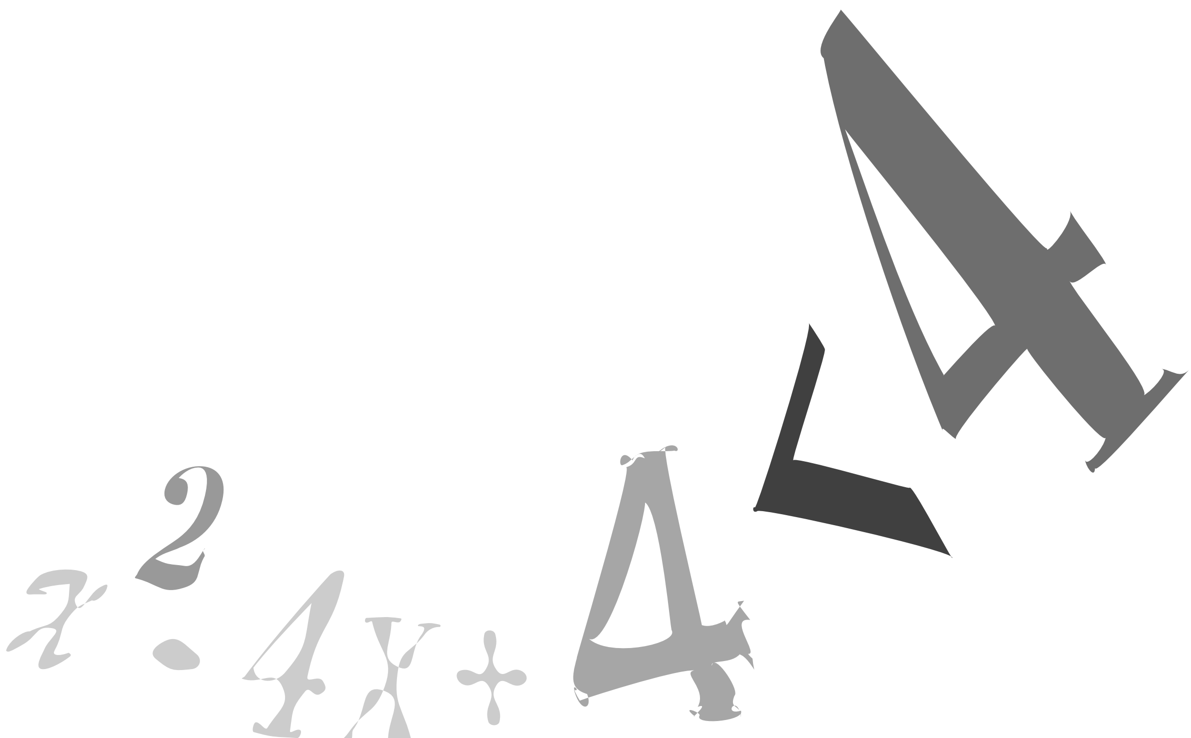
LESSON 13.3 – INEQUALITIES



OVERVIEW

Here's what you'll learn in this lesson:

Quadratic Inequalities

a. Solving quadratic inequalities

Rational Inequalities

a. Definition of a rational inequality

b. Solving rational inequalities

An ice cream company is considering producing several new special-edition flavors, and the members of the market research team are hard at work. They are analyzing each of the potential new flavors, and trying to determine how much of each the company would have to sell to make a profit. They are also trying to estimate the most economical number of production runs the company should make for each flavor.

To do these tasks, the members of the market research team are setting up and solving quadratic and rational inequalities. In this lesson, you will learn how to solve such inequalities.

EXPLAIN

QUADRATIC INEQUALITIES

Summary

Graphing Method

When you graph the function $y = ax + b$, the x-coordinate of the point at which the line crosses the x-axis is the solution of the equation $ax + b = 0$. Similarly, when you graph the function $y = ax^2 + bx + c$, the x-coordinates of the points at which the parabola crosses the x-axis are the solutions of the equation $ax^2 + bx + c = 0$.

You can also use graphing to solve quadratic inequalities.

If you graph the function corresponding to...	The solutions are ...
$ax^2 + bx + c > 0$	the x-coordinates of all the points on the graph that lie **above** the x-axis.
$ax^2 + bx + c \geq 0$	the x-coordinates of all the points on the graph that lie **on and above** the x-axis.
$ax^2 + bx + c < 0$	the x-coordinates of all the points on the graph that lie **below** the x-axis.
$ax^2 + bx + c \leq 0$	the x-coordinates of all the points on the graph that lie **on and below** the x-axis.

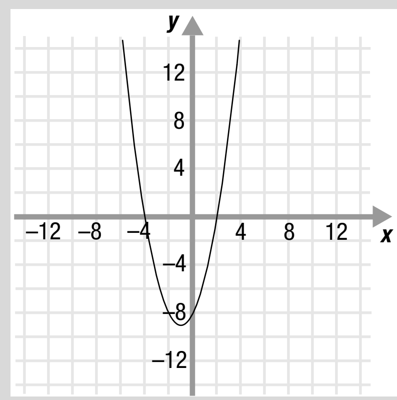


Figure 13.3.1

To find the solutions of a quadratic inequality by graphing:

1. Graph the corresponding parabola, $y = ax^2 + bx + c$.
2. Find the x-coordinates of the appropriate points on the graph, as indicated in the table above.

For example, to solve the inequality $x^2 + 2x - 8 < 0$ by graphing:

1. Graph the parabola $y = x^2 + 2x - 8$. See Figure 13.3.1.
2. Find the x-coordinates of the points on the graph that lie **below** the x-axis. See Figure 13.3.2.

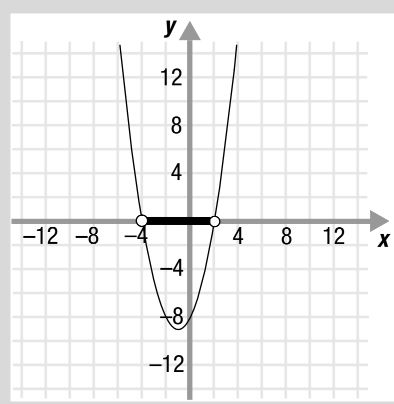


Figure 13.3.2

So $x^2 + 2x - 8 < 0$ when $-4 < x < 2$.

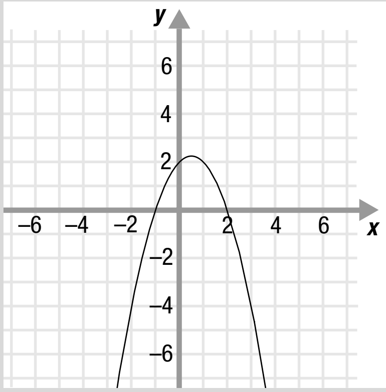


Figure 13.3.3

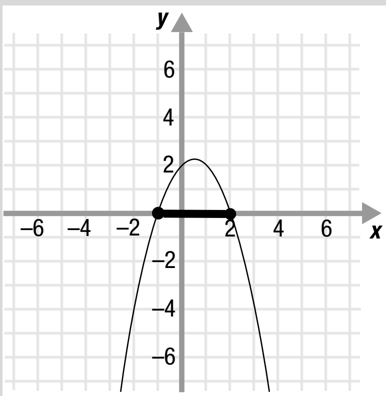


Figure 13.3.4

As another example, to solve the inequality $-x^2 + x + 2 \geq 0$ by graphing:

1. Graph the parabola $y = -x^2 + x + 2$. See Figure 13.3.3.
2. Find the x-coordinates of the points on the graph that lie **on and above** the x-axis. See Figure 13.3.4.

So $-x^2 + x + 2 \geq 0$ when $-1 \leq x \leq 2$.

Test Point Method

When you use graphing to solve a quadratic inequality, you look at the x-coordinates of the points on the graph of the corresponding function. You can also solve a quadratic inequality by finding the sign of the corresponding function at different intervals on the x-axis.

When a quadratic function crosses the x-axis, the value of the function either goes from positive to negative or from negative to positive. A function crosses the x-axis when it is equal to 0, that is, at the solutions of the corresponding equation. So the solutions of the corresponding equation are the dividing points where the function changes sign.

Since the value of a quadratic function is always positive or negative in an interval between dividing points, you can evaluate the function at any "test point" in the interval to find out whether all the values for points in that interval are positive or negative.

To find the solutions of a quadratic inequality without graphing:

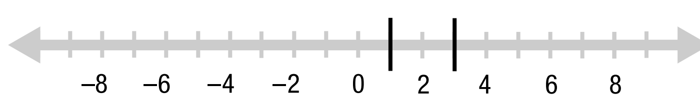
1. Solve the corresponding equation, $ax^2 + bx + c = 0$.
2. Use the solutions to divide the number line into intervals.
3. Substitute a test point from each interval into $ax^2 + bx + c$ to determine its sign.
4. Find the intervals that satisfy the original inequality.

