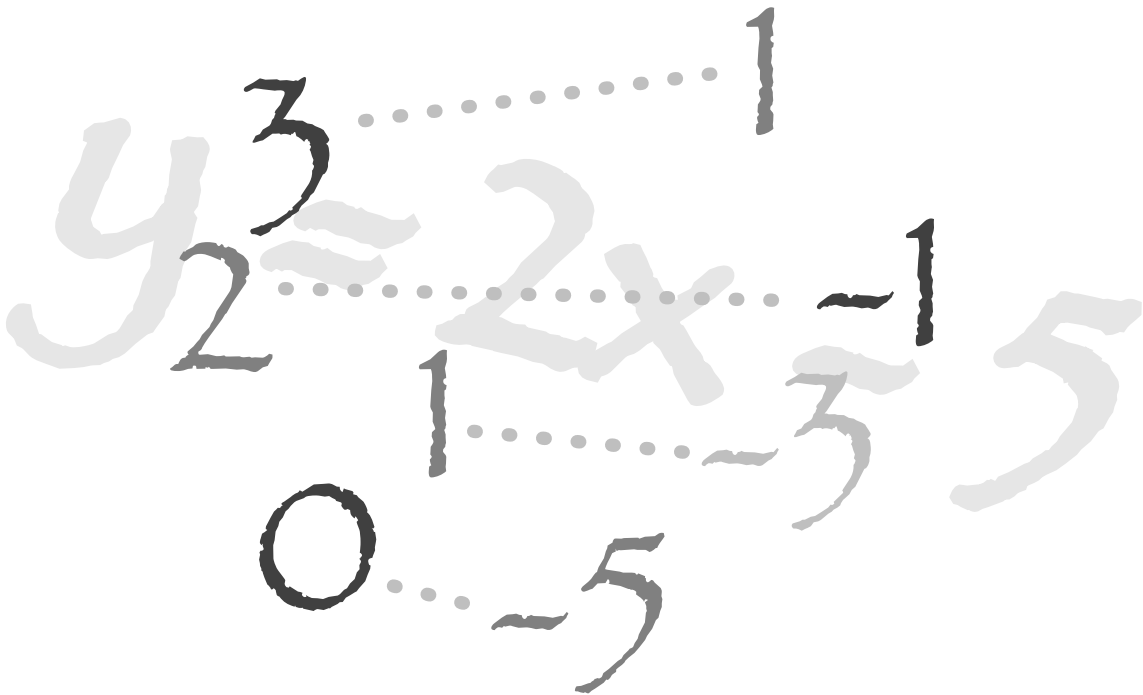


# LESSON 11.1 – FUNCTIONS

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## OVERVIEW

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**Here's what you'll learn in this lesson:**

### **Functions and Graphs**

- a. Definition of a function
- b. Function as an ordered pair of numbers
- c. Finding function values given a formula
- d. Function notation:  $y = f(x)$
- e. Graphing simple functions
- f. Domain and range of a function
- g. The vertical line test

### **Linear Functions**

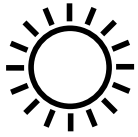
- a. Graphs of linear functions
- b. Graphs of absolute value functions

### **Quadratic Functions**

- a. Graphs of quadratic functions
- b. Intercepts of quadratic functions

For centuries, artists from many cultures have used patterns to create beautiful mosaics, textiles, and stained glass windows. Many of these patterns can be described mathematically using functions.

In this lesson, you will learn about functions. You will learn how to graph a function, how to identify the domain and the range of a function, and how to use the notation for functions. In addition, you will study three types of functions that you will see again in your study of algebra: linear functions, absolute value functions, and quadratic functions.



## FUNCTIONS AND GRAPHS

### Summary

#### Definition of a Function

Rules such as  $y = 3x + 1$  and  $y = 2x - 7$  are called functions. These rules assign to each real number  $x$  exactly one real number  $y$ .

Some examples of functions are:

$$y = x + 1$$

$$y = x^2$$

$$y = 4 - 2x$$

$$y = \frac{1}{x}$$

Sometimes letters such as  $f$ ,  $g$ , or  $h$  are used to denote functions. When  $x$  is the input number, the output number is written as  $f(x)$ ,  $g(x)$ , or  $h(x)$ .

For example, you might write the above functions as:

$$f(x) = x + 1$$

$$g(x) = 4 - 2x$$

$$h(x) = x^2$$

$$k(x) = \frac{1}{x}$$

To find the value of a function at a given point,  $x$ :

1. Substitute the given  $x$ -value into the function.
2. Simplify to get the output value.

For example, to find the value of  $f(x) = 3x - 7$  at  $x = 1$ :

1. Substitute 1 for  $x$ .  $f(1) = 3(1) - 7$

2. Simplify.  $= 3 - 7$

$$= -4$$

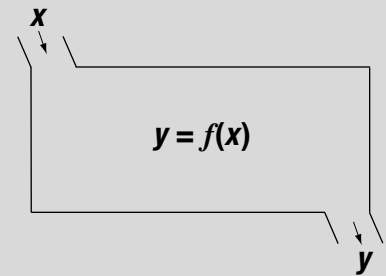
As another example, to find the value of  $k(x) = \frac{1}{2x}$  at  $x = -7$ :

1. Substitute  $-7$  for  $x$ .  $f(-7) = \frac{1}{2(-7)}$

2. Simplify.  $= \frac{1}{-14}$

$$= -\frac{1}{14}$$

*You can think of a function as a machine that assigns to each input number,  $x$ , a unique output number,  $y$ .*



*You read " $f(x)$ " as "f of x."*

The domain is the  $x$ -values. The range is the  $y$ -values. Here's a way to help you remember which is which: **domain** comes before **range** in alphabetical order just like  $x$  comes before  $y$ .

In interval notation:

- A square bracket, “[ ” or “ ] ”, means the number **is** included in the interval.
- A parenthesis, “ ( ” or “ ) ” means the number is **not** included in the interval.

To help you find the domain, start with all real numbers and eliminate those numbers for which the function does not make sense.

The domain is all real numbers since  $(x - 1)$  is defined for any real number. The range is all real numbers since  $(x - 1)$  can equal any real number as  $x$  runs through all real numbers.

The domain can't include 2 or  $-2$  because substituting either of these numbers makes the denominator equal to 0. The range can't include those real numbers which are greater than  $-\frac{1}{4}$  and less than or equal to 0. There are no real numbers you can substitute for  $x$  that will make  $\frac{1}{x^2 - 4}$  equal to numbers which are greater than  $-\frac{1}{4}$  and less than or equal to 0.

## Domain and Range of a Function

The **domain** of a function is all of the real numbers,  $x$ , for which the function is defined and produces a real number,  $y$ . The **range** of a function is all of the possible  $y$ -values you can get from the  $x$ -values.

The domain and range of a function don't always include all the real numbers.

- When a function includes a square root, the domain does not include any numbers that make the value under the square root negative.

For example, in the function  $y = \sqrt{x - 1}$  you need to make sure that the  $(x - 1)$  under the square root sign is never negative. So the domain includes all nonnegative real numbers greater than or equal to 1. In interval notation, the domain is the interval  $[1, \infty)$ .

- When a function includes a fraction, the domain does not include any numbers that make the denominator of the fraction zero.

For example, in the function  $y = \frac{1}{x + 2}$  you need to make sure that the  $(x + 2)$  in the denominator is never zero. So the domain includes all real numbers except  $-2$ .

To find the domain and range of a function:

1. Find all of the  $x$ -values for which the function is defined and produces a real number. This is the domain of the function.
2. Find all possible  $y$ -values for all  $x$ -values in the domain. This is the range of the function.

For example, to find the domain and range of the function  $y = x - 1$ :

1. Find all the  $x$ -values for which the function is defined. All real numbers; the interval  $(-\infty, \infty)$ .
2. Find all possible  $y$ -values for all  $x$ -values in the domain. All real numbers; the interval  $(-\infty, \infty)$ .

As another example, to find the domain and range of the function  $y = \frac{1}{x^2 - 4}$ :

1. Find all the  $x$ -values for which the function is defined. Any real number except  $x = 2$  or  $x = -2$ . This is the set of real numbers,  $x$ , which lie in either the interval  $(-\infty, -2)$  or in the interval  $(-2, 2)$  or in the interval  $(2, \infty)$ . This does not include the numbers  $-2$  or  $2$ .
2. Find all possible  $y$ -values for all  $x$ -values in the domain. All real numbers except those that are greater than  $-\frac{1}{4}$  and less than or equal to 0. This is the set of real numbers,  $y$ , which lie in either the interval  $(-\infty, -\frac{1}{4}]$  or in the interval  $(0, \infty)$ .

## Graphing Functions

When you are given a function, you can find ordered pairs of real numbers that satisfy the function and then use these ordered pairs to graph the function.

To find an ordered pair that satisfies a function  $y = f(x)$ :

1. Pick a value for  $x$ .
2. Substitute your chosen value,  $x$ , into the function to solve for the other variable,  $y$ .
3. Write the ordered pair as  $(x, y)$ .

For example, to find an ordered pair that satisfies the function  $y = 2x - 5$ :

1. Pick a value for  $x$ .  $x = 3$
2. Substitute  $x = 3$  into  $y = 2x - 5$  and solve for  $y$ .  
 $y = 2x - 5$   
 $y = 2(3) - 5$   
 $y = 6 - 5$   
 $y = 1$
3. Write the ordered pair as  $(x, y)$ .  $(3, 1)$

In general, there are infinitely many ordered pairs that satisfy a function. For the function  $y = 2x - 5$ , for any  $x$  you choose you can find a  $y$ . Once you have several ordered pairs that satisfy the function, you can plot the points on a grid to graph the function.

To graph a function:

1. Find several ordered pairs that satisfy the function.
2. Plot these points on a grid.
3. Use these points to sketch the graph of the function.

For example, to graph the function  $y = 2x - 5$ :

1. Find several ordered pairs that satisfy the function.

$x$	$y$
3	1
2	-1
1	-3
0	-5
2. Plot these points on a grid.
3. Use these points to sketch the graph of the function. See Figure 11.1.1.

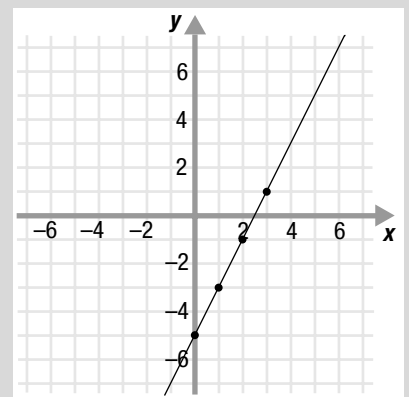


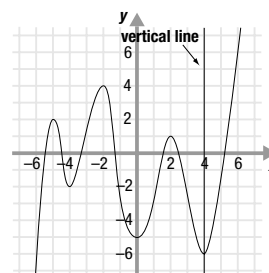
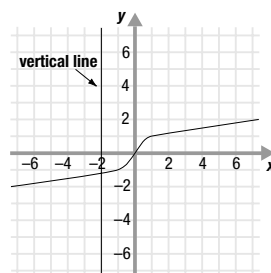
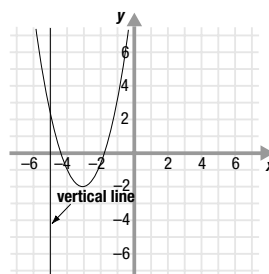
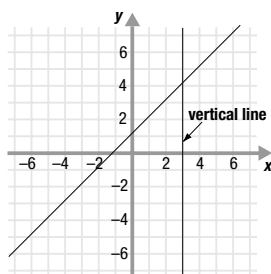
Figure 11.1.1

## Vertical Line Test

Remember, a function is a relation that has a unique value,  $y$ , for any value of  $x$ . Therefore, you can tell if a graph is the graph of a function by using the vertical line test: if you draw a vertical line anywhere through the graph of a function, it will intersect the graph in only one place. Likewise, if a vertical line intersects a graph in more than one place then the graph is not the graph of a function.

