LESSON EII.C – EQUATIONS AND INEQUALITIES

Here's what you'll learn in this lesson:

Linear

a. Solving linear equations

b. Solving linear inequalities

OVERVIEW

Once you know ways to simplify algebraic expressions, you can use these techniques as you solve equations and inequalities. Solving equations and inequalities is an important part of algebra.

In this lesson, you will solve linear equations and inequalities.

EXPLAIN

LINEAR

Summary

Equations and Inequalities

You have already learned how to simplify expressions such as $7(x + 5)$ and $3x - 2 + 5x$. When you relate expressions to each other with an equals sign or an inequality sign, you create an equation or an inequality.

An example of an equation is: $7(x + 5) = 3x - 2 + 5x$.

An example of an inequality is: $7(y + 5) > 3y - 2 + 5y$.

In this lesson you will review how to solve linear equations and linear inequalities.

Solving Linear Equations

To solve a linear equation, you must isolate the variable—that is, you must get the variable by itself on one side of the equation. Some equations are simple enough that you can find the solution just by looking at the equation. Other equations are more complicated and you need a systematic approach to find the solution.

Here's a way to solve a linear equation:

1. Remove any parentheses using the distributive property.
2. Combine like terms on each side of the equation.
3. Then do the following, as necessary:
   - Add the same quantity to both sides of the equation.
   - Subtract the same quantity from both sides of the equation.
   - Multiply or divide both sides of the equation by the same nonzero quantity.
4. Check the solution.

For example, to solve the equation $3x + 7 - 6x = -2$:

1. Combine the like terms, $3x$ and $-6x$. $\quad -3x + 7 = -2$
2. Subtract 7 from both sides. $\quad -3x + 7 - 7 = -2 - 7$

   $-3x = -9$
3. Divide both sides by –3. $\quad \frac{-3x}{-3} = \frac{-9}{-3}$

   $x = 3$

The distributive property states that for all real numbers a, b, and c:

$a(b + c) = a \cdot b + a \cdot c$

For example:

$3(4 + 5) = 3 \cdot 4 + 3 \cdot 5$

$= 12 + 15$

$= 27$

4. Check the solution, $x = 3$.

$$\begin{aligned} \text{Is } 3(3) + 7 - 6(3) &= -2? \\ \text{Is } 9 + 7 - 18 &= -2? \\ \text{Is } -2 &= -2? \text{ Yes.} \end{aligned}$$

So, the solution of the equation $3x + 7 - 6x = -2$ is $x = 3$.

Here is another example. To solve the equation $5(x + 1) - 2 = 18$:

1. Distribute the 5. $\quad 5 \cdot x + 5 \cdot 1 - 2 = 18$

   $5x + 5 - 2 = 18$

2. Combine like terms on the left side. $\quad 5x + 3 = 18$

3. Subtract 3 from both sides. $\quad 5x + 3 - 3 = 18 - 3$

   $5x = 15$

4. Divide both sides by 5. $\quad \frac{5x}{5} = \frac{15}{5}$

   $x = 3$

5. Check the solution, $x = 3$.

$$\begin{aligned} \text{Is } 5(3 + 1) - 2 &= 18? \\ \text{Is } 5(4) - 2 &= 18? \\ \text{Is } 20 - 2 &= 18? \\ \text{Is } 18 &= 18? \text{ Yes.} \end{aligned}$$

So, the solution of the equation $5(x + 1) - 2 = 18$ is $x = 3$.

Solving Linear Equations that Contain Fractions

When a linear equation contains fractions, it is often easier to solve the equation if you first clear the fractions. Then you can use the steps you just learned to solve equations without fractions.

To clear the fractions, multiply both sides of the equation by the least common denominator (LCD) of the fractions.

Sometimes you can figure out the LCD just by looking at the denominators. Try this method on the first example below.

To find the LCD using a formal method:

1. Factor each denominator into its prime factors.
2. List each prime factor the greatest number of times it appears in any one of the denominators.
3. Multiply the prime factors in the list.

For example, to solve $\frac{1}{2}x = \frac{1}{3}(x - 4)$:

1. Find the LCD of the fractions.

   • Factor the denominators. $2 = 2$
   $3 = 3$

   • List each factor the greatest number of times it appears in any one of the denominators. 2, 3

   • Multiply the factors in the list. $\text{LCD} = 2 \cdot 3 = 6$

2. Multiply by 6 to clear the fractions.
$$6 \cdot \frac{1}{2}x = 6 \cdot \frac{1}{3}(x - 4)$$
$$3x = 2(x - 4)$$

3. Distribute the 2.
$$3x = 2 \cdot x - 2 \cdot 4$$
$$3x = 2x - 8$$

4. Subtract $2x$ from both sides.
$$3x - 2x = 2x - 8 - 2x$$
$$x = -8$$

5. Check the solution.
$$\text{Is } \frac{1}{2}(-8) = \frac{1}{3}(-8 - 4)?$$
$$\text{Is } \quad -4 = \frac{1}{3}(-12) \quad ?$$
$$\text{Is } \quad -4 = -4 \quad ? \text{ Yes.}$$

So, $x = -8$ is the solution of the equation $\frac{1}{2}x = \frac{1}{3}(x - 4)$.