For example, to solve $x^2 - 4x + 3 < 0$ without graphing:

1. Solve the equation $x^2 - 4x + 3 = 0$.

$$x^2 - 4x + 3 = 0$$
$$(x - 1)(x - 3) = 0$$
$$x - 1 = 0 \text{ or } x - 3 = 0$$
$$x = 1 \text{ or } x = 3$$

2. Divide the number line into intervals, using $x = 1$ and $x = 3$ as the dividing points.



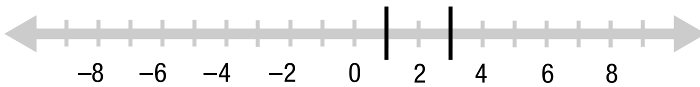
3. Substitute a test point from each interval into $x^2 - 4x + 3$ to determine its sign; for example, $x = 0$, $x = 2$, and $x = 5$.

Test $x = 0$:
$(0)^2 - 4(0) + 3$
$0 - 0 + 3 = 3$ is **positive**

Test $x = 2$:
$(2)^2 - 4(2) + 3$
$4 - 8 + 3 = -1$ is **negative**

Test $x = 5$:
$(5)^2 - 4(5) + 3$
$25 - 20 + 3 = 8$ is **positive**

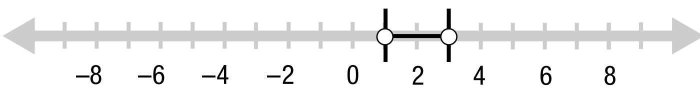
+++++++++++++++++++++0−−−0+++++++++++++



4. Find the intervals where $x^2 - 4x + 3 < 0$.

*You want the value of $x^2 - 4x + 3$ to be **less than 0**, that is, **negative**.*

+++++++++++++++++++++0−−−0+++++++++++++



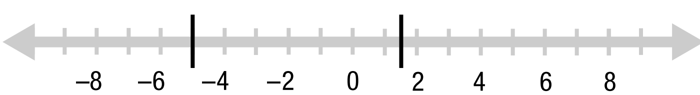
So $x^2 - 4x + 3 < 0$ when $1 < x < 3$.

As another example, to solve $2x^2 + 7x - 15 \geq 0$ without graphing:

1. Solve the equation $2x^2 + 7x - 15 = 0$.

$$2x^2 + 7x - 15 = 0$$
$$(x + 5)(2x - 3) = 0$$
$$x + 5 = 0 \quad \text{or} \quad 2x - 3 = 0$$
$$x = -5 \quad \text{or} \quad x = \frac{3}{2}$$

2. Divide the number line into intervals, using $x = -5$ and $x = \frac{3}{2}$ as the dividing points.



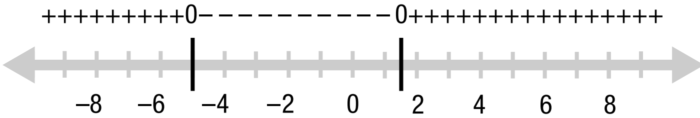
3. Substitute a test point from each interval into $2x^2 + 7x - 15$ to determine its sign; for example, $x = -7$, $x = 0$, and $x = 6$.

Test $x = -7$:
$2(-7)^2 + 7(-7) - 15$
$98 - 49 - 15 = 34$ is **positive**

Test $x = 0$:
$2(0)^2 + 7(0) - 15$
$0 + 0 - 15 = -15$ is **negative**

Test $x = 6$:
$2(6)^2 + 7(6) - 15$
$72 + 42 - 15 = 99$ is **positive**

+++++++++0−−−−−−−−−−0++++++++++++++++

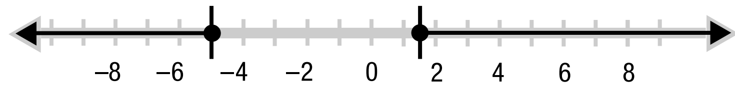


*You want the value of $2x^2 + 7x - 15$ to be **greater than or equal to 0**, that is, **nonnegative**.*

Notice that you include the endpoints, $x = -5$ and $x = \frac{3}{2}$, in the solution because the inequality is "≥," not ">."

4. Find the intervals where $2x^2 + 7x - 15 \geq 0$.

+++++++++0– – – – – – – – – – 0++++++++++++++++



So $2x^2 + 7x - 15 \geq 0$ when $x \leq -5$ or $x \geq \frac{3}{2}$.

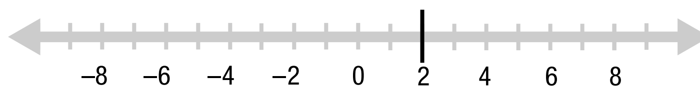
Although in the two preceding examples the number line was divided into three intervals, this is not always the case.

For example, to solve $x^2 - 4x + 4 < 0$ without graphing:

1. Solve the equation $x^2 - 4x + 4 = 0$.

$$x^2 - 4x + 4 = 0$$
$$(x - 2)(x - 2) = 0$$
$$x - 2 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = 2 \quad \text{or} \quad x = 2$$

2. Divide the number line into intervals, using $x = 2$ as the dividing point.

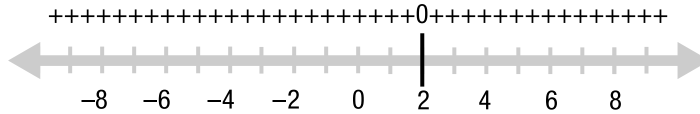


3. Substitute a test point from each interval into $x^2 - 4x + 4$ to determine its sign; for example, $x = 0$ and $x = 3$.

Test $x = 0$:
$(0)^2 - 4(0) + 4$
$0 - 0 + 4 = 4$ is **positive**

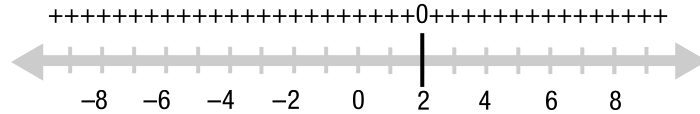
Test $x = 3$:
$(3)^2 - 4(3) + 4$
$9 - 12 + 4 = 1$ is **positive**

+++++++++++++++++++++++0+++++++++++++++



4. Find the intervals where $x^2 - 4x + 4 < 0$.

+++++++++++++++++++++++0+++++++++++++++

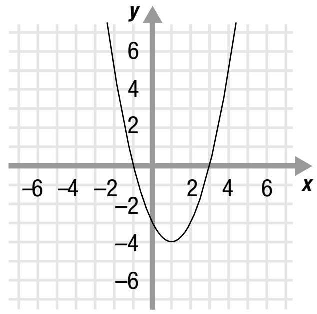


So $x^2 - 4x + 4 < 0$ is **never true** since $x^2 - 4x + 4$ is positive in both intervals, and 0 at $x = 2$.

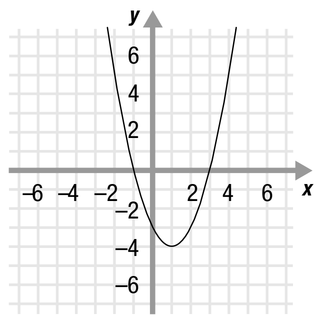
Sample Problems

1. Find the solution of $x^2 - 2x - 3 \leq 0$ by graphing the equation $y = x^2 - 2x - 3$.

   ✔ a. Graph the equation $y = x^2 - 2x - 3$.

   ☐ b. Find the x-coordinates of the points on the graph that lie **on and below** the x-axis.

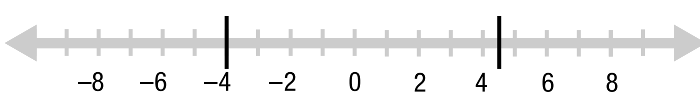
   ☐ c. Write the solution.

2. Solve $2x^2 - x - 36 > 0$ using the test point method.

   ✔ a. Solve the equation $2x^2 - x - 36 = 0$.

   $$2x^2 - x - 36 = 0$$
   $$(x + 4)(2x - 9) = 0$$
   $$x + 4 = 0 \quad \text{or} \quad 2x - 9 = 0$$
   $$x = -4 \quad \text{or} \quad x = \frac{9}{2}$$

   ✔ b. Use the solutions to divide the number line into intervals.

   ☐ c. Substitute a test point from each interval into the expression $2x^2 - x - 36$ to determine its sign, for example, $x = -5$, $x = 0$, and $x = 6$.

   Test $x = -5$:
   $2(-5)^2 - (-5) - 36$
   $50 + 5 - 36 = 19$ is **positive**

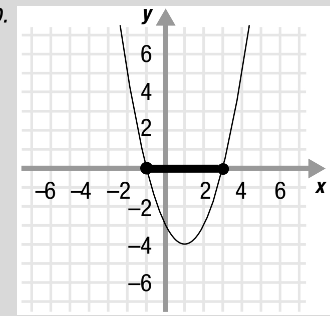
   Test $x = 0$:

   Test $x = 6$:

   ☐ d. Find the intervals that satisfy the original inequality.

Answers to Sample Problems

b.



c. $-1 \leq x \leq 3$

c. $2(0)^2 - (0) - 36$
$0 - 0 - 36 = -36$ is ***negative***

$2(6)^2 - 6 - 36$
$72 - 6 - 36 = 30$ is ***positive***

d. $x < -4$ or $x > \frac{9}{2}$

RATIONAL INEQUALITIES

Summary

Graphing Method

Just as you can use graphing to solve linear or quadratic inequalities, you can also use it to solve rational inequalities.