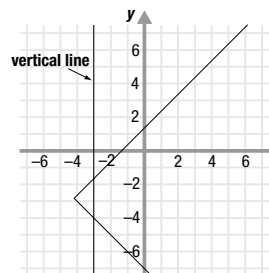
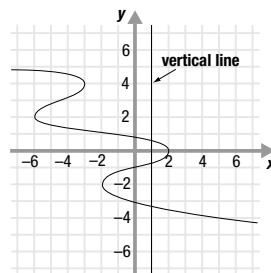
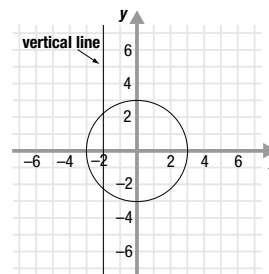
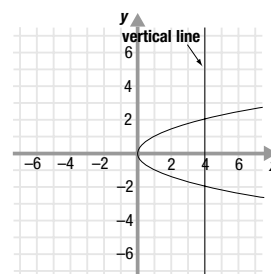
For example, here are the graphs of some functions:

*Any vertical line intersects each graph in one and only one place.*



Here are some graphs that are not functions:

*There is a vertical line that intersects these graphs in more than one place.*



## Sample Problems

1. For the function  $g(x) = 4x - 5$ , calculate:

a.  $g(0)$

b.  $g(2)$

c.  $g(-5)$

a. Substitute 0 for  $x$  and simplify.

$$\begin{aligned}g(0) &= 4(0) - 5 \\&= 0 - 5 \\&= -5\end{aligned}$$

b. Substitute 2 for  $x$  and simplify.

$$\begin{aligned}g(2) &= 4(2) - 5 \\&= \underline{\hspace{2cm}} \\&= \underline{\hspace{2cm}}\end{aligned}$$

c. Substitute  $-5$  for  $x$  and simplify.

$$\begin{aligned}g(\underline{\hspace{1cm}}) &= 4(\underline{\hspace{1cm}}) - 5 \\&= \underline{\hspace{2cm}} \\&= \underline{\hspace{2cm}}\end{aligned}$$

2. Find the domain and range for each of the functions below.

a.  $y = x - 7$

b.  $y = \frac{3}{2x}$

c.  $y = \sqrt{x - 2}$

a. Find all of the  $x$ -values for which the function is defined.

domain: all real numbers

Find all possible  $y$ -values, given the possible  $x$ -values.

range: all real numbers

b. Find all of the  $x$ -values for which the function is defined.

domain: \_\_\_\_\_

Find all possible  $y$ -values, given the possible  $x$ -values.

range: \_\_\_\_\_

c. Find all of the  $x$ -values for which the function is defined.

domain: \_\_\_\_\_

Find all possible  $y$ -values, given the possible  $x$ -values.

range: \_\_\_\_\_

## Answers to Sample Problems

b.  $8 - 5$

3

c.  $-5, -5$   
 $-20 - 5$   
 $-25$

b. *all real numbers except  $x = 0$*   
( $x \neq 0$ )

*all real numbers except  $y = 0$*   
( $y \neq 0$ )

c. *all real numbers greater than or equal to 2 ( $x \geq 2$ )*

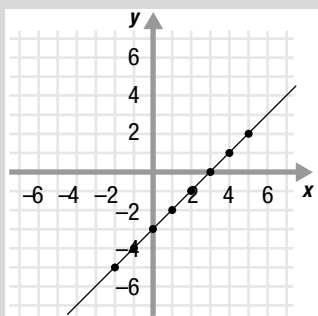
*all real numbers greater than or equal to 0 ( $y \geq 0$ )*

**Answers to Sample Problems**

a.

- 1
- 0
- 1
- 2

b., c.

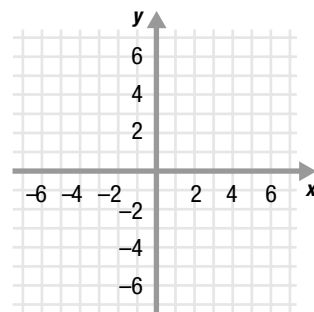


3. Complete the table below for the function  $y = x - 3$ . Then use the table to graph the function on the grid.

a. Complete the table.

$x$	$x - 3$
-2	-5
-1	-4
0	-3
1	-2
2	_____
3	_____
4	_____
5	_____

b. Plot the points.



c. Sketch the function.



# LINEAR FUNCTIONS

## Summary

### Linear Functions

A linear function is a function whose graph is a straight line.

The equation of a linear function may be written in the form:

$$y = Ax + B \quad \text{or} \quad f(x) = Ax + B$$

For example, the following functions are linear functions:

$$y = x \quad \text{or} \quad f(x) = x$$

$$y = 3x + 1 \quad \text{or} \quad f(x) = 3x + 1$$

$$y = -2x + 5 \quad \text{or} \quad f(x) = -2x + 5$$

These functions are **not** linear functions:

$$y = x^2 + 6 \quad \text{or} \quad f(x) = x^2 + 6$$

$$y = \frac{1}{x} \quad \text{or} \quad f(x) = \frac{1}{x}$$

The output value,  $y$ , **depends** on the input value,  $x$ , so  $y$  is called the **dependent** variable. Because you pick the number  $x$  (**independent** of anything else) it is called the **independent** variable.

### Domain and Range of a Linear Function

For any linear function  $y = Ax + B$ ,  $A \neq 0$ :

- The domain is all real numbers. That's the interval  $(-\infty, \infty)$ .
- The range is all real numbers. That's the interval  $(-\infty, \infty)$ .

### Graphs of Linear Functions

When you have two or more linear functions of the form  $y = Ax + B$  with the same coefficient  $A$  of  $x$ , the graphs of the functions look almost the same. The lines have the same steepness but are shifted up or down depending on the value of the constant,  $B$ .

For example, look at the graphs of these functions in Figure 11.1.2:

$$y = 4x$$

$$y = 4x + 2$$

$$y = 4x - 3$$

Notice:

- The line  $y = 4x + 2$  is the line  $y = 4x$  shifted up 2 units.
- The line  $y = 4x - 3$  is the line  $y = 4x$  shifted down 3 units.

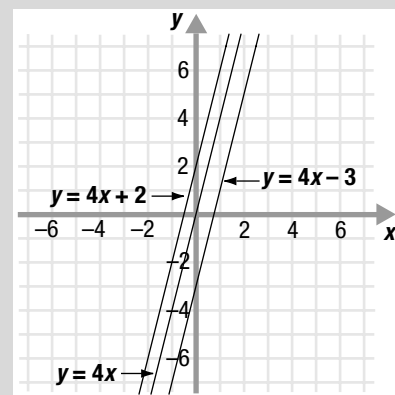


Figure 11.1.2

Here the coefficient of  $x$  in each function is 4.

## Graphing Linear Functions

If you know the equation that describes a function, you can use the equation to graph the function.

To graph a function of the form  $f(x) = Ax + B$ :

1. Find several ordered pairs that satisfy the function.
2. Plot these points on a grid.
3. Use these points to sketch the graph of the function.

For example, to graph the function  $y = f(x) = 2x + 1$ :

1. Pick several values for  $x$  and find the corresponding values for  $f(x)$ .

$x$	$f(x)$
-3	-5
-1	-1
0	1
2	5

2. Plot the ordered pairs and draw a line through them. See Figure 11.1.3.

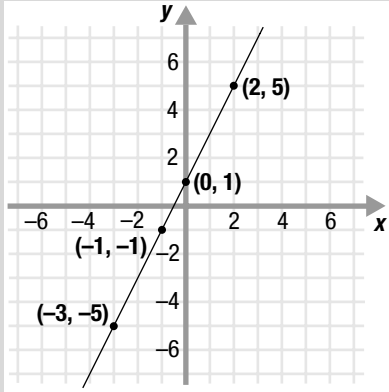


Figure 11.1.3

## Absolute Value Functions

Here are four absolute value functions that are related to the linear function  $y = Ax + B$ :

$$y = |Ax| + B$$

$$y = -|Ax| + B$$

$$y = |Ax + B|$$

$$y = -|Ax + B|$$

*If there is a negative sign in front of the absolute value then the “V” is inverted.*

The graphs of these absolute value functions are in the shape of a “V” that is either upright or inverted.

## Domain and Range of Absolute Value Functions

For an absolute value function based on a linear function:

- The domain is all real numbers.
- For functions of the form  $y = |Ax| + B$ , the range is all real numbers  $\geq B$ . That’s the interval  $[B, \infty)$ .
- For functions of the form  $y = -|Ax| + B$  the range is all real numbers  $\leq B$ . That’s the interval  $(-\infty, B]$ .
- For functions of the form  $y = |Ax + B|$  the range is all real numbers  $\geq 0$ . That’s the interval  $[0, \infty)$ .
- For functions of the form  $y = -|Ax + B|$  the range is all real numbers  $\leq 0$ . That’s the interval  $(-\infty, 0]$ .

For example, the domain and range of each of the functions below are:

$y = |3x| + 1$       domain: all real numbers; the interval  $(-\infty, \infty)$   
 range: real numbers  $\geq 1$ ; the interval  $[1, \infty)$ .

$y = -|3x| + 1$       domain: all real numbers; the interval  $(-\infty, \infty)$   
 range: real numbers  $\leq 1$ ; the interval  $(-\infty, 1]$ .

$y = |3x + 1|$       domain: all real numbers; the interval  $(-\infty, \infty)$   
 range: real numbers  $\geq 0$ ; the interval  $[0, \infty)$ .

$y = -|3x + 1|$       domain: all real numbers; the interval  $(-\infty, \infty)$   
 range: real numbers  $\leq 0$ ; the interval  $(-\infty, 0]$ .

## Graphs of Absolute Value Functions

When you have two or more absolute value functions each based on a linear function and each having the same coefficient of  $x$  with no leading negative sign, their graphs look almost the same. Their only difference is the “V’s” are shifted up or down depending on the constant term.

For example, look at the graphs of these absolute value functions in Figure 11.1.4:

$$y = |x|$$

$$y = |x| + 2$$

$$y = |x| - 3$$

Notice:

- The graph of  $y = |x| + 2$  is the graph of  $y = |x|$  shifted up 2 units.
- The graph of  $y = |x| - 3$  is the graph of  $y = |x|$  shifted down 3 units.

When there is a negative sign in front of the absolute value sign, the rules above still hold but the graphs of the functions are inverted “V’s” rather than upright “V’s.”

For example, look at the graphs of the functions in Figure 11.1.5:

$$y = -|x|$$

$$y = -|x| + 2$$

$$y = -|x| - 3$$

Notice:

- The graph of  $y = -|x| + 2$  is the graph of  $y = -|x|$  shifted up 2 units.
- The graph of  $y = -|x| - 3$  is the graph of  $y = -|x|$  shifted down 3 units.

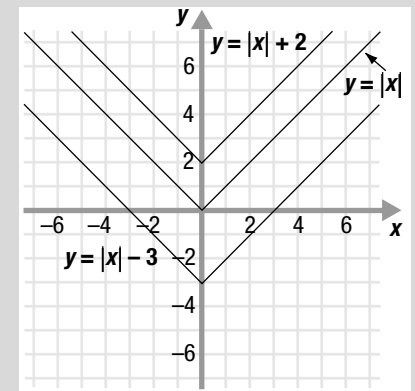


Figure 11.1.4

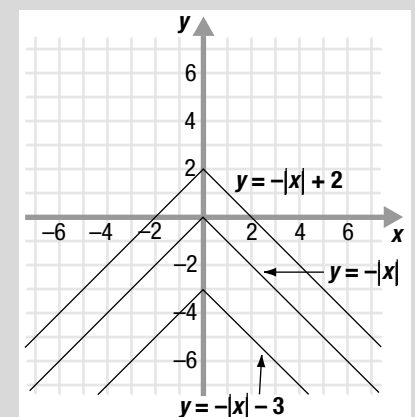


Figure 11.1.5

## Sample Problems

1. Find the domain and range of each of the functions below.  
In b., write the domain and the range in interval notation.

a.  $y = x - 1$

b.  $y = |x + 2|$

c.  $y = 5 - 3x$

- a. The domain is all the possible values for  $x$ . domain: all real numbers

The range is all the possible values for  $y$ . range: all real numbers

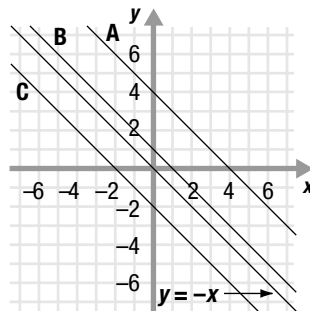
- b. The domain is all the possible values for  $x$ . domain: \_\_\_\_\_

The range is all the possible values for  $y$ . range: \_\_\_\_\_

- c. The domain is all the possible values for  $x$ . domain: \_\_\_\_\_

The range is all the possible values for  $y$ . range: \_\_\_\_\_

2. The graph of the function  $y = -x$  is shown below. Determine the equations of the other lines shown.



- a. Find the equation of line A. This line is shifted **up** 4 units, so its equation is  $y = -x + 4$ .