Consider another example. To solve $\frac{x}{8} = \frac{1}{20}x + \frac{1}{10}$:

1. Find the LCD of the fractions.

   • Factor the denominators.
   $8 = 2 \cdot 2 \cdot 2$
   $10 = 2 \cdot 5$
   $20 = 2 \cdot 2 \cdot 5$

   • List each factor the greatest number of times it appears in any one of the denominators. 2, 2, 2, 5

   • Multiply the factors in the list. $\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 5 = 40$

2. Multiply by 40 to clear the fractions.
$$40 \cdot \frac{x}{8} = 40 \cdot \left(\frac{1}{20}x + \frac{1}{10}\right)$$
$$5x = 40 \cdot \frac{1}{20}x + 40 \cdot \frac{1}{10}$$
$$5x = 2x + 4$$

3. Subtract $2x$ from both sides.
$$5x - 2x = 2x + 4 - 2x$$
$$3x = 4$$

4. Divide both sides by 3.

$$\frac{3x}{3} = \frac{4}{3}$$
$$x = \frac{4}{3}$$

5. Check the solution.

$$\text{Is } \frac{\frac{4}{3}}{8} = \frac{1}{20}\left(\frac{4}{3}\right) + \frac{1}{10}?$$
$$\text{Is } \frac{4}{24} = \frac{1}{15} + \frac{1}{10} \quad ?$$
$$\text{Is } \frac{1}{6} = \frac{2+3}{30} \quad ?$$
$$\text{Is } \frac{1}{6} = \frac{1}{6} \quad \text{? Yes.}$$

So, $x = \frac{4}{3}$ is the solution of the equation $\frac{x}{8} = \frac{1}{20}x + \frac{1}{10}$.

Solving Equations with Multiple Variables

Often formulas or equations have more than one variable. You can solve for one of the variables in terms of the others.

This is the formula for the circumference of a circle. C is the circumference, r is the radius, and π is a number approximately equal to 3.14.

For example, to solve the formula $C = 2\pi r$ for r:

1. Divide both sides by 2.

$$\frac{C}{2} = \frac{2\pi r}{2}$$
$$\frac{C}{2} = \pi r$$

2. Divide both sides by π.

$$\frac{\frac{C}{2}}{\pi} = \frac{\pi r}{\pi}$$
$$\frac{C}{2\pi} = r$$

So, $r = \frac{C}{2\pi}$.

As another example, to solve $w = xy + 5z$ for x:

1. Subtract $5z$ from both sides.

$$w - 5z = xy + 5z - 5z$$
$$w - 5z = xy$$

2. Divide both sides by y.

$$\frac{w-5z}{y} = \frac{xy}{y}$$
$$\frac{w-5z}{y} = x$$

So, $x = \frac{w-5z}{y}$.

Solving Linear Inequalities

Remember there are equations with no solutions. There are also equations, called identities, where every value of the variable is a solution.

While the linear equations you have seen have only one solution, linear inequalities usually have an infinite number of solutions. But you use the same strategy to solve both. However, it is important to remember one key difference: when you multiply or divide an inequality by a negative number, you must reverse the direction of the inequality.

For example, to solve $6(3 - x) > 7$:

1. Distribute the 6. $\quad 6 \cdot 3 - 6 \cdot x > 7$

$$18 - 6x > 7$$

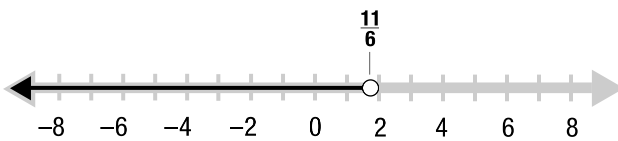
2. Subtract 18 from both sides. $\quad 18 - 6x - 18 > 7 - 18$

$$-6x > -11$$

3. Divide both sides by -6. $\quad \frac{-6x}{-6} < \frac{-11}{-6}$

$$x < \frac{11}{6}$$

So all real numbers less than $\frac{11}{6}$ satisfy this inequality. You can graph the solution on a number line.



*If you're having trouble remembering when to reverse the direction of a sign, try this: **always** reverse the direction of the sign when you multiply or divide both sides by a negative number, even with an equality sign. Just notice that a flipped equality sign doesn't change the solution.*

As another example, to solve $-7(3 - x) \geq 5 + 7x$:

1. Distribute the –7. $\quad -7 \cdot 3 - (-7) \cdot x \geq 5 + 7x$

$$-21 + 7x \geq 5 + 7x$$

2. Add 21 to both sides. $\quad -21 + 7x + 21 \geq 5 + 7x + 21$

$$7x \geq 7x + 26$$

3. Subtract $7x$ from both sides. $\quad 7x - 7x \geq 7x + 26 - 7x$

$$0 \geq 26 \quad \text{No!}$$

Since 0 is not greater than or equal to 26, this inequality has no solution.

Remember, you do not reverse the inequality sign when you distribute a negative number. Only flip the sign when you multiply or divide both sides of the inequality by a negative number.

Another way to solve a compound inequality is to break up the compound inequality and solve the two simple inequalities separately.

Solving Compound Linear Inequalities

A compound inequality is a shorthand way of writing two inequalities. When you solve a compound inequality and you add, subtract, multiply or divide, you must do so to all sides of the inequality.

Another way to solve a compound inequality is to break up the compound inequality and solve the two simple inequalities separately.

For example, to solve $9 \geq 1 - 4y > 1$:

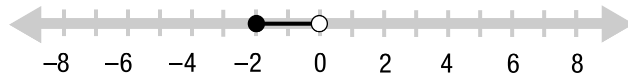
1. Subtract 1 from all sides. $\quad 9 - 1 \geq 1 - 4y - 1 > 1 - 1$

$$8 \geq -4y > 0$$

2. Divide all sides by -4. $\quad \frac{8}{-4} \leq \frac{-4y}{-4} < \frac{0}{-4}$

$$-2 \leq y < 0$$

The graph of this solution is shown below.



Here is another example. To solve $3 + x < 4x - 6 < 9 + x$:

1. Subtract x from all sides. $\quad 3 + x - x < 