If you graph the function corresponding to...	The solutions are ...
$\frac{ax + b}{cx + d} > 0$	the x-coordinates of all the points on the graph that lie **above** the x-axis.
$\frac{ax + b}{cx + d} \geq 0$	the x-coordinates of all the points on the graph that lie **on and above** the x-axis.
$\frac{ax + b}{cx + d} < 0$	the x-coordinates of all the points on the graph that lie **below** the x-axis.
$\frac{ax + b}{cx + d} \leq 0$	the x-coordinates of all the points on the graph that lie **on and below** the x-axis.

To find the solutions of a rational inequality by graphing:

1. Graph the corresponding function, $y = \frac{ax + b}{cx + d}$.
2. Find the x-coordinates of the appropriate points on the graph, as indicated in the table above.

For example, to solve the rational inequality $\frac{x - 4}{x + 2} \geq 0$ by graphing:

1. Graph the function $y = \frac{x - 4}{x + 2}$. See Figure 13.3.5.
2. Find the x-coordinates of the points on the graph that lie **on and above** the x-axis. See Figure 13.3.6.

So $\frac{x - 4}{x + 2} \geq 0$ when $x < -2$ or $x \geq 4$.

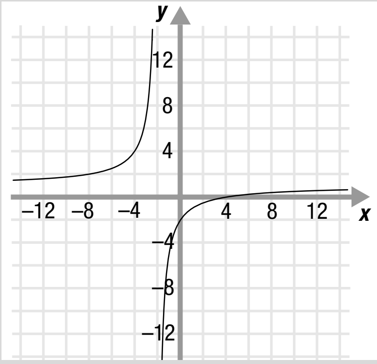


Figure 13.3.5

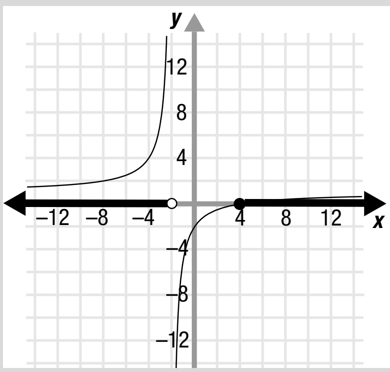


Figure 13.3.6

Notice $x = -2$ is not included as part of the solution. This is because the function is undefined at $x = -2$.

As another example, to solve the rational inequality $\frac{x + 3}{x - 5} < 0$ by graphing:

1. Graph the function $y = \frac{x + 3}{x - 5}$. See Figure 13.3.7.
2. Find the x-coordinates of the points on the graph that lie **below** the x-axis. See Figure 13.3.8.

So $\frac{x + 3}{x - 5} < 0$ when $-3 < x < 5$.

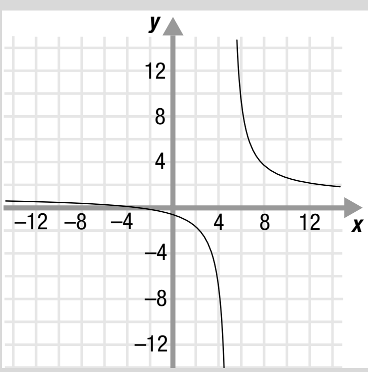


Figure 13.3.7

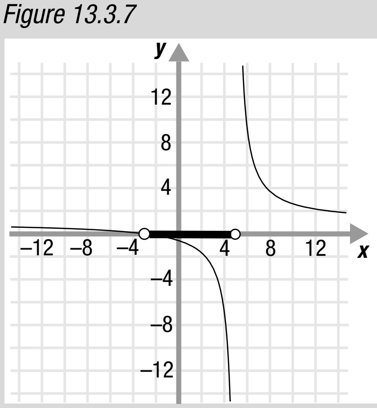


Figure 13.3.8

Test Point Method

Instead of using graphing to find the solutions of a rational inequality, you can look at the value of the corresponding function at different intervals on the *x*-axis.

The dividing points of a rational function occur when either the numerator or denominator of the function equals 0. When the numerator of the function is 0, the value of the function is 0. When the denominator of the function is 0, the function has a break since division by 0 is undefined.

Since the value of a rational function is always positive or negative in an interval between dividing points, you can evaluate the function at any "test point" in the interval to find out whether all the values for points in that interval are positive or negative.

To find the solutions of a rational inequality without graphing:

1. Find where the numerator of the function equals 0 and where the denominator of the function equals 0.
2. Use these values to divide the number line into intervals.
3. Substitute a test point from each interval into $\frac{ax + b}{cx + d}$ to determine its sign.
4. Find the intervals that satisfy the original inequality.

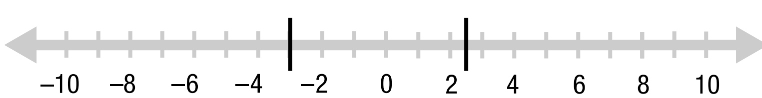
For example, to solve the rational inequality $\frac{x + 3}{2x - 5} \geq 0$ without graphing:

1. Find where the numerator equals 0. Find where the denominator equals 0.

   $x + 3 = 0$ or $2x - 5 = 0$

   $x = -3$ or $x = \frac{5}{2}$

2. Divide the number line into intervals, using $x = -3$ and $x = \frac{5}{2}$ as the dividing points.



3. Substitute a test point from each interval into $\frac{x + 3}{2x - 5}$ to determine its sign; for example, $x = -10$, $x = 0$, and $x = 10$.

   Test $x = -10$:

   $\frac{-10 + 3}{2(-10) - 5}$

   $= \frac{-7}{-25}$

   $= \frac{7}{25}$ is **positive**

   Test $x = 0$:

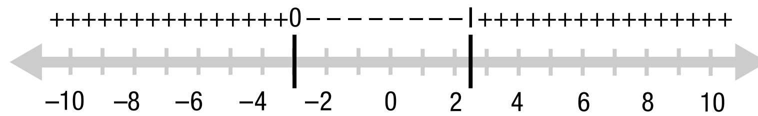
   $\frac{0 + 3}{2(0) - 5}$

   $= \frac{3}{-5}$ is **negative**

Test $x = 10$:

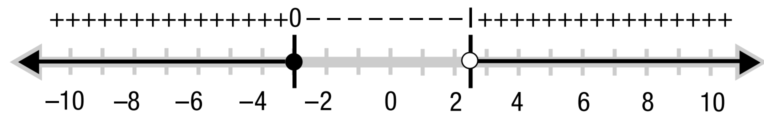
$\frac{10 + 3}{2(10) - 5}$

$= \frac{13}{15}$ is **positive**



You want the value of the function to be ***greater than or equal to 0****, that is,* ***nonnegative****.*

4. Find the intervals where $\frac{x + 3}{2x - 5} \geq 0$.



The point $x = \frac{5}{2}$ is ***not*** *included in the solution because $\frac{x + 3}{2x - 5}$ is undefined at this point.*

So $\frac{x + 3}{2x - 5} \geq 0$ when $x \leq -3$ or $x > \frac{5}{2}$.

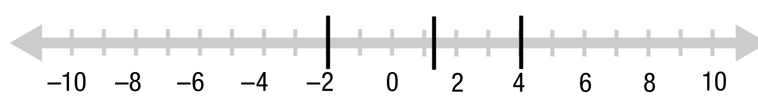
If the expression in the numerator is not linear, you can still solve the rational inequality without graphing.

For example, to solve $\frac{3x^2 + 2x - 8}{x - 4} < 0$ without graphing:

1. Find where the numerator equals 0. Find where the denominator equals 0.

$$3x^2 + 2x - 8 = 0 \quad \text{or} \quad x - 4 = 0$$
$$(x + 2)(3x - 4) = 0 \quad \text{or} \quad x = 4$$
$$x + 2 = 0 \quad \text{or} \quad 3x - 4 = 0$$
$$x = -2 \quad \text{or} \quad x = \frac{4}{3} \quad \text{or} \quad x = 4$$

2. Divide the number line into intervals, using $x = -2$, $x = \frac{4}{3}$, and $x = 4$ as the dividing points.



3. Substitute a test point from each interval into $\frac{3x^2 + 2x - 8}{x - 4}$ to determine its sign; for example $x = -10$, $x = 0$, $x = 3$, and $x = 10$.