- b. Find the equation of line B. The equation is: \_\_\_\_\_

- c. Find the equation of line C. The equation is: \_\_\_\_\_

b. the interval  $(-\infty, \infty)$

the interval  $[0, \infty)$

c. all real numbers

all real numbers

b.  $y = -x + 1$

c.  $y = -x - 2$

# QUADRATIC FUNCTIONS

## Summary

### Quadratic Functions

A quadratic function is a function whose graph is a parabola.

### Equations of Quadratic Functions

The equation of a quadratic function may be written in the form:

$$y = Ax^2 + Bx + C$$

or  $f(x) = Ax^2 + Bx + C$

where  $A \neq 0$ .

For example, the following functions are quadratic functions:

$$y = 4x^2 + 5$$

$$y = 4x^2 + 0x + 5$$

$$f(x) = -3x^2 + 2x - 8$$

$$f(x) = -3x^2 + 2x + (-8)$$

These functions are **not** quadratic functions:

$$y = \frac{4}{x^2}$$

$$y = x + 7$$

$$y = x^3 + 5x^2 - 2$$

### The Graph of $y = Ax^2 + Bx + C$ when $A > 0$

The graph of a quadratic function is a parabola that opens up or down depending on the coefficient of the  $x^2$ -term. If the coefficient of the  $x^2$ -term is positive, the parabola opens up.

For example, look at the graphs of these parabolas with positive  $x^2$ -terms. See Figure 11.1.6:

$$y = x^2$$

$$y = x^2 + 2$$

$$y = x^2 - 3$$

Notice:

- All of the graphs open up.
- The graph of  $y = x^2 + 2$  is the graph of  $y = x^2$  shifted up 2 units.
- The graph of  $y = x^2 - 3$  is the graph of  $y = x^2$  shifted down 3 units.

*Notice that in each case the coefficient of the  $x^2$ -term is nonzero.*

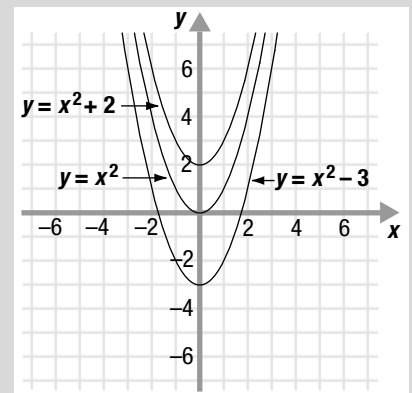


Figure 11.1.6

*When the coefficient of the  $x^2$ -term is positive, the vertex of the parabola is the low point.*

## The Graph of $y = Ax^2 + Bx + C$ when $A < 0$

The graph of a quadratic function is a parabola that opens up or down depending on the coefficient of the  $x^2$ -term. If the coefficient of the  $x^2$ -term is negative, the parabola opens down.

For example, look at the graphs of these parabolas where the coefficient of  $x^2$  is negative. See Figure 11.1.7:

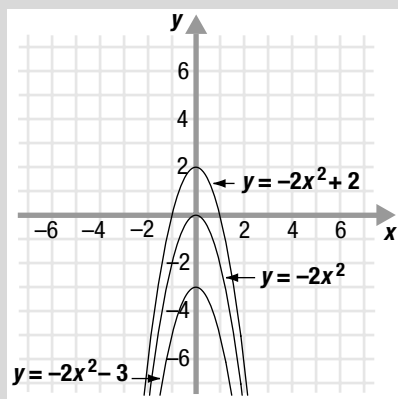


Figure 11.1.7

When the coefficient of the  $x^2$ -term is negative, the vertex of the parabola is the high point.

$$y = -2x^2$$

$$y = -2x^2 + 2$$

$$y = -2x^2 - 3$$

Notice:

- All of the graphs open down.
- The graph of  $y = -2x^2 + 2$  is the graph of  $y = -2x^2$  shifted up 2 units.
- The graph of  $y = -2x^2 - 3$  is the graph of  $y = -2x^2$  shifted down 3 units.

## Domain and Range of Quadratic Functions

For a quadratic function  $y = Ax^2 + Bx + C$ :

- The domain is all real numbers.
- When  $A > 0$ , the range is all real numbers greater than or equal to the  $y$ -coordinate of the vertex of the parabola (the lowest point).
- When  $A < 0$ , the range is all real numbers less than or equal to the  $y$ -coordinate of the vertex of the parabola (the highest point).

For example, the domain and range of each of the functions below are:

$$y = x^2 \quad \begin{array}{l} \text{domain: all real numbers; the interval } (-\infty, \infty) \\ \text{range: all real numbers } \geq 0; \text{ the interval } [0, \infty) \end{array}$$

$$y = x^2 + 2 \quad \begin{array}{l} \text{domain: all real numbers; the interval } (-\infty, \infty) \\ \text{range: all real numbers } \geq 2; \text{ the interval } [2, \infty) \end{array}$$

$$y = 2x^2 - 3 \quad \begin{array}{l} \text{domain: all real numbers; the interval } (-\infty, \infty) \\ \text{range: all real numbers } \geq -3; \text{ the interval } [-3, \infty) \end{array}$$

$$y = -5x^2 \quad \begin{array}{l} \text{domain: all real numbers; the interval } (-\infty, \infty) \\ \text{range: all real numbers } \leq 0; \text{ the interval } (-\infty, 0] \end{array}$$

$$y = -x^2 + 2 \quad \begin{array}{l} \text{domain: all real numbers; the interval } (-\infty, \infty) \\ \text{range: all real numbers } \leq 2; \text{ the interval } (-\infty, 2] \end{array}$$

$$y = -x^2 - 3 \quad \begin{array}{l} \text{domain: all real numbers; the interval } (-\infty, \infty) \\ \text{range: all real numbers } \leq -3; \text{ the interval } (-\infty, -3] \end{array}$$

## Intercepts

### The $y$ -Intercept

The  $y$ -intercept of a function is the point where the graph crosses the  $y$ -axis; that is, the point where  $x$  is 0. A parabola of the form  $y = Ax^2 + Bx + C$  always has one  $y$ -intercept.

To find the  $y$ -intercept of a function without a graph:

1. Set  $x$  equal to 0.
2. Solve for  $y$ .
3. Write the ordered pair as  $(0, y\text{-value})$ .

For example, to find the  $y$ -intercept of  $y = x^2 - 3x - 10$ :

1. Set  $x$  equal to 0.  $y = (0)^2 - 3(0) - 10$
2. Solve for  $y$ .  $y = 0 - 0 - 10$   
 $y = -10$
3. Write the ordered pair.  $(0, -10)$

So the  $y$ -intercept of  $y = x^2 - 3x - 10$  is  $(0, -10)$ .

In general, if  $y = Ax^2 + Bx + C$ , then the  $y$ -intercept of the function is  $(0, C)$ .

### The $x$ -Intercept

The  $x$ -intercepts of a function are the points where the graph crosses the  $x$ -axis; that is, the points where  $y$  is 0. A parabola can have zero, one, or two  $x$ -intercepts (because a parabola can cross the  $x$ -axis in zero, one, or two places).

To find the  $x$ -intercept(s) of a function  $y = f(x)$  without its graph:

1. Set  $y$  equal to 0.
2. Solve for  $x$ .
3. Write the ordered pair(s) as  $(x\text{-value}, 0)$ .

For example, to find the  $x$ -intercepts of  $y = x^2 - 3x - 10$ :

1. Set  $y$  equal to 0.  $0 = x^2 - 3x - 10$
2. Solve for  $x$ .  $(x + 2)(x - 5) = 0$   
 $x + 2 = 0$  or  $x - 5 = 0$   
 $x = -2$  or  $x = 5$
3. Write the ordered pairs.  $(-2, 0)$  and  $(5, 0)$

So the two  $x$ -intercepts of  $y = x^2 - 3x - 10$  are  $(-2, 0)$  and  $(5, 0)$ .

As another example, to find the  $x$ -intercept of  $y = x^2 + 2x + 1$ :

1. Set  $y$  equal to 0.  $0 = x^2 + 2x + 1$
2. Solve for  $x$ .  $(x + 1)(x + 1) = 0$   
 $x + 1 = 0$   
 $x = -1$
3. Write the ordered pair.  $(-1, 0)$

So the only  $x$ -intercept of  $y = x^2 + 2x + 1$  is  $(-1, 0)$ .

As a third example, to find the  $x$ -intercepts of  $y = x^2 + 9$ :

1. Set  $y$  equal to 0.  $0 = x^2 + 9$
2. Solve for  $x$ .  $x^2 = -9$   
 $x = \pm\sqrt{-9}$   
 $x = \pm 3i$

This equation has two imaginary solutions, so the graph doesn't cross the  $x$ -axis and it has no  $x$ -intercepts.

## The Vertex of a Parabola

When the parabola given by the quadratic function

$$y = Ax^2 + Bx + C$$

opens up ( $A > 0$ ), its vertex is its low point.

When it opens down ( $A < 0$ ), its vertex is its high point.

The  $x$ -coordinate of the vertex is given by:

$$x = -\frac{B}{2A}$$

To find the  $y$ -coordinate of the vertex, substitute  $x = -\frac{B}{2A}$  into the equation  $y = Ax^2 + Bx + C$ .

For example, to find the vertex of the parabola given by the quadratic function

$$y = x^2 - 3x - 10 \quad (\text{Here, } A = 1, B = -3, C = -10)$$

1. Find the  $x$ -coordinate of the vertex  $x = -\frac{B}{2A}$   
 $= \frac{-(-3)}{2(1)}$   
 $= \frac{3}{2}$
2. Find the  $y$ -coordinate of the vertex  $y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 10$   
 $= \frac{9}{4} - \frac{9}{2} - 10$   
 $= -\frac{49}{4}$

So the coordinates of the vertex are  $\left(\frac{3}{2}, -\frac{49}{4}\right)$ .

*Notice that  $\frac{3}{2}$ , the  $x$ -coordinate of the vertex, lies halfway between the  $x$ -coordinates of the two  $x$ -intercepts,  $x = -2$  and  $x = 5$ .*



As another example, to find the vertex of the parabola given by the quadratic function

$$y = x^2 + 2x + 1 \quad (\text{Here, } A = 1, B = 2, C = 1)$$

1. Find the  $x$ -coordinate of the vertex

$$\begin{aligned}x &= \frac{-B}{2A} \\ &= \frac{-2}{2(1)} \\ &= -1\end{aligned}$$

2. Find the  $y$ -coordinate of the vertex

$$\begin{aligned}y &= (-1)^2 + 2(-1) + 1 \\ &= 1 - 2 + 1 \\ &= 0\end{aligned}$$

So the coordinates of the vertex are  $(-1, 0)$ .