Test $x = -10$:

$\frac{3(-10)^2 + 2(-10) - 8}{-10 - 4}$

$= \frac{300 - 20 - 8}{-14}$

$= \frac{272}{-14}$ is **negative**

Test $x = 0$:

$\frac{3(0)^2 + 2(0) - 8}{0 - 4}$

$= \frac{-8}{-4}$

$= 2$ is **positive**

Test $x = 3$:

$\frac{3(3)^2 + 2(3) - 8}{3 - 4}$

$= \frac{27 + 6 - 8}{-1}$

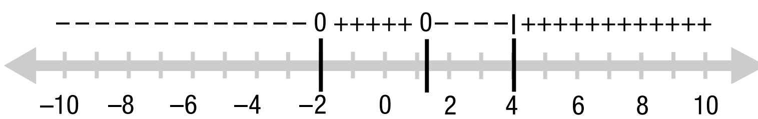
$= \frac{25}{-1}$ is **negative**

Test $x = 10$:

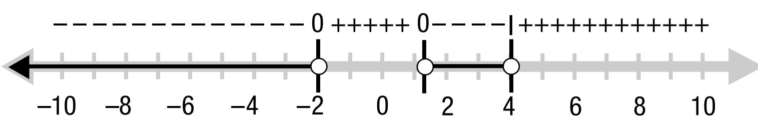
$\frac{3(10)^2 + 2(10) - 8}{10 - 4}$

$= \frac{300 + 20 - 8}{6}$

$= \frac{312}{6}$ is **positive**



4. Find the intervals where $\frac{3x^2 + 2x - 8}{x - 4} < 0$.



So $\frac{3x^2 + 2x - 8}{x - 4} < 0$ when $x < -2$ or $\frac{4}{3} < x < 4$.

When working with a rational inequality, make sure the right side of the inequality is 0. Otherwise you can't use this technique. If the right side is not 0, first rewrite the inequality so that the right side is 0, then follow the steps above.

For example, to solve $\frac{2x + 5}{x - 3} > 1$ without graphing:

1. Rewrite the inequality so the right side is 0.

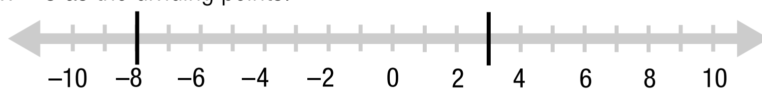
$$\frac{2x + 5}{x - 3} - 1 > 1 - 1$$
$$\frac{2x + 5}{x - 3} - \frac{x - 3}{x - 3} > 0$$
$$\frac{2x + 5 - (x - 3)}{x - 3} > 0$$
$$\frac{x + 8}{x - 3} > 0$$

2. Find where the numerator equals 0. Find where the denominator equals 0.

$x + 8 = 0$ or $x - 3 = 0$

$x = -8$ or $x = 3$

3. Divide the number line into intervals, using $x = -8$ and $x = 3$ as the dividing points.



4. Substitute a test point from each interval into $\frac{x+8}{x-3}$ to determine its sign; for example $x = -10$, $x = 0$, and $x = 10$.

Test $x = -10$:

$\frac{-10+8}{-10-3}$

$= \frac{-2}{-13}$

$= \frac{2}{13}$ is **positive**

Test $x = 0$:

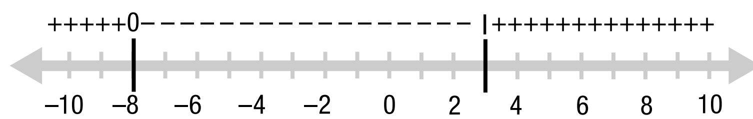
$\frac{0+8}{0-3}$

$= \frac{8}{-3}$ is **negative**

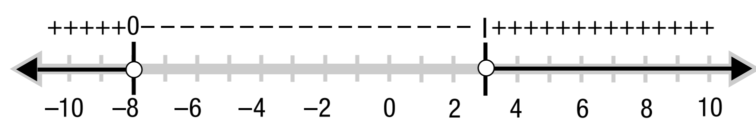
Test $x = 10$:

$\frac{10+8}{10-3}$

$= \frac{18}{7}$ is **positive**



5. Find the intervals where $\frac{x+8}{x-3} > 0$.



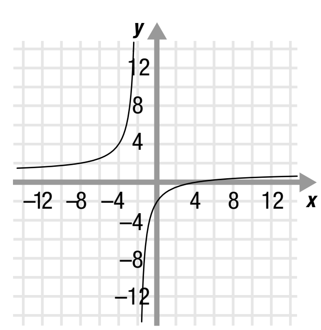
Since $\frac{x+8}{x-3} > 0$ is just $\frac{2x+5}{x-3} > 1$ rewritten, $\frac{2x+5}{x-3} > 1$ when $x < -8$ or $x > 3$.

So $\frac{x+8}{x-3} > 0$ when $x < -8$ or $x > 3$.

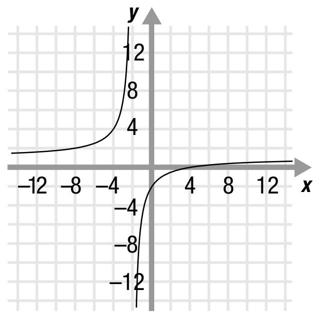
Sample Problems

1. Solve $\frac{x-4}{x+2} > 0$ by graphing.

   ✔ a. Graph the corresponding function, $y = \frac{x-4}{x+2}$.



☐ b. Find the x-coordinates of the points on the graph that lie above the x-axis.



☐ c. Write the solution.

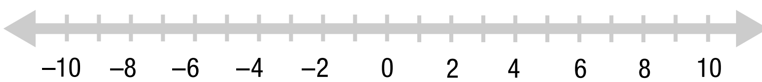
2. Solve $\frac{3x + 2}{x - 5} \leq 0$ without graphing.

☐ a. Find where the numerator equals 0. Find where the denominator equals 0.

$3x + 2 = 0$ or $x - 5 = 0$

$x =$ ______ or $x =$ ______

☐ b. Use these values to divide the number line into intervals.



☐ c. Substitute a test point from each interval into $\frac{3x + 2}{x - 5}$ to determine its sign, for example, $x = -10$, $x = 0$, and $x = 10$.

Test $x = -10$:

$\frac{3(-10) + 2}{-10 - 5}$

$= \frac{-28}{-15}$

$= \frac{28}{15}$ is **positive**

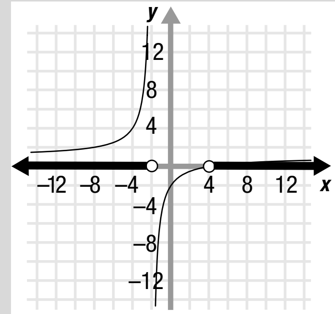
Test $x = 0$:

Test $x = 10$:

☐ d. Determine which intervals satisfy the original inequality.

Answers to Sample Problems

b.



c. $x < -2$ or $x > 4$

a. $-\frac{2}{3}$, 5

b.

c. $\frac{3(0) + 2}{0 - 5}$

$= \frac{2}{-5}$ *is* ***negative***

$\frac{3(10) + 2}{10 - 5}$

$= \frac{32}{5}$ *is* ***positive***

d. $-\frac{2}{3} \leq x < 5$

3. Solve $\frac{5x+9}{x-1} \geq 2$ without graphing.

✔ a. Rewrite the inequality so the right side is 0.

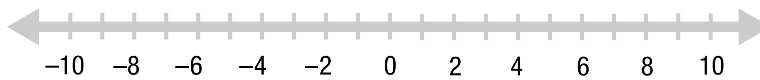
$$\frac{5x+9}{x-1} - 2 \geq 2 - 2$$
$$\frac{5x+9}{x-1} - \frac{2(x-1)}{x-1} \geq 0$$
$$\frac{5x+9}{x-1} - \frac{2x-2}{x-1} \geq 0$$
$$\frac{5x-2x+9+2}{x-1} \geq 0$$
$$\frac{3x+11}{x-1} \geq 0$$

☐ b. Find where the numerator equals 0. Find where the denominator equals 0.

$3x + 11 = 0$ or $x - 1 = 0$

$x =$ ______ or $x =$ ______

☐ c. Use these values to divide the number line into intervals.



☐ d. Substitute a test point from each interval into $\frac{3x+11}{x-1}$ to determine its sign, for example, $x = -10$, $x = 0$, and $x = 10$.

Test $x = -10$:

$\frac{3(-10)+11}{-10-1}$

$= \frac{-19}{-11}$

$= \frac{19}{11}$ is **positive**

Test $x = 0$:

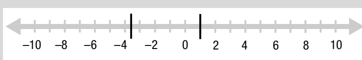
Test $x = 10$:

☐ e. Determine which intervals satisfy the original inequality.

Answers to Sample Problems

b. $-\frac{11}{3}$, 1

c.



d. $\frac{3(0)+11}{0-1}$

$= \frac{11}{-1}$ *is* ***negative***

$\frac{3(10)+11}{10-1}$

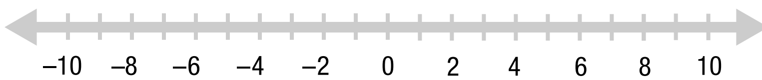
$= \frac{41}{9}$ *is* ***positive***

e. $x \leq -\frac{11}{3}$ *or* $x > 1$

4. Solve $\frac{x^2 + 4x - 21}{x + 5} > 0$ without graphing.

☐ a. Find where the numerator equals 0. Find where the denominator equals 0. $x =$ ____ or $x =$ ____ or $x =$ ____

☐ b. Use these values to divide the number line into intervals.



☐ c. Substitute a test point from each interval into $\frac{x^2 + 4x - 21}{x + 5}$ to determine its sign; for example, $x = -10$, $x = -6$, $x = 0$, and $x = 10$.

Test $x = -10$:

$\frac{(-10)^2 + 4(-10) - 21}{-10 + 5}$

$= \frac{100 - 40 - 21}{-5}$

$= \frac{39}{-5}$ is **negative**

Test $x = -6$:

Test $x = 0$:

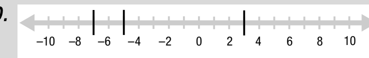
Test $x = 10$:

☐ d. Determine which intervals satisfy the original inequality.

Answers to Sample Problems

a. $-7, 3, -5$

b.



c. $\frac{(-6)^2 + 4(-6) - 21}{-6 + 5}$

$= \frac{36 - 24 - 21}{-1} = \frac{-9}{-1}$

$= 9$ *is* ***positive***

$\frac{(0)^2 + 4(0) - 21}{0 + 5}$

$= \frac{-21}{5}$ *is* ***negative***

$\frac{(10)^2 + 4(10) - 21}{10 + 5}$

$= \frac{100 + 40 - 21}{15}$

$= \frac{119}{15}$ *is* ***positive***

d. $-7 < x < -5$ or $x > 3$

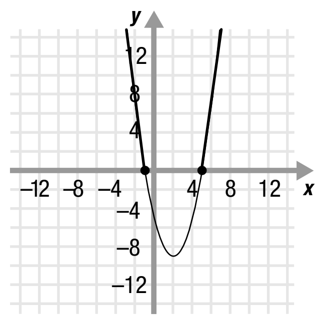
EXPLORE

Sample Problems

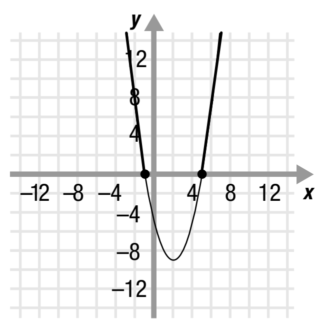
On the computer you used the Grapher to solve some inequalities by investigating the graphs of their corresponding functions.

1. The graph of $y = x^2 - 4x - 5$ is shown below. Use the graph to find the solutions of $x^2 - 4x - 5 \geq 0$.

   ☑ a. Find where the graph lies on or above the x-axis.



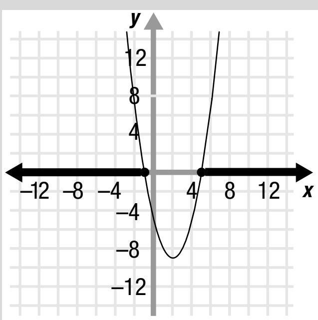
   ☐ b. Find the x-coordinates of the points on the graph that make the inequality $x^2 - 4x - 5 \geq 0$ true.



   ☐ c. Write the solution.

Answers to Sample Problems

b.



c $x \leq -1$ *or* $x \geq 5$

2. Given the function $y = \frac{x^2 + 1}{x - 2}$, for each value of x below, determine whether y is positive, negative, 0, or undefined.

a. –5

b. 0

c. 2

d. 7

☑ a. Substitute $x = -5$ into $y = \frac{x^2 + 1}{x - 2}$.

$$y = \frac{(-5)^2 + 1}{-5 - 2}$$

$$= \frac{26}{-7} \text{ is } \mathbf{negative}$$

☐ b. Substitute $x = 0$ into $y = \frac{x^2 + 1}{x - 2}$.

☐ c. Substitute $x = 2$ into $y = \frac{x^2 + 1}{x - 2}$.

☐ d. Substitute $x = 7$ into $y = \frac{x^2 + 1}{x - 2}$.

Answers to Sample Problems

b. $y = \frac{0^2 + 1}{0 - 2} = -\frac{1}{2}$ *is* ***negative***

c. $y = \frac{2^2 + 1}{2 - 2} = \frac{5}{0}$ *is* ***undefined***

d. $y = \frac{7^2 + 1}{7 - 2} = \frac{50}{5}$ *is* ***positive***

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Quadratic Inequalities

1. Use the equation $y = x^2 - 4x$ to find the solution of $x^2 - 4x < 0$. See Figure 13.3.9.

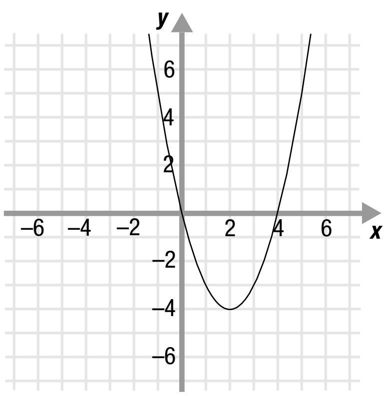


Figure 13.3.9

2. Solve $x^2 + 3x - 10 \geq 0$ using the test point method.

3. Solve $x^2 + 4x + 4 > 0$ using the test point method.

4. Use the equation $y = x^2 + 6x + 8$ to find the solution of $x^2 + 6x + 8 < 0$. See Figure 13.3.10.

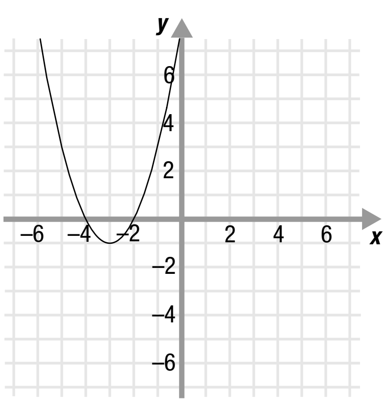


Figure 13.3.10

5. Solve $x^2 - 11x + 28 > 0$ using the test point method.

6. Solve $x^2 - 16 \leq 0$ using the test point method.

7. Use the equation $y = x^2 - 2x + 1$ to find the solution of $x^2 - 2x + 1 \leq 0$. See Figure 13.3.11.

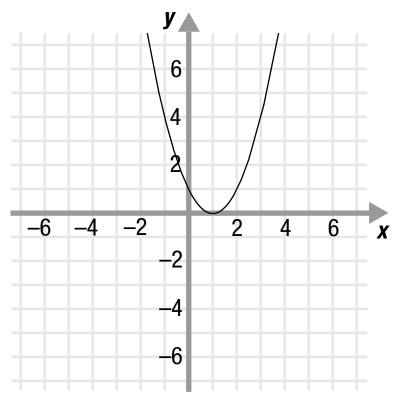


Figure 13.3.11

8. Solve $x^2 - 7x - 18 > 0$ using the test point method.

9. The profit, y, that a widget company can earn by selling x widgets can be expressed by the equation $y = -0.01x^2 + 5x - 400$. How many widgets does the company need to sell to make a positive profit?

10. The velocity, V, in feet per second, of a given object at time t seconds can be expressed by the equation $V = 3t^2 - 39t + 108$. Find the times when the object has a positive velocity.

11. Solve $x^3 + 2x^2 - 11x - 12 < 0$ using the test point method.

    Hint: $x^3 + 2x^2 - 11x - 12 = (x + 1)(x + 4)(x - 3)$

12. Solve $x^3 - 9x \geq 0$ using the test point method.

Rational Inequalities

13. The graph of the function $y = \frac{x}{x+3}$ is shown on the grid in Figure 13.3.13.

    Use this graph to solve the inequality $\frac{x}{x+3} < 0$.

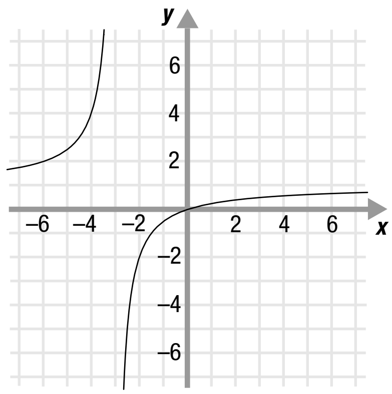


Figure 13.3.13

14. Solve $\frac{x+7}{x-4} > 0$ using the test point method.

15. The graph of the function $y = \frac{x-3}{x+1}$ is shown on the grid in Figure 13.3.14.

    Use this graph to solve the inequality $\frac{x-3}{x+1} < 0$.

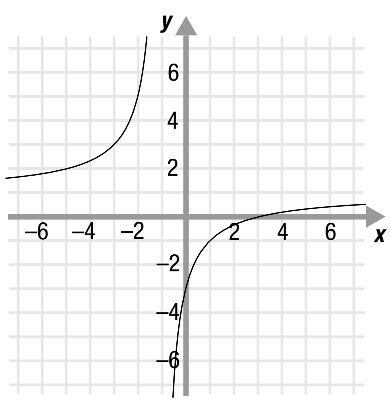


Figure 13.3.14

16. Solve $\frac{6-x}{6+x} \le 0$ using the test point method.

17. The graph of the function $y = \frac{3x+2}{x+4}$ is shown on the grid in Figure 13.3.15.

    Use this graph to solve the inequality $\frac{3x+2}{x+4} > 0$.