## Sample Problems

1. Find the domain and range of each of the functions below.

In b., write the domain and the range in interval notation.

a.  $y = x^2$

b.  $y = x^2 + 4$

c.  $y = x^2 - 1$

a. The domain is all the possible values for  $x$ . domain: all real numbers

The range is all the possible values for  $y$ . range:  $y \geq 0$

b. The domain is all the possible values for  $x$ . domain: \_\_\_\_\_

The range is all the possible values for  $y$ . range: \_\_\_\_\_

c. The domain is all the possible values for  $x$ . domain: \_\_\_\_\_

The range is all the possible values for  $y$ . range: \_\_\_\_\_

*Notice that the vertex,  $(-1, 0)$ , is the same as the one and only one  $x$ -intercept.*

## Answers to Sample Problems

b. *the interval  $(-\infty, \infty)$*

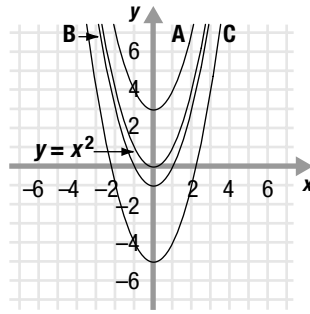
*the interval  $[4, \infty)$*

c. *all real numbers*

*all real numbers greater than or equal to  $-1$  ( $y \geq -1$ )*

**Answers to Sample Problems**

2. The graph of the function  $y = x^2$  is shown below. Determine the equations of the other parabolas shown by finding the number of units each parabola is shifted up or down relative to  $y = x^2$ .



b.  $y = x^2 - 1$

c. 5 units

$y = x^2 - 5$

b. 8

c. (0, 8)

b. -2, -4

c. (-2, 0), (-4, 0)

b. -1

c. (-3, -1)

- a. Find the number of units parabola A is shifted up from  $y = x^2$ . Shifted up: 3 units  
Write the equation of parabola A. The equation is:  $y = x^2 + 3$ .
- b. Find the number of units parabola B is shifted down from  $y = x^2$ . Shifted down: 1 unit  
Write the equation of parabola B. The equation is: \_\_\_\_\_
- c. Find the number of units parabola C is shifted down from  $y = x^2$ . Shifted down: \_\_\_\_\_  
Write the equation of parabola C. The equation is: \_\_\_\_\_

3. Find the  $y$ -intercept of the parabola  $y = x^2 + 6x + 8$ .

- a. Set  $x$  equal to 0.  $y = (0)^2 + 6(0) + 8$
- b. Solve for  $y$ .  $y = \underline{\hspace{2cm}}$
- c. Write the ordered pair.  $(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$

4. Find the  $x$ -intercepts of the parabola  $y = x^2 + 6x + 8$ .

- a. Set  $y$  equal to 0.  $0 = x^2 + 6x + 8$
- b. Solve for  $x$ .  $x = \underline{\hspace{1cm}}$  or  $x = \underline{\hspace{1cm}}$
- c. Write the ordered pairs.  $(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$  or  $(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$

5. Find the vertex of the parabola  $y = x^2 + 6x + 8$ .

- a. Find the  $x$ -coordinate of the vertex.  $x = -\frac{B}{2A}$   
 $= -\frac{6}{2(1)}$   
 $= -3$
- b. Find the  $y$ -coordinate of the vertex.  $y = \underline{\hspace{2cm}}$
- c. The coordinates of the vertex are.  $(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$



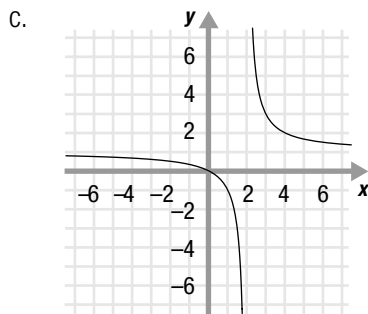
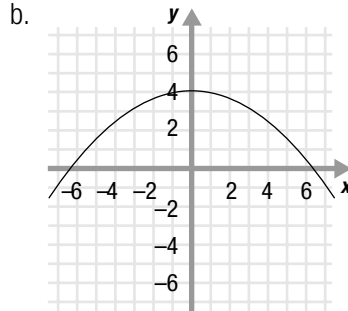
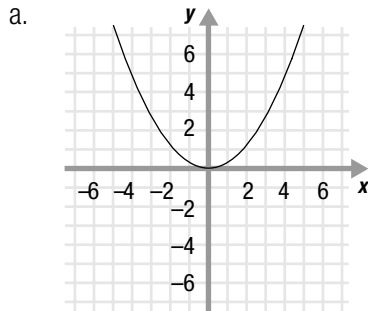
# EXPLORE

## Sample Problems

On the computer you used the Grapher to explore the relationship between a function and its graph.

1. Determine the domain and range of each function graphed below.

In b., write the domain and the range in interval notation.



- a. The domain is all the possible values for  $x$ . domain: all real numbers  
The range is all the possible values for  $y$ . range:  $y \geq 0$
- b. The domain is all the possible values for  $x$ . domain: \_\_\_\_\_  
The range is all the possible values for  $y$ . range: \_\_\_\_\_
- c. The domain is all the possible values for  $x$ . domain: \_\_\_\_\_  
The range is all the possible values for  $y$ . range: \_\_\_\_\_

2a. If  $f(x) = x + 8$ , find  $f(7)$ .

b. If  $f(x) = |x - 9|$ , find  $f(-3)$ .

c. If  $f(x) = 2x + 4$ , find  $f(5)$ .

- a. Substitute  $x = 7$  into  $f(x) = x + 8$  and simplify.
 
$$\begin{aligned} f(x) &= x + 8 \\ f(7) &= 7 + 8 \\ &= 15 \end{aligned}$$

b. Substitute  $x = -3$  into  $f(x) = |x - 9|$  and simplify.

c. Substitute  $x = 5$  into  $f(x) = 2x + 4$  and simplify.

## Answers to Sample Problems

b. the interval  $(-\infty, \infty)$

the interval  $(-\infty, 4]$

c. all real numbers except  $x = 2$

all real numbers except  $y = 1$

b.  $f(x) = |x - 9|$

$$f(-3) = |-3 - 9|$$

$$= |-12|$$

$$= 12$$

c.  $f(x) = 2x + 4$

$$f(5) = 2(5) + 4$$

$$= 10 + 4$$

$$= 14$$



## Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.



### Explain

### Functions and Graphs

1. Complete the table below for the function  $y = x + 7$ .

$x$	$y$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	

2. For the function  $f(x) = 2x + 3$ , calculate:

- $f(0)$
- $f(-4)$
- $f(2)$
- $f(-1)$
- $f(6)$

3. Find the domain and range of each of the functions below.

In d., write the domain and the range in interval notation.

- $y = x - 8$
- $y = 4x + 7$
- $y = x^2$
- $y = x^2 + 1$
- $y = \frac{1}{x}$

4. Complete the table below for the function  $y = x + 2$ . Then use the table to graph the function.

$x$	$y$
-3	
-2	
-1	
0	
1	
2	
3	
4	

5. For the function  $g(x) = x^2 - 5$ , calculate:

- $g(0)$
- $g(4)$
- $g(7)$
- $g(-2)$
- $g(-6)$

6. Find the domain of each of the functions below. In a., b., and c., also find the range.

- $y = 3x - 4$
- $y = \frac{x + 9}{7}$
- $y = x^3 + 5$
- $y = \frac{7}{x + 9}$
- $y = \frac{3}{x^2 - 4}$

7. For the function  $h(x) = x^3 + 1$ , calculate:

- a.  $h(2)$
- b.  $h(-1)$
- c.  $h(0)$
- d.  $h(-3)$
- e.  $h(1)$

8. Make a table of at least seven ordered pairs that satisfy the equation  $y = 2x - 5$ . Then use the table to graph the function.

9. The world population increases at a net rate of approximately 3 people per second. This increase can be written as a function:  $p(x) = 3x$ , where  $x$  represents seconds. Find the number of people by which the world's population will increase in:

- a. 1 day (86,400 seconds)
- b. 1 week (604,800 seconds)
- c. 1 year (31,536,000 seconds)

10. America's population increases at a net rate of approximately 1 person every 14 seconds. This increase can be written as a function;  $p(x) = \frac{x}{14}$ , where  $x$  represents seconds. Find the number of people by which America's population will increase in:

- a. 1 day (86,400 seconds)
- b. 1 week (604,800 seconds)
- c. 1 year (31,536,000 seconds)

11. Find the domain for each of the functions below. In c. and e., also find the range.

- a.  $f(x) = \frac{x}{x-3}$
- b.  $g(x) = \frac{2}{(x+1)(x+2)}$
- c.  $h(z) = \sqrt{z^3 + 8}$
- d.  $h(w) = \frac{(w+4)}{(w+4)(w-8)(w-11)}$
- e.  $g(q) = \sqrt{4 - q^2}$

12. Make a table with at least seven ordered pairs that satisfy the equation  $y = x^2 - 3$ . (Use both positive and negative values.) Then use your table to graph the function.

## Linear Functions

13. Circle the functions below that are linear.

$$y = 2x + 1$$

$$y = x^3 + 4x^2 + 2x + 5$$

$$y = 5 - 8x$$

$$y = -7x^2 + 11x + 2$$

$$y = 1 + 5x + 2x^4$$

14. The graph of the function  $y = |x|$  is shown on the grid in Figure 11.1.8. Determine the equations of the other lines shown.

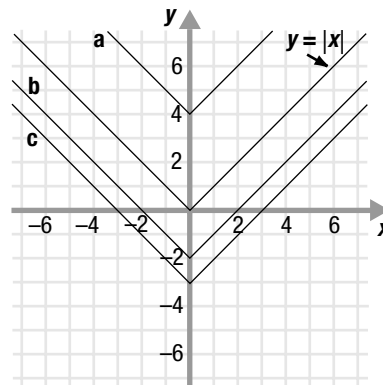


Figure 11.1.8

15. Find the domain and range of each of the functions below. In d., write the domain and the range in interval notation.

a.  $y = x + 3$

b.  $y = |x|$

c.  $y = 4x - 11$

d.  $y = |x - 4|$

16. The graph of the function  $y = 2x$  is shown in Figure 11.1.9. Determine the equations of the other lines shown.

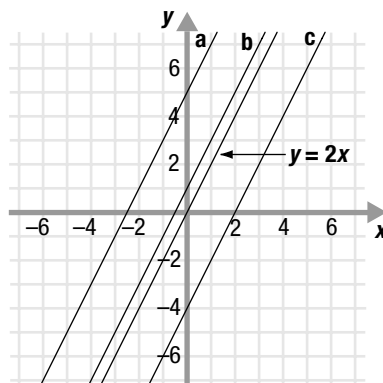


Figure 11.1.9

17. Find the domain and range of each of the functions below. In b., write the domain and the range in interval notation.
- $y = x - 5$
  - $y = |x| - 5$
  - $y = |x - 5|$

18. Graph the line  $y = x - 2$ .

19. The graph of the function  $y = |x + 1|$  is shown in Figure 11.1.10. Determine the equations of the other functions shown.

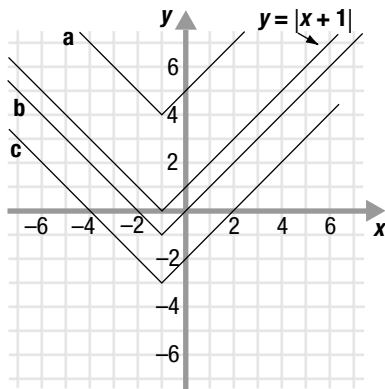


Figure 11.1.10

20. Find the domain and range of each of the functions below. In b., write the domain and the range in interval notation.
- $y = |4x + 3|$
  - $y = |2 - x|$
  - $y = 3 - 8x$

21. When Katy goes for a walk, she walks 2 miles then turns around and walks back home. The graph in Figure 11.1.11 shows her distance from home as she walks. Determine the domain and range of this function.

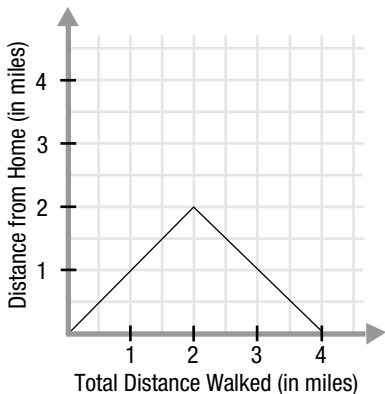


Figure 11.1.11

22. When Katy decides to go for a walk alone, she leaves immediately and walks at the rate of 4 miles an hour. When her dog Swirly comes, it takes them 15 minutes to get ready to leave, but they walk at the same speed. The graph in Figure 11.1.12 shows Katy's progress when she walks alone. On this grid, graph the line that shows her progress when she walks with her dog.

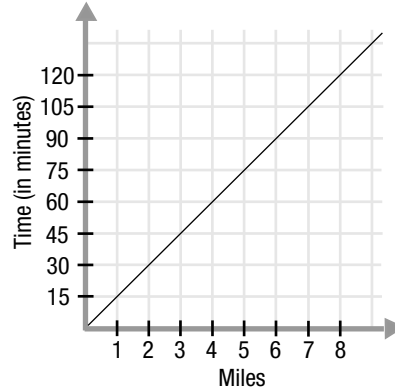


Figure 11.1.12

23. The graph of the line  $y = -2x$  is shown on the grid in Figure 11.1.13. Graph the other lines whose equations are shown.
- $y = -2x + 5$
  - $y = -2x + 1$
  - $y = -2x - 4$

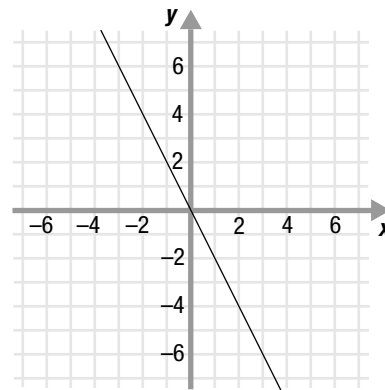


Figure 11.1.13

24. Find the domain and range of each of the functions below. In a., write the domain and the range in interval notation.
- $y = |2x| - 3$
  - $y = 2|x| - 3$
  - $y = |2x - 3|$

## Quadratic Functions

25. Circle the functions below that are quadratic.

$$y = 7 - x$$

$$y = 2x^3 + x^2 - 3x + 6$$

$$y = 4 - 2x + 6x^2$$

$$y = 8x^2 + 9x - 1$$

$$y = 4x - 11$$

26. Determine whether each of the following parabolas opens up or down.

a.  $y = x^2$

b.  $y = 3x^2 - 5$

c.  $y = -x^2 + 3x + 2$

d.  $y = 4 - x^2$

e.  $y = 2x^2 + 7x - 1$

f.  $y = -5x^2 + 4x - 3$

27. Find the domain and range of each of the functions below. In b., write the domain and the range in interval notation.

a.  $y = x^2 + 3$

b.  $y = x^2 - 5$

c.  $y = 3x^2$

d.  $y = x^2 + 4x + 4$

28. The graph of the function  $y = -x^2$  is shown on the grid in Figure 11.1.14. Determine the equations of the other parabolas shown.

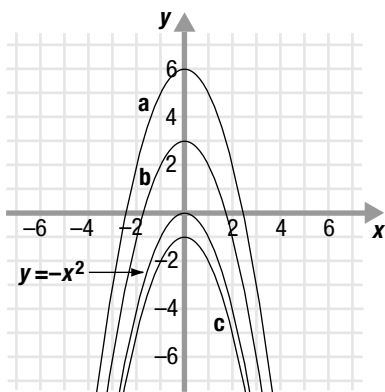


Figure 11.1.14

29. Find the  $x$ - and  $y$ -intercepts of each of the functions below.

a.  $y = x^2$

b.  $y = x^2 + 4x + 3$

c.  $y = x^2 + 8x + 16$

30. Find the domain and range of each of the functions below. In b., write the domain and the range in interval notation.

a.  $y = x^2 - 4$

b.  $y = -x^2 - 4$

c.  $y = x^2 + 2$

d.  $y = -x^2 + 2$

31. The graph of the function  $y = \frac{1}{2}x^2$  is shown on the grid in Figure 11.1.15. Determine the equations of the other parabolas shown.

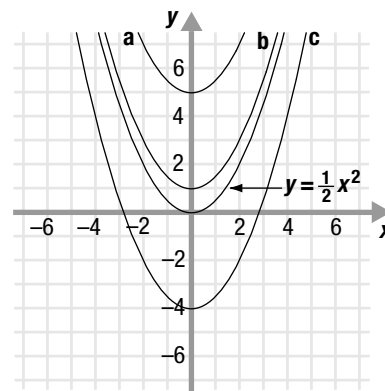


Figure 11.1.15

32. Find the vertex of the parabola given by each quadratic function below.

a.  $y = x^2 + 3x - 10$

b.  $y = x^2 - 2x + 1$

c.  $y = x^2 + 4$

33. Duncan threw a ball in the air. The height of this ball over time is graphed on the grid in Figure 11.1.16. Use the graph to determine the domain (time) and range (height) of this function.

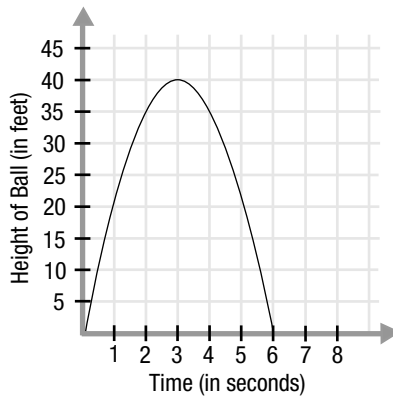


Figure 11.1.16

34. Duncan threw a ball in the air. The height of this ball over time is graphed on the grid in Figure 11.1.17. Next he climbed on top of his car (whose roof was 5 feet off the ground) and threw the ball the same way again. On this grid, draw the graph that shows the progress of the ball when it is thrown from the roof of the car.

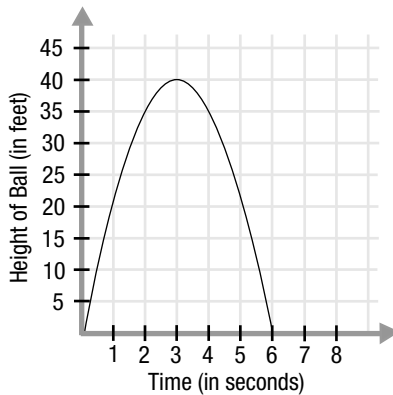


Figure 11.1.17

35. The graph of the function  $y = x^2$  is shown on the grid in Figure 11.1.18. Graph the other parabolas whose equations are shown below.

- $y = x^2 - 3$
- $y = x^2 - 6$
- $y = x^2 + 2$

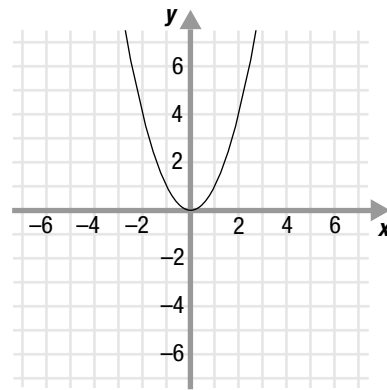


Figure 11.1.18

36. Find the domain and range of each of the functions below.

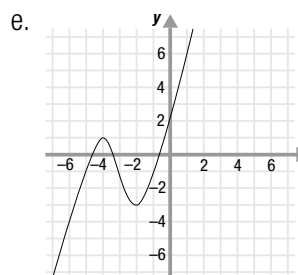
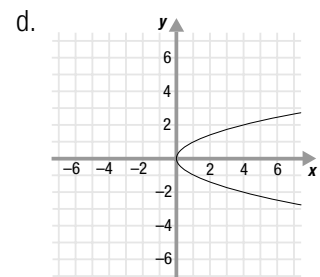
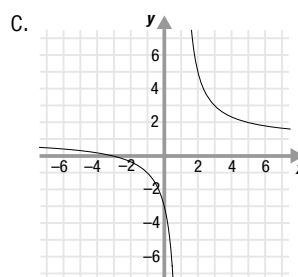
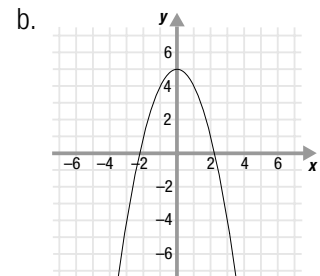
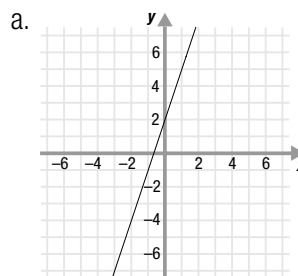
a.  $y = x^2 - 2$

b.  $y = (x - 2)^2$



## Explore

37. Determine which of the graphs below are functions by using the vertical line test.





38a. If  $f(x) = x$ , find  $f(6)$ .

b. If  $f(x) = x + 2$ , find  $f(3)$ .

c. If  $f(x) = 1 - x$ , find  $f(-2)$ .

d. If  $f(x) = 2x + 7$ , find  $f(-4)$ .

e. If  $f(x) = |x - 1|$ , find  $f(5)$ .

39. Two reservoirs are filled at the rates shown in Figure 11.1.19. After 5 days, which reservoir has received the most water?

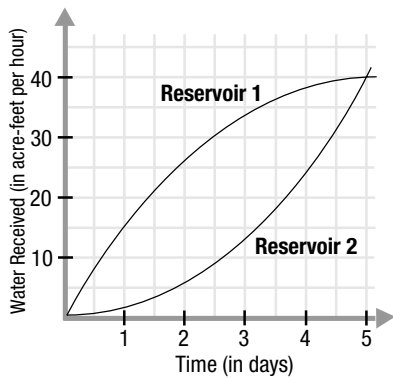
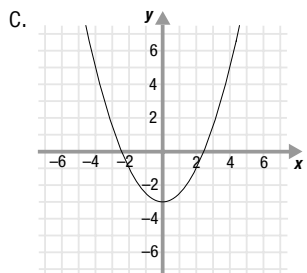
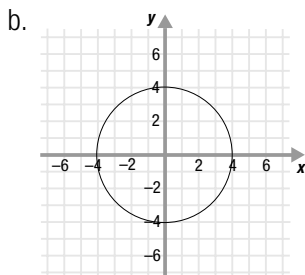
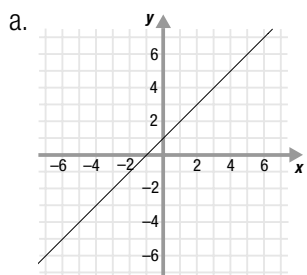


Figure 11.1.19

40. Determine the domain and range of each of the functions whose graphs are shown. In b., write the domain and the range in interval notation.



41a. If  $f(x) = 3 - x$ , find  $f(3)$ .

b. If  $f(x) = x^2 + 4$ , find  $f(1)$ .

c. If  $f(x) = |5 - x|$ , find  $f(8)$ .

d. If  $f(x) = 2 - x^3$ , find  $f(-2)$ .

e. If  $f(x) = 2$ , find  $f(7)$ .

42. The two curves in Figure 11.1.20 represent the speed of two cars in the first minute of travel. After one minute, is Car 1 ahead of Car 2 or is Car 2 ahead of Car 1?

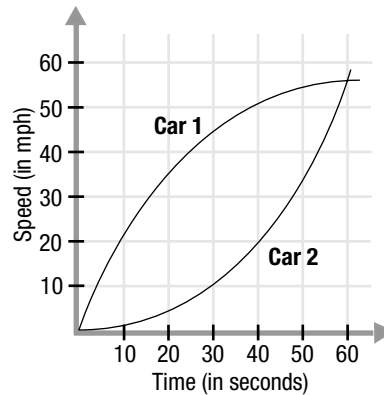


Figure 11.1.20



## Practice Problems

Here are some additional practice problems for you to try.

### Functions and Graphs

1. Which of the points below satisfy the function  $y = 4x + 5$ ?

(0, -5)            (1, 9)  
(-1, -9)             $(-\frac{5}{4}, 0)$   
(2, 13)

2. Which of the points below satisfy the function  $y = 3x - 6$ ?

(2, 0)            (0, -6)  
(3, -3)            (-1, -3)  
(1, -3)

3. Which of the points below satisfy the function  $y = 6x - 2$ ?

(0, 2)            (1, 4)  
(-1, -4)             $(\frac{1}{3}, 0)$   
(2, 10)

4. Which of the points below satisfy the equation  
 $y = x^2 - 4x + 2$ ?

(0, 2)            (1, 1)  
(0, 0)            (-1, 7)  
(2, -2)

5. Which of the points below satisfy the equation  
 $y = -x^2 + 3x - 5$

(0, 5)            (1, -3)  
(-1, -3)            (-1, -9)  
(2, -3)

6. Which of the points below satisfy the function  
 $g(x) = x^2 - 2x + 3$ ?

(0, 0)            (1, 2)  
(-1, 4)            (0, 3)  
(2, 11)

7. Make a table of at least three ordered pairs that satisfy the equation  $y = 3x - 2$ . Then use your table to graph the function.

8. Make a table of at least three ordered pairs that satisfy the equation  $y = -x + 3$ . Then use your table to graph the function.

9. Make a table of at least three ordered pairs that satisfy the equation  $y = 2x - 4$ . Then use your table to graph the function.

10. Make a table of at least seven ordered pairs that satisfy the equation  $y = \frac{1}{3}x^2 - 3$ . Then use your table to graph the function.

11. Make a table of at least seven ordered pairs that satisfy the equation  $y = -x^2 + 4$ . Then use your table to graph the function.

12. Make a table of at least seven ordered pairs that satisfy the equation  $y = \frac{1}{2}x^2 - 2$ . Then use your table to graph the function.