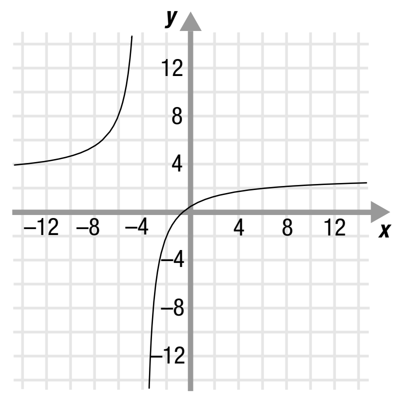


Figure 13.3.15

18. Solve $\frac{4x+2}{x-7} \ge 3$ using the test point method.

19. The graph of the function $y = \frac{x+1}{x-2}$ is shown on the grid in Figure 13.3.16.

    Use this graph to solve the inequality $\frac{x+1}{x-2} \le 0$.

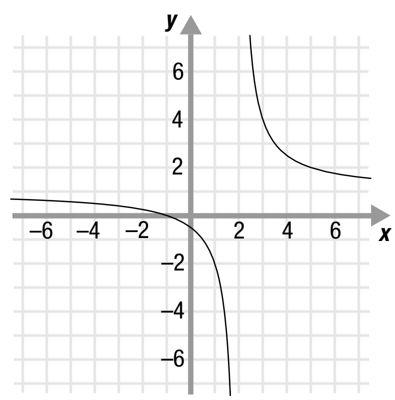


Figure 13.3.16

20. Solve $\frac{3x^2 + 13x + 4}{x+6} > 0$ using the test point method.

21. In a geometric sequence, 1, r, r^2, r^3, ... (where each term is the product of the previous term and a given number, r), you can find the sum of the first n terms using the formula $S_n = \frac{1-r^n}{1-r}$. For what value(s) of r can you not use this formula?

22. Given a cube with sides of length e, when is the surface area of the cube larger than the volume of the cube?

    Hint: If x is the length of a side of a cube, surface area = $6x^2$, and volume = x^3.

23. The graph of the function $y = \frac{x-3}{2x+1}$ is shown in Figure 13.3.17.

    Use this graph to solve the inequality $\frac{x-3}{2x+1} \geq 0$.

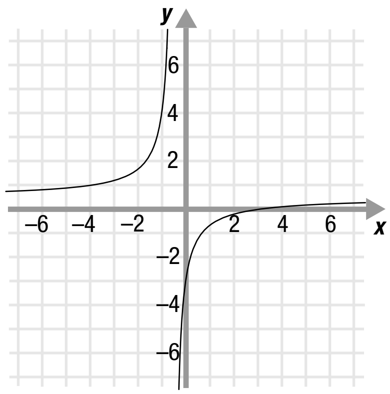


Figure 13.3.17

24. Solve $\frac{2x^2 + 30x + 91}{x + 5} \leq 7$ using the test point method.

Explore

25. The graph of the function $y = x^2 + 8x + 15$ is shown on the grid in Figure 13.3.18. Use the graph of the function to find the solutions of $x^2 + 8x + 15 < 0$.

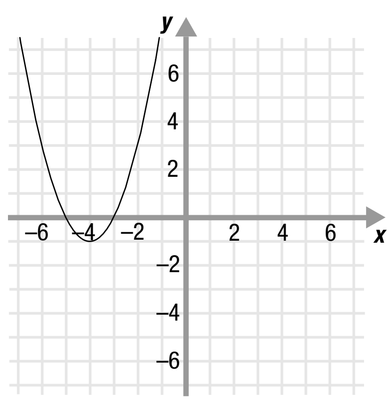


Figure 13.3.18

26. The graph of the function $y = \frac{x}{x-1}$ is shown on the grid in Figure 13.3.19. Use the graph of the function to find the solutions of $\frac{x}{x-1} > 0$.

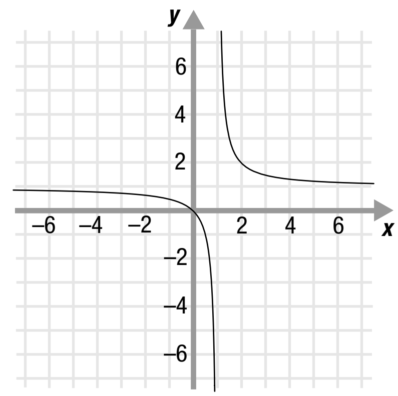


Figure 13.3.19

27. Given the function $y = 3x^4 - 7x^3 - 39x^2 + 55x + 84$, determine for each value of x below whether y is positive, negative, 0, or undefined.

    a. –10

    b. –2

    c. –1

    d. 0

    e. 4

    f. 17

28. The graph of the function $y = x^3 - x^2 - 6x$ is shown on the grid in Figure 13.3.20. Use the graph of the function to find the solutions of $x^3 - x^2 - 6x > 0$.

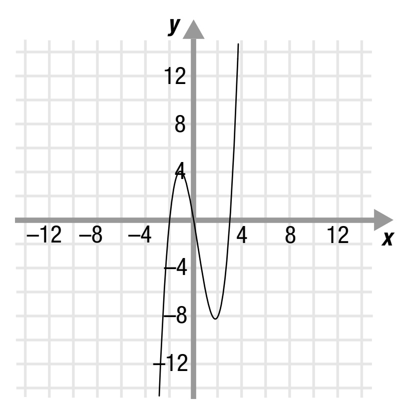


Figure 13.3.20

29. The graph of the function $y = \frac{x^2 + 2x - 8}{2x}$ is shown on the grid in Figure 13.3.21. Use the graph of the function to find the solutions of $\frac{x^2 + 2x - 8}{2x} \le 0$.

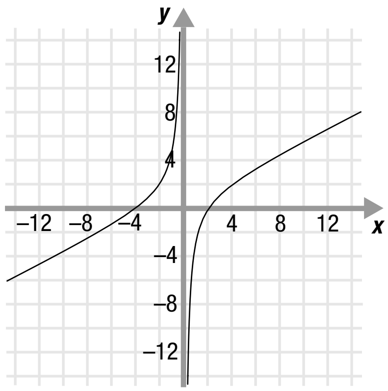


Figure 13.3.21

30. Given the function $y = \frac{0.2x^3 - 3x^2 + 11x - 6}{4x}$, determine for each value of x below whether y is positive, negative, 0, or undefined.

a. –10

b. –1

c. 0

d. 2

e. 7

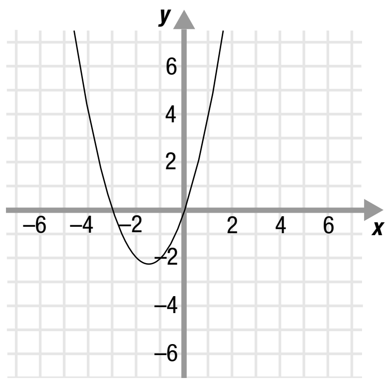
f. 15

APPLY

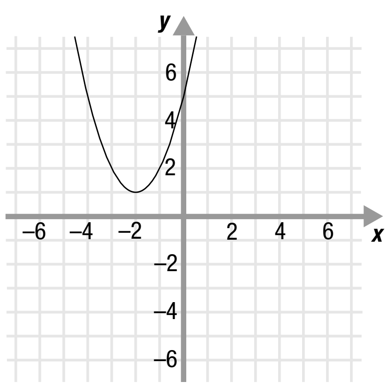
Practice Problems

Here are some additional practice problems for you to try.

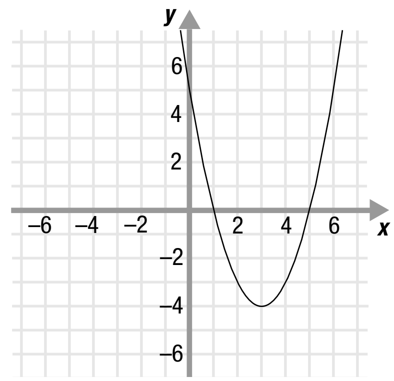
1. The graph of the function $y = x^2 + 3x$ is shown on the grid below. Use this graph to solve the inequality $x^2 + 3x < 0$.



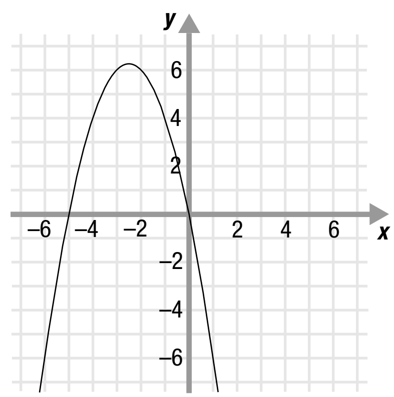
2. The graph of the function $y = x^2 + 4x + 5$ is shown on the grid below. Use this graph to solve the inequality $x^2 + 4x + 5 > 0$.



3. The graph of the function $y = x^2 - 6x + 5$ is shown on the grid below. Use this graph to solve the inequality $x^2 - 6x + 5 \geq 0$.



4. The graph of the function $y = -x^2 - 5x$ is shown on the grid below. Use this graph to solve the inequality $-x^2 - 5x < 0$.