13. For the function  $f(x) = 2x^2 - 3x$ , calculate:

a.  $f(-2)$   
b.  $f(-1)$   
c.  $f(0)$   
d.  $f(1)$   
e.  $f(2)$

14. For the function  $g(t) = -3t^2 + t - 4$ , calculate:

- $g(-2)$
- $g(-1)$
- $g(0)$
- $g(1)$
- $g(2)$

15. For the function  $g(t) = 2t^2 + 5t$ , calculate:

- $g(-2)$
- $g(-1)$
- $g(0)$
- $g(1)$
- $g(2)$

16. For the function  $h(s) = \sqrt{3s + 6}$ , calculate:

- $h(-2)$
- $h(0)$
- $h(1)$
- $h(2)$
- $h(10)$

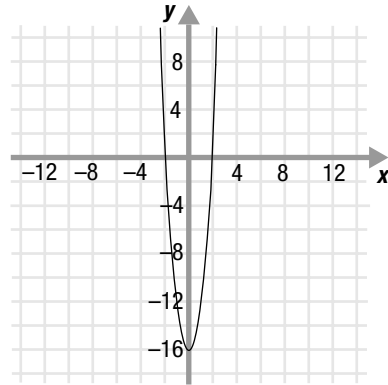
17. For the function  $f(x) = \sqrt{2x + 4}$ , calculate:

- $f(0)$
- $f(-1)$
- $f(-2)$
- $f(2)$
- $f(4)$

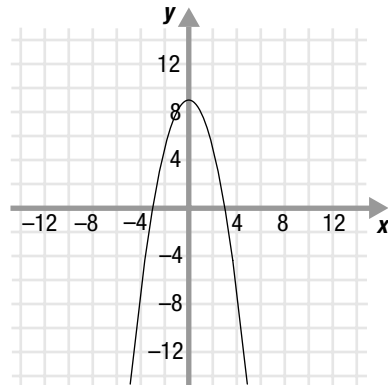
18. For the function  $f(x) = \frac{2x-1}{x+3}$ , calculate:

- $f\left(\frac{1}{2}\right)$
- $f(0)$
- $f(3)$
- $f(-1)$
- $f(-3)$

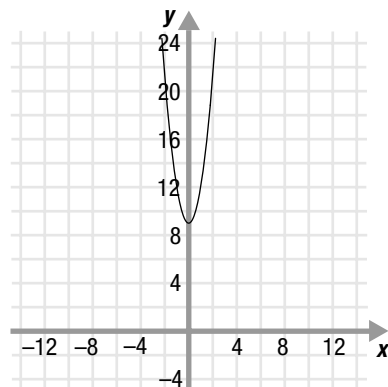
19. Use the graph of the function  $y = 4x^2 - 16$ , shown on the grid below, to find its domain and range.



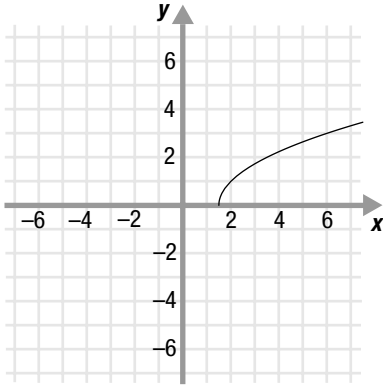
20. Use the graph of the function  $y = -x^2 + 9$ , shown on the grid below, to find its domain and range.



21. Use the graph of the function  $y = 3x^2 + 9$ , shown on the grid below, to find its domain and range. Write the domain and the range in interval notation.



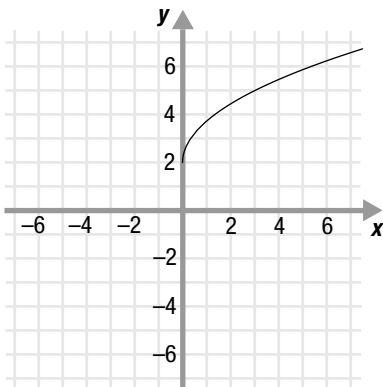
22. Use the graph of the function  $y = \sqrt{2x-3}$ , shown on the grid below, to find its domain and range.



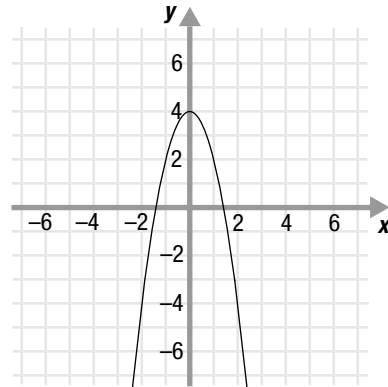
25. Use the graph of the function  $y = -3x^2 + 3$ , shown on the grid below, to find its domain and range.



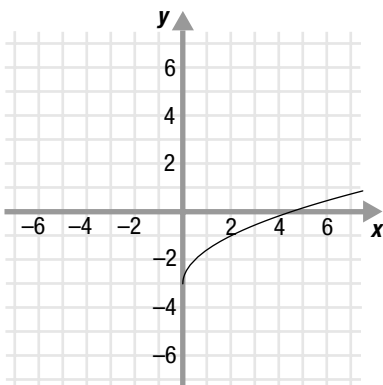
23. Use the graph of the function  $y = \sqrt{3x} + 2$ , shown on the grid below, to find its domain and range. Write the domain and the range in interval notation.



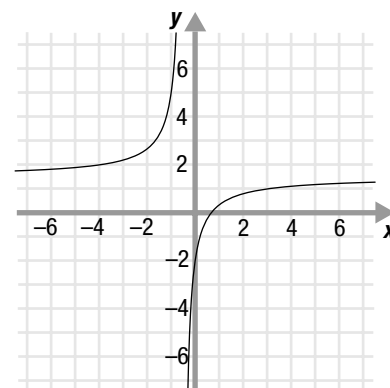
26. Use the graph of the function  $y = -2x^2 + 4$ , shown on the grid below, to find its domain and range. Write the domain and the range in interval notation.



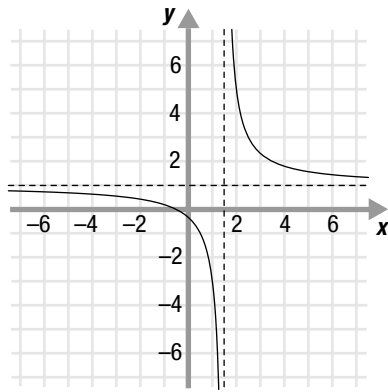
24. Use the graph of the function  $y = \sqrt{2x} - 3$ , shown on the grid below, to find its domain and range. Write the domain and the range in interval notation.



27. Use the graph of the function  $y = \frac{3x-2}{2x+1}$ , shown on the grid below, to find its domain and range.



28. Use the graph of the function  $y = \frac{2x+1}{2x-3}$ , shown on the grid below, to find its domain and range.



## Linear Functions

29. Circle the functions below that are linear.

$$f(x) = x^2 - 1 \quad f(x) = 3x + 5 \quad f(x) = 5^x$$

$$f(x) = \frac{x}{3} + 1 \quad f(x) = 7 - \frac{3}{x}$$

30. Circle the functions below that are linear.

$$f(x) = 7 - 4x \quad f(x) = x^5 + 7x - 2$$

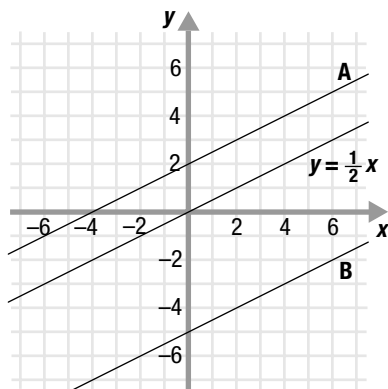
$$f(x) = 2^{x+1} \quad f(x) = x^{-1} + 3 \quad f(x) = \frac{x}{5} - 3$$

31. Circle the functions below that are linear.

$$f(x) = 2x - 1 \quad f(x) = x^4 - 15x^2 - 16$$

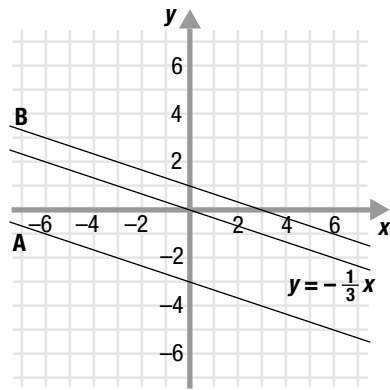
$$f(x) = \frac{x}{2} \quad f(x) = 10^x \quad f(x) = \frac{4}{x} + 3$$

32. The graph of the function  $y = \frac{1}{2}x$  is shown on the grid below.



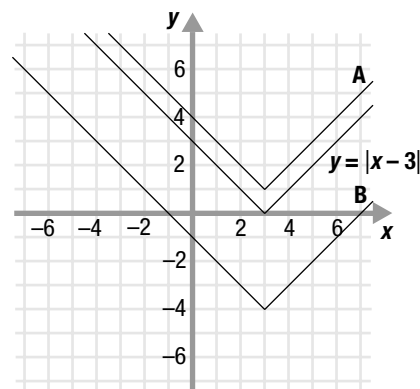
- a. Find the equation of line A.  
b. Find the equation of line B.

33. The graph of the function  $y = -\frac{1}{3}x$  is shown on the grid below.



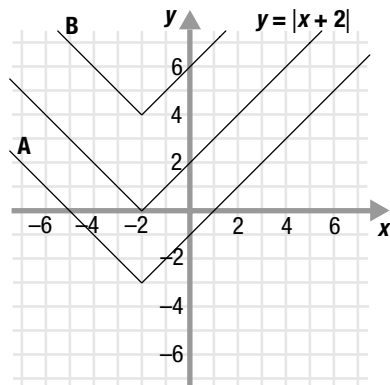
- a. Find the equation of line A.  
b. Find the equation of line B.

34. The graph of the function  $y = |x - 3|$  is shown on the grid below.



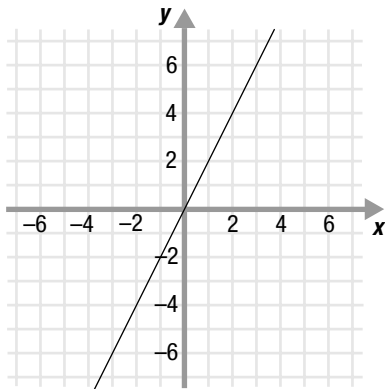
- a. Find the equation of A.  
b. Find the equation of B.

35. The graph of the function  $y = |x + 2|$  is shown on the grid below.

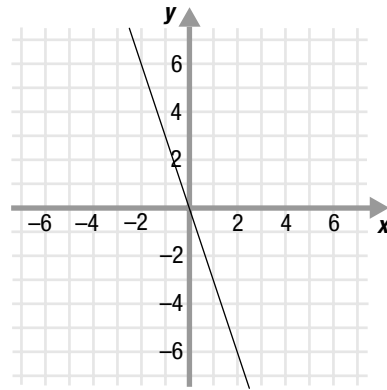


- a. Find the equation of A.  
b. Find the equation of B.

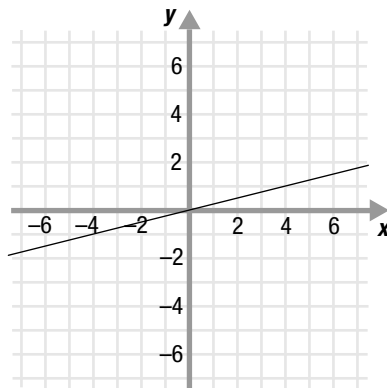
36. Graph the line  $y = 2x + 4$ .
37. Graph the line  $y = -\frac{1}{3}x - 4$ .
38. Graph the line  $y = -\frac{1}{2}x + 3$ .
39. Graph the function  $y = |x| + 2$ .
40. Graph the function  $y = -|x| - 3$ .
41. Graph the function  $y = -|x| + 1$ .
42. Graph the function  $y = |x + 1|$ .
43. Graph the function  $y = -|x - 2|$ .
44. Graph the function  $y = |x - 3|$ .
45. Find the domain and range of the function  $y = 7x + 9$ .
46. Find the domain and range of the function  $y = 10 - \frac{1}{5}x$ .
47. Find the domain and range of the function  $y = 5x - 2$ .  
Write them in interval notation.
48. Find the domain and range of the function  $y = |x| + 5$ .
49. Find the domain and range of the function  $y = -|x + 4|$ .  
Write them in interval notation.
50. Find the domain and range of the function  $y = -|x| - 1$ .
51. The graph of the line  $y = 2x$  is shown on the grid below. Graph the line whose equation is  $y = 2x + 1$  on the same grid.



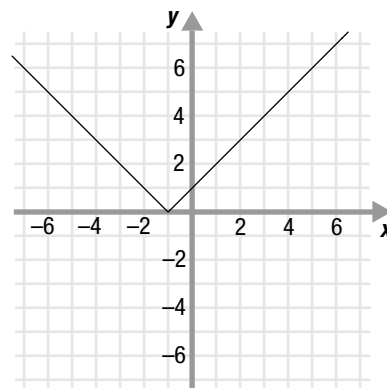
52. The graph of the line  $y = -3x$  is shown on the grid below. Graph the line whose equation is  $y = -3x + 2$  on the same grid.