5. The graph of $y = -x^2 + 6x - 11$ is shown below. Use this graph to decide whether each statement is true or false.

   a. The solution of the inequality $-x^2 + 6x - 11 > 0$ is all real numbers.

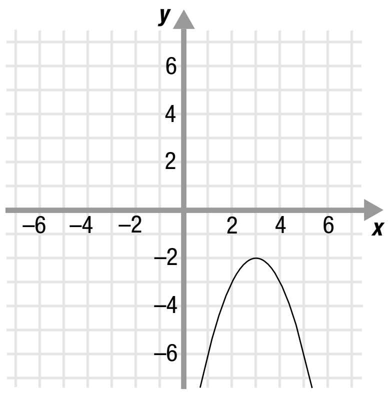
   b. The solution of the inequality $-x^2 + 6x - 11 < 0$ is all real numbers.

   c. The solution of the equation $-x^2 + 6x - 11 = 0$ is all real numbers.

   d. The inequality $-x^2 + 6x - 11 > 0$ has no solutions.

   e. The inequality $-x^2 + 6x - 11 < 0$ has no solutions.

f. The equation $-x^2 + 6x - 11 = 0$ has no real solutions.



6. Solve the inequality $x^2 + 2x - 15 \geq 0$ using the test point method.

7. Solve the inequality $x^2 - 10x < -24$ using the test point method.

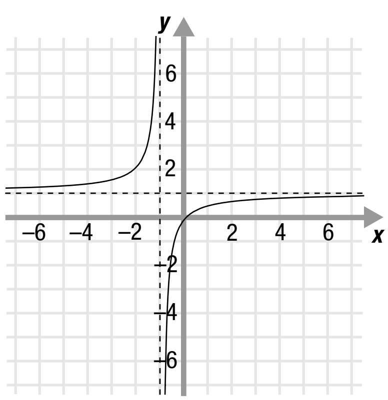
8. Solve the inequality $x^2 - 10 \leq -3x$ using the test point method.

9. Solve the inequality $x^2 - 8x < -16$ using the test point method.

10. Solve the inequality $x^2 - 49 \geq 0$ using the test point method.

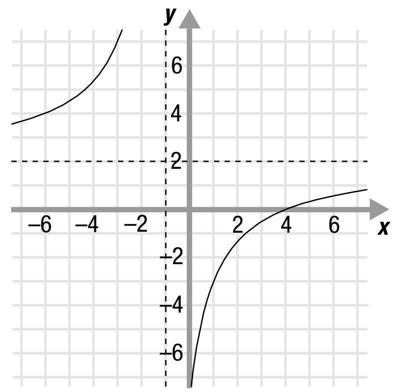
11. The graph of the function $y = \frac{x}{x+1}$ is shown on the grid below.

    Use this graph to solve the inequality $\frac{x}{x+1} < 0$.



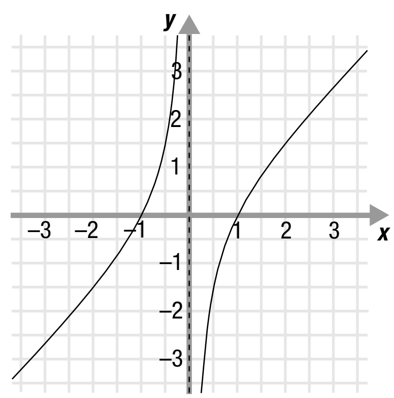
12. The graph of the function $y = \frac{2x-8}{x+1}$ is shown on the grid below.

    Use this graph to solve the inequality $\frac{2x-8}{x+1} > 0$.



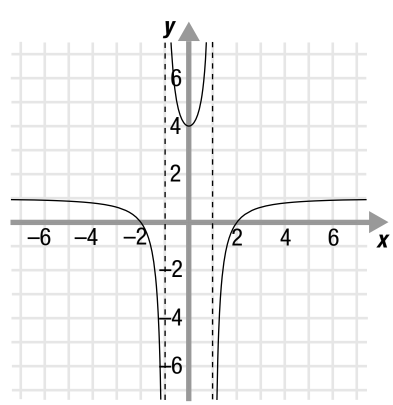
13. The graph of the function $y = \frac{x^2-1}{x}$ is shown on the grid below.

    Use this graph to solve the inequality $\frac{x^2-1}{x} \leq 0$.



14. The graph of the function $y = \frac{x^2-4}{x^2-1}$ is shown on the grid below.

    Use this graph to solve the inequality $\frac{x^2-4}{x^2-1} \geq 0$.



15. Solve the rational inequality $\frac{x+1}{x-3} \geq 0$ using the test point method.

16. Solve the rational inequality $\frac{3x-9}{x+1} \leq 0$ using the test point method.

17. Solve the rational inequality $\frac{(x+1)(x-2)}{x-1} \geq 0$ using the test point method.

18. Solve the rational inequality $\frac{x+2}{x^2+2x+1} < 0$ using the test point method.

19. Solve the rational inequality $\frac{2x+4}{x-8} > 0$.

20. Solve the rational inequality $\frac{x^2-7x+10}{x+3} > 0$.

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. The graph of the function $y = \frac{1}{2}x^2 + \frac{5}{2}x - 3$ is shown on the grid in Figure 13.3.22. Use this graph to solve the inequality $\frac{1}{2}x^2 + \frac{5}{2}x - 3 < 0$.

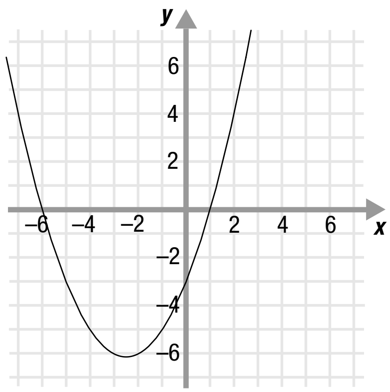


Figure 13.3.22

2. Solve the inequality $x^2 - 10 \leq 3x$ using the test point method.

3. Solve the inequality $x^2 - 8x \geq -16$ using the test point method.

4. The graph of $y = x^2 + 5x + 8$ is shown on the grid in Figure 13.3.23. Use this graph to decide whether each statement below is true or false.

   a. The solution of the inequality $x^2 + 5x + 8 > 0$ is all real numbers.

   b. The solution of the inequality $x^2 + 5x + 8 < 0$ is all real numbers.

   c. The solution of the equation $x^2 + 5x + 8 = 0$ is all real numbers.

   d. The inequality $x^2 + 5x + 8 > 0$ has no solutions.

   e. The inequality $x^2 + 5x + 8 < 0$ has no solutions.

   f. The equation $x^2 + 5x + 8 = 0$ has no real solutions.

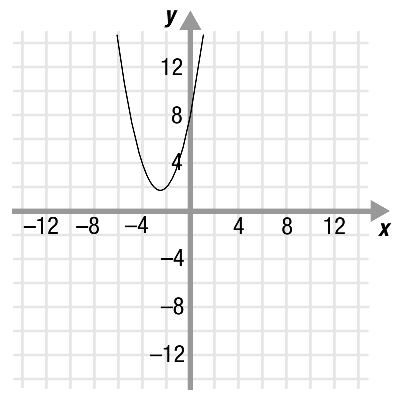


Figure 13.3.23

5. Solve the rational inequality $\frac{x - 2}{x + 3} > 0$ using the test point method.

6. Solve the rational inequality $\frac{1 - 2x}{x + 9} \leq 0$ using the test point method.

7. Solve the rational inequality $\frac{x^2 - 4}{x + 5} < 0$.

8. The graph of the function $y = \frac{x^2 + 2}{x + 1}$ is shown on the grid in Figure 13.3.24.

   Find the solution of the inequality $\frac{x^2 + 2}{x + 1} \geq 0$.

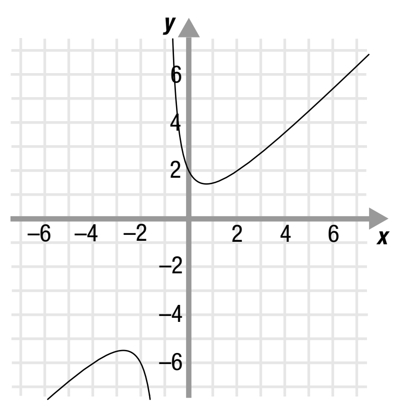


Figure 13.3.24

9. The graphs of $y = x^2 - 2x - 8$ and $y = 2x^2 - 4x - 16$ are shown on the grid in Figure 13.3.25. Find the values of x that satisfy both $x^2 - 2x - 8 \geq 0$ and $2x^2 - 4x - 16 \geq 0$.

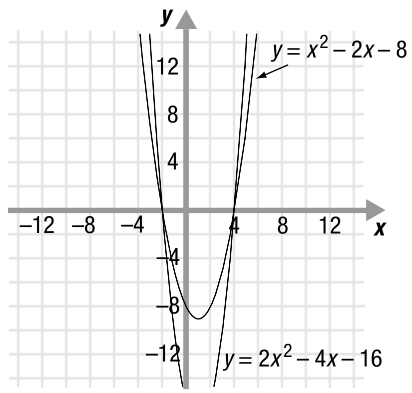


Figure 13.3.25

10. The graph of $y = \frac{1}{64}x^4 - \frac{5}{16}x^2 + 1$ is shown on the grid in Figure 13.3.26.

Find where $\frac{1}{64}x^4 - \frac{5}{16}x^2 + 1 < 0$.