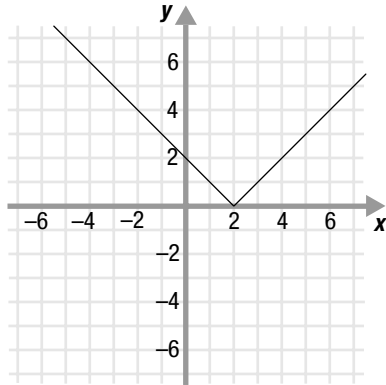
53. The graph of the line  $y = \frac{1}{4}x$  is shown on the grid below. Graph the line whose equation is  $y = \frac{1}{4}x - 5$  on the same grid.



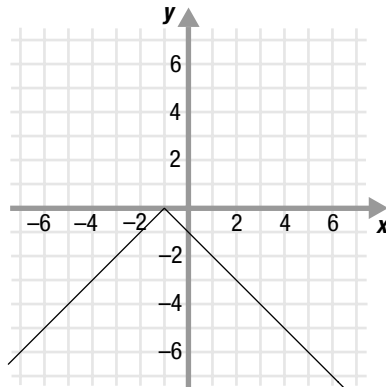
54. The graph of the function  $y = |x + 1|$  is shown on the grid below. Graph the function  $y = |x + 1| - 2$  on the same grid.



55. The graph of the function  $y = |x - 2|$  is shown on the grid below. Graph the function  $y = |x - 2| + 3$  on the same grid.



56. The graph of the function  $y = -|x + 1|$  is shown on the grid below. Graph the function  $y = -|x + 1| - 2$  on the same grid.



57. Circle the functions below that are quadratic.

$$y = x^2 - 5$$

$$y = -x^2$$

$$y = 3x + 7$$

$$y = x^3 + 2x^2 - 6$$

$$y = 9 - x^2$$

58. Circle the functions below that are quadratic.

$$y = x + 10$$

$$y = -5x^2 + 3x - 7$$

$$y = 3x^2 + 2$$

$$y = 7 - 2x - 10x^2$$

$$y = 4 - x$$

59. Circle the functions below that are quadratic.

$$y = x^2 + 4$$

$$y = -5x^2 - 9$$

$$y = 2x - 6$$

$$y = 2x^3 + 4x^2 + x + 1$$

$$y = 3 - 2x - x^2$$

60. Determine whether each of the following parabolas opens up or down.

a.  $y = 3x^2$

b.  $y = -\frac{1}{2}x^2$

c.  $y = 8 - 3x + 7x^2$

d.  $y = 25x^2 - 16$

e.  $y = 12 - 3x^2$

61. Determine whether each of the following parabolas opens up or down.

a.  $y = \frac{6}{7}x^2$

b.  $y = 2.34x^2 - 3.7x$

c.  $y = 9 + 4x - 2x^2$

d.  $y = 49x^2 - 64$

e.  $y = 32 - 2x^2$

62. Determine whether each of the following parabolas opens up or down.

a.  $y = -2x^2$

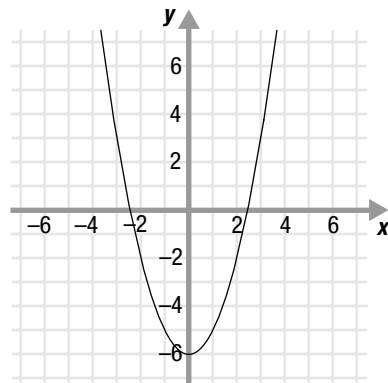
b.  $y = \frac{5}{7}x^2 - 3x + 7$

c.  $y = 4 - 3x - 5x^2$

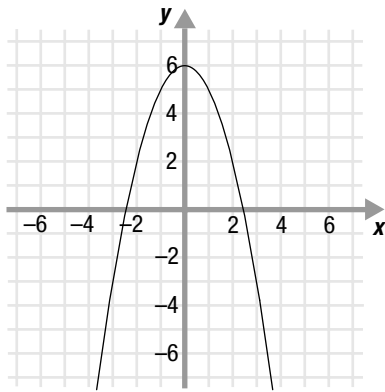
d.  $y = 6x^2 - 9$

e.  $y = 2x - x^2$

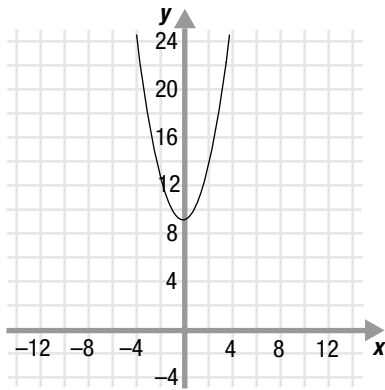
63. Use the graph of the function  $y = x^2 - 6$ , shown on the grid below, to find its domain and range. Write the domain and the range in interval notation.



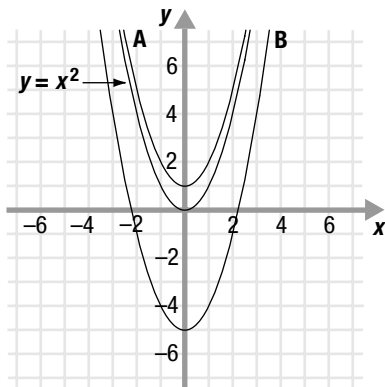
64. Use the graph of the function  $y = 6 - x^2$ , shown on the grid below, to find its domain and range. Write the domain and the range in interval notation.



65. Use the graph of the function  $y = x^2 + 9$ , shown on the grid below, to find its domain and range.

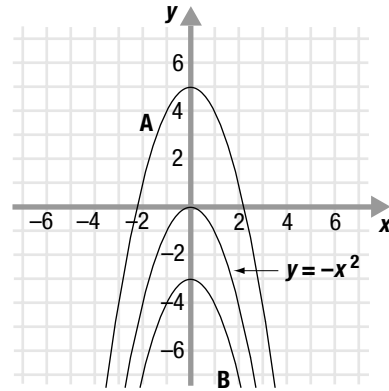


66. The graph of the function  $y = x^2$  is shown on the grid below.

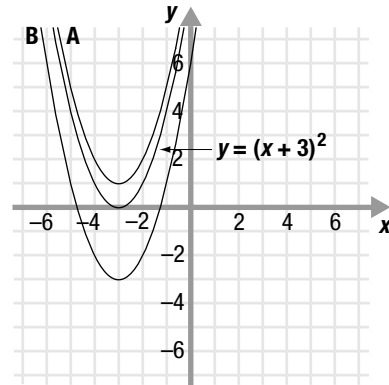


- Find the equation of parabola A.
- Find the equation of parabola B.

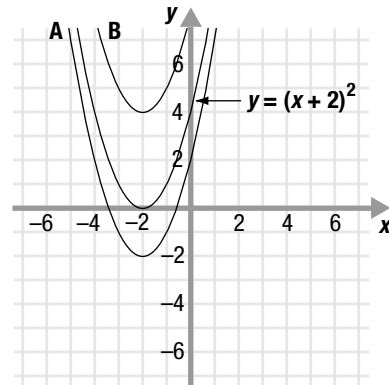
67. The graph of the function  $y = -x^2$  is shown on the grid that follows.



- Find the equation of parabola A.
  - Find the equation of parabola B.
68. The graph of the function  $y = (x + 3)^2$  is shown on the grid below.



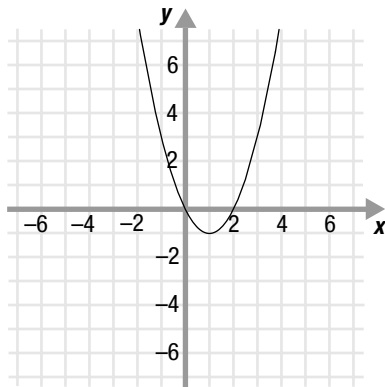
- Find the equation of parabola A.
  - Find the equation of parabola B.
69. The graph of the function  $y = (x + 2)^2$  is shown on the grid below.



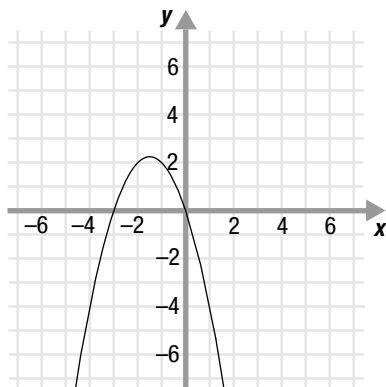
- Find the equation of parabola A.
- Find the equation of parabola B.



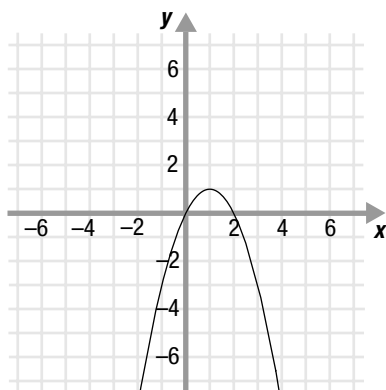
70. The graph of the function  $y = x^2 - 2x$  is shown on the grid below. On the same grid, graph the parabola whose equation is  $y = x^2 - 2x + 3$ .



71. The graph of the function  $y = -x^2 - 3x$  is shown on the grid below. On the same grid, graph the parabola whose equation is  $y = -x^2 - 3x + 4$ .



72. The graph of the function  $y = -x^2 + 2x$  is shown on the grid below. On the same grid, graph the parabola whose equation is  $y = -x^2 + 2x - 3$ .



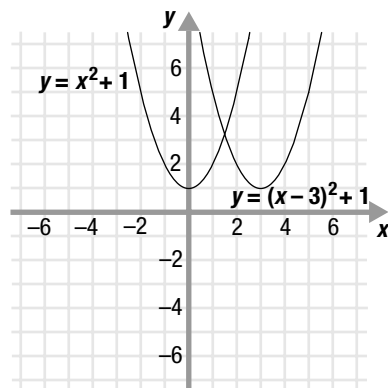
73. Find the  $x$ - and  $y$ -intercepts of the function  $y = x^2 + 3x - 10$ .

74. Find the  $x$ - and  $y$ -intercepts of the function  $y = 2x^2 - 18$ .

75. Find the  $x$ - and  $y$ -intercepts of the function  $y = x^2 + 2x - 15$ .

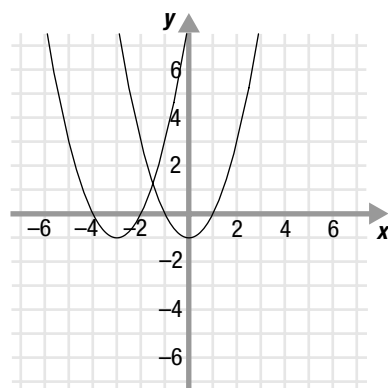
76. The graphs of the parabolas  $y = x^2 + 1$  and  $y = (x - 3)^2 + 1$  are shown on the grid below. Use these graphs to decide which of the statements below are true.

- Both functions have the same domain.
- Both functions have the same range.
- Both graphs have the same vertex.
- Both graphs have the same  $y$ -intercepts.



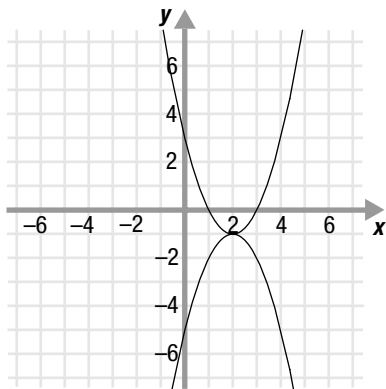
77. The graphs of the parabolas  $y = x^2 - 1$  and  $y = (x + 3)^2 - 1$  are shown on the grid below. Use these graphs to decide which of the statements below are true.

- Both functions have the same domain.
- Both functions have the same range.
- Both graphs have the same vertex.
- Both graphs have the same  $y$ -intercepts.

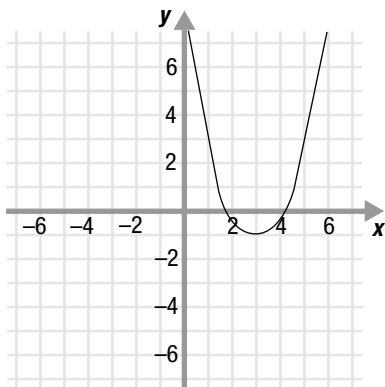


78. The graphs of the parabolas  $y = -(x-2)^2 - 1$  and  $y = (x-2)^2 - 1$  are shown on the grid below. Use these graphs to decide which of the statements below are true.