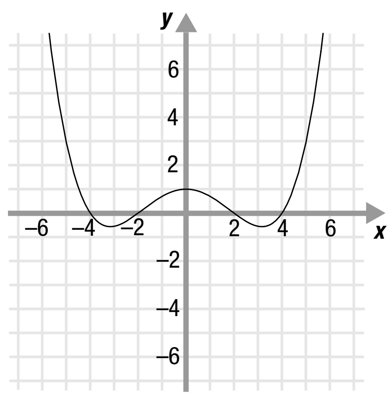


Figure 13.3.26

11. The graph of $y = \frac{x-1}{x^2 + x - 20}$ is shown on the grid in Figure 13.3.27. Use the graph of the function to find the solution of the inequality $\frac{x-1}{x^2 + x - 20} \geq 0$.

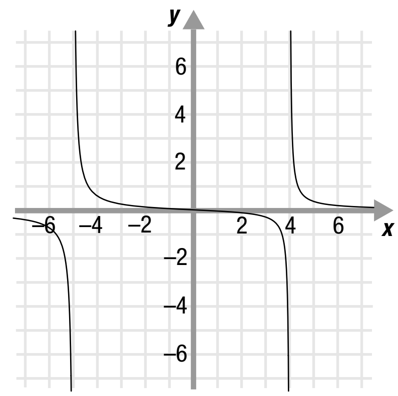


Figure 13.3.27

12. Given the function $y = \frac{x^2 - 9}{x - 5}$, for each value of x below, determine whether y is positive, negative, 0, or undefined.

a. –10

b. –3

c. 0

d. 5

e. 14

TOPIC 13 CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic. Or you may wish to do these problems to review for a test.

1. Find: $4 - 2[2 - 3(4 - 1)]$
2. Find the x– and y–intercepts of each of the functions below.
   a. $y = x^2$
   b. $y = x^2 + 2x - 8$
   c. $y = x^2 + 9$
3. Solve for x: $\sqrt{x^2 + 5} + x = 1$
4. Find the slope intercept form of the equation of the line that passes through the point (–3, 7) and is parallel to the line $y = -2x + 9$.
5. Solve for x: $\log_x 16 = 2$
6. Solve for y: $\frac{1}{2}y + 3 = \frac{1}{4}y - 8$
7. Given $f(x) = x^2 + 2$ and $g(x) = 3x - 4$, find:
   a. $(f \circ g)(x)$
   b. $(f \circ g)(-2)$
8. Solve for t: $\sqrt{t+6} - \sqrt{t+1} = 3$
9. Rewrite in logarithmic form: $6^x = 20$
10. Simplify: $\frac{x + 3}{5xy} \cdot \frac{7yz}{x^2 + 5x + 6} \div \frac{4}{x + 2}$
11. Find: $\log_5 119$
    Approximate your answer to two decimal places.
12. The equation $y = -x^2 + 5x$ is graphed in Figure 13.1. Use the graphto find the solution of $-x^2 + 5x < 0$.

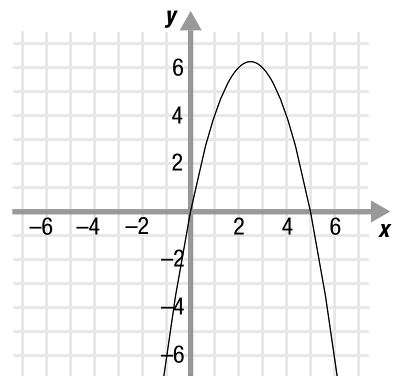


Figure 13.1

13. Solve for x: $|x + 3| - 4 = 10$
14. Given $g(x) = 3x + 1$, graph the function $y = g$ and its inverse, $y = g^{-1}$.
15. Solve for x: $|-3x + 6| > 9$
16. Factor: $5x^2y - 40xy + 35y$
17. Solve for x: $x^2 - 3x - 10 \leq 0$
18. Graph the function $y = \left(\frac{1}{2}\right)^x$.
19. Find: $(2\sqrt{7} + 1)(2\sqrt{7} - 1)$
20. Solve for b: $b^3 + 2b^2 = 16b + 32$
21. Solve for x: $(x - 2)^2 = 196$
22. Simplify: $4\sqrt{3y} - 2\sqrt{75y} + 5$
23. Solve for x: $x^2 + 16 = 0$
    (Here, x can be an imaginary number.)

24. For the function $f(x) = 3x^2 + x - 1$, calculate:

a. $f(3)$

b. $f(0)$

c. $f(-2)$

d. $f(5)$

e. $f(-10)$

25. Use the graphs in Figure 13.2 to find the number of solutions to this nonlinear system:

$$y = -x^2$$
$$x^2 + y^2 = 16$$

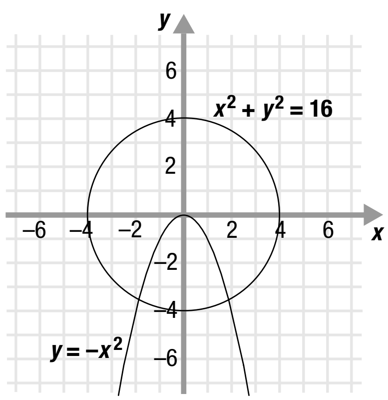


Figure 13.2

26. Solve for x: $\frac{4x}{5} - 2 = \frac{4}{3}$

27. The graph of the function $y = |x|$ is shown on the grid in Figure 13.3.
Determine the equations of the other lines shown.

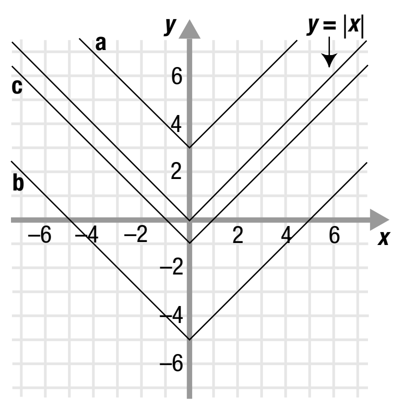


Figure 13.3

28. Solve $x^2 + 7x = 0$ by factoring.

29. Given $f(x) = 10x - 3$, find f^{-1}.

30. Solve for x: $\frac{x-2}{x+5} < 0$

31. Solve for t: $(3^2)^{t-2} = (3^3)^{4t}$

32. Solve for x: $x^2 + x - 9 = 0$

33. Simplify: $\sqrt[7]{b} \cdot \sqrt{b}$

34. Find the distance, d, between the point $(2, -2)$ and the point $(-1, 2)$.

35. Find: $(a + 5b)(2a - 3b + c)$

36. Solve for x: $2e^{x-1} = 8$

37. Find the domain and the range of each of the functions below.

a. $f(x) = \frac{x}{x-2}$

b. $f(x) = |x|$

c. $f(x) = |3x| - 2$

38. Find: $\log_9 729$

39. Solve for x: $2x^2 + 6x = 3$

40. Solve for x: $\log_4(x + 1) = 1 + \log_4 x$

41. Simplify: $\sqrt{\frac{25a^4b^2}{48c^3}}$

42. Rewrite as a single logarithm: $\log 6 + 3[\log(x + 5)]$

43. Solve for k: $\left(\frac{k+2}{k}\right)^2 - 8\left(\frac{k+2}{k}\right) + 15 = 0$

44. Solve for x: $\frac{2}{x^2 + 3x} = \frac{5}{x^2 - x} + \frac{3}{x^2 + 2x - 3}$

45. Rewrite as a single logarithm: $5\log_3 x - \log_3 25$

46. Given $f(x) = x^3 + 2$ and $g(x) = x^3 - 2$, find:

a. $(f + g)(x)$

b. $(f - g)(3)$

c. $(f \cdot g)(x)$

d. $\left(\frac{f}{g}\right)(-1)$

47. Simplify using the properties of exponents:

$\left[\frac{(3a)^2}{(2b)^3}\right]^2 \cdot a^5b^2$

48. Solve this nonlinear system:

$$y = x^2 + 2x - 8$$

$$y + 2x + 8 = 0$$

49. Solve for b: $\frac{a - b}{c} + d = 0$

50. The equation $y = \frac{x}{x + 2}$ is graphed in Figure 13.4. Use the graph to find the solution of $\frac{x}{x + 2} \geq 0$.

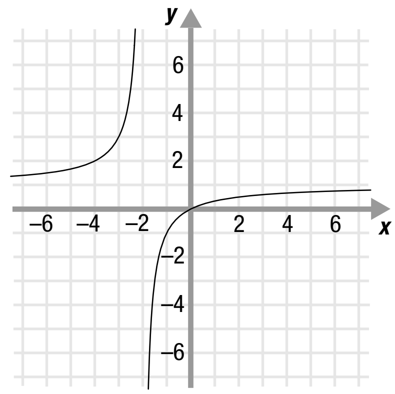


Figure 13.4