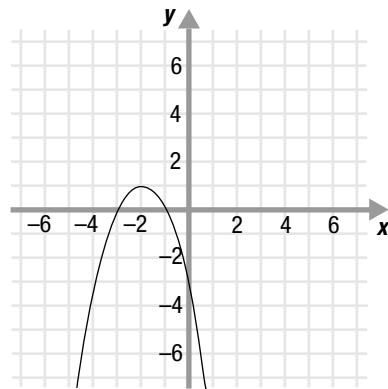
- Both functions have the same domain.
- Both functions have the same range.
- Both graphs have the same vertex.
- Both graphs have the same  $y$ -intercepts.



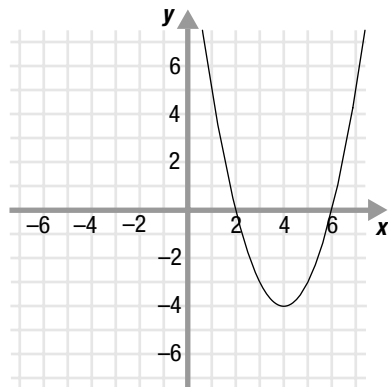
79. The graph of the parabola  $y = x^2 - 6x + 8$  is shown on the grid below. If the graph is moved down 4 units, what is the vertex of the new parabola?



80. The graph of the parabola  $y = -x^2 - 4x - 3$  is shown on the grid below. If the graph is moved up 3 units, what is the vertex of the new parabola?



81. The graph of the parabola  $y = x^2 - 8x + 12$  is shown on the grid below. If the graph is moved up 3 units, what is the vertex of the new parabola?



82. If the graph of the parabola  $y = x^2 - 6x + 5$  is moved down 3 units, what is the equation of the new parabola?

83. If the graph of the parabola  $y = x^2 + 4x + 3$  is moved up 3 units, what is the equation of the new parabola?

84. If the graph of the parabola  $y = -x^2 + 6x - 5$  is moved down 2 units, what is the equation of the new parabola?

## Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Which of the points below satisfy the function  $y = 3x^2 + 1$ ?

(0, 1)                      (-2, 13)

(3, 8)                      (1, 4)

(-4, 11)

2. Given the function  $f(x) = x^2 + 3x$ , find:

a.  $f(0)$

b.  $f(3)$

c.  $f(-2)$

d.  $f(-5)$

3. Use the graph of the function  $y = x^2 - 2$ , shown on the grid in Figure 11.1.21, to find its domain and range. Write the domain and the range in interval notation.

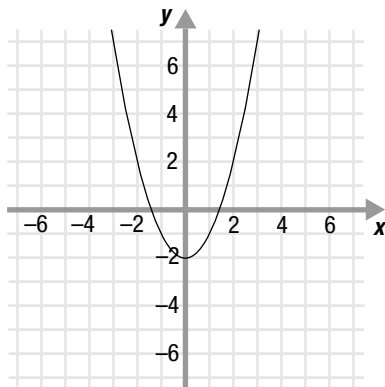


Figure 11.1.21

4. Given the functions  $f(x) = 4x - 5$  and  $g(x) = 1 - 3x$ , find:

a.  $f(7)$

b.  $g(7)$

c.  $f(-2)$

d.  $g(-2)$

5. The graph of the function  $y = 2x$  is shown on the grid in Figure 11.1.22.

a. Find the equation of line A.

b. Find the equation of line B.

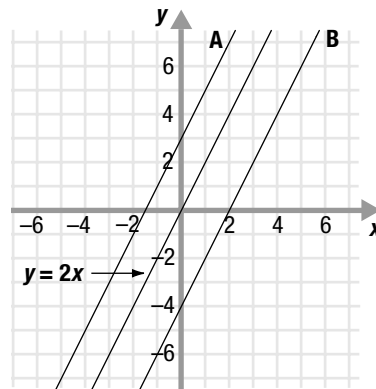


Figure 11.1.22

6. The graph of the function  $y = |x - 5|$  is shown on the grid in Figure 11.1.23. Use this graph to find the domain and range of  $y = |x - 5|$ . Write the domain and the range in interval notation.

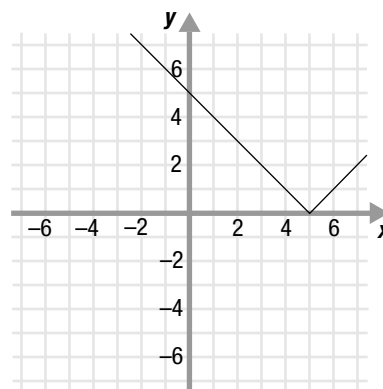


Figure 11.1.23

7. The graphs of the functions  $y = |x| + 3$  and  $y = -|x| + 3$  are shown on the grid in Figure 11.1.24. Use these graphs to decide which of the statements below are true.

Both functions have the same domain.

Both functions have the same range.

The point  $(0, 3)$  satisfies both equations.

The point  $(3, 0)$  satisfies both equations.

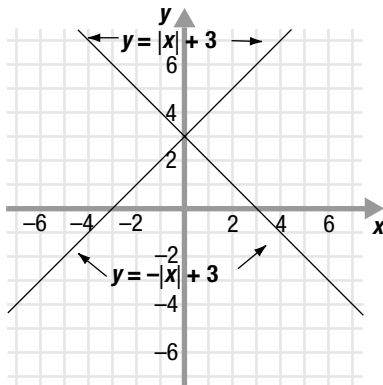


Figure 11.1.24

8. The graph of the function  $y = x$  is shown on the grid in Figure 11.1.25. Circle the points below that lie on the graph of  $y = x - 4$ .

$(5, 1)$

$(2, -2)$

$(1, 3)$

$(0, -4)$

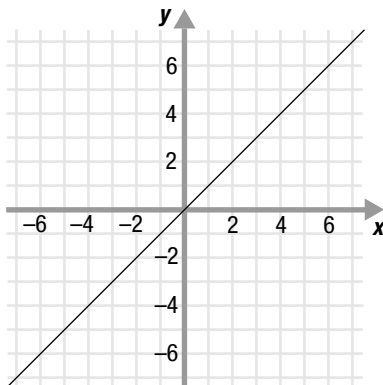


Figure 11.1.25

9. The graph of the parabola  $y = x^2 - 4x - 5$  is shown on the grid in Figure 11.1.26. Use this graph to help you find the  $x$ -intercepts of the function.

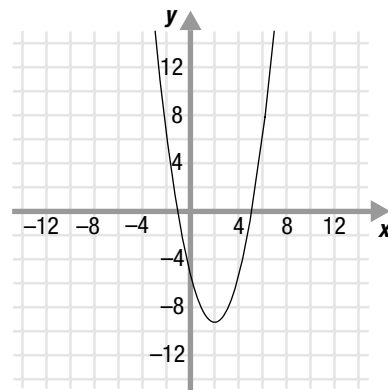


Figure 11.1.26

10. The graphs of the parabolas  $y = x^2 - 4$  and  $y = 2x^2 - 8$  are shown on the grid in Figure 11.1.27. Use these graphs to decide which of the statements below are true.

Both functions have the same domain.

Both functions have the same range.

Both graphs have the same vertex.

Both graphs have the same  $x$ -intercepts.

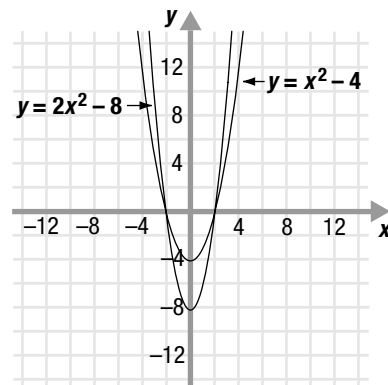


Figure 11.1.27

11. Circle the functions below that are **not** quadratic functions.

$y = 2x^2$

$y = 4x + 1$

$f(x) = -3x + 7$

$y = x^2 - 5x + 8$

$f(x) = -6 + x^2$

12. The graph of the parabola  $y = x^2 + 4x + 3$  is shown on the grid in Figure 11.1.28. If the graph is moved up 3 units, what is the vertex of the new parabola?

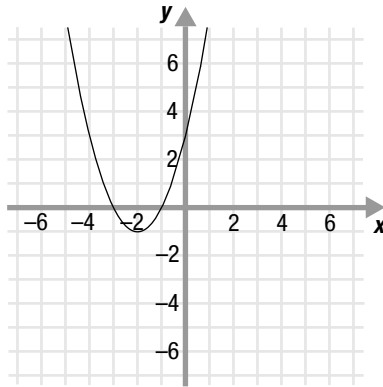


Figure 11.1.28

13. The graph of the function  $f(x) = \frac{1}{4}x^3 - 2$  is shown on the grid in Figure 11.1.29. Use this graph to find the values of:

- $f(2)$
- $f(0)$
- $f(-2)$

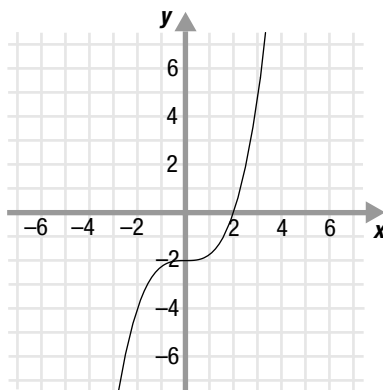


Figure 11.1.29

14. The graph of  $y = 2|x - 1| + 4$  is shown on the grid in Figure 11.1.30. Use the graph to decide which of the statements below are true.

The domain of  $y = 2|x - 1| + 4$  is all real numbers.

The range of  $y = 2|x - 1| + 4$  is the interval  $[4, \infty)$ .

For each input value,  $x$ , there is only one output value  $y$ .

For each output value,  $y$ , there is only one input value  $x$ .

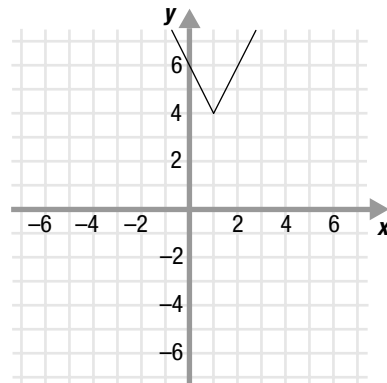
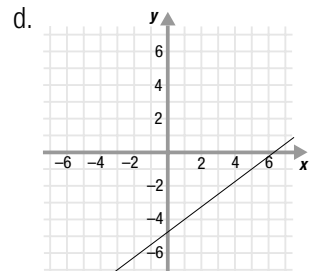
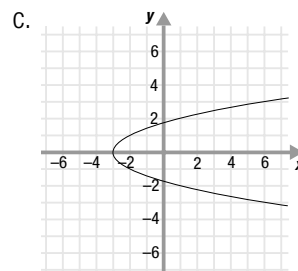
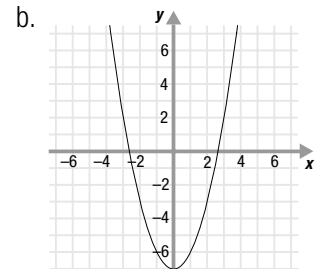
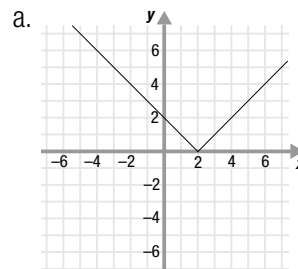


Figure 11.1.30

15. Determine which of the graphs below are functions by using the vertical line test.



16. The graph of the function  $y = -x^2 - 4x + 5$  is shown on the grid in Figure 11.1.31. Use this graph to:

- Find the domain of the function  $y = -x^2 - 4x + 5$ .
- Find the range of the function  $y = -x^2 - 4x + 5$ .
- Find the vertex of the function  $y = -x^2 - 4x + 5$ .

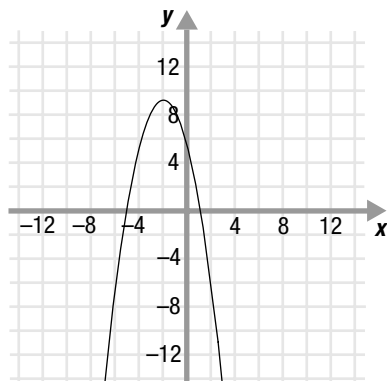


Figure 11.1.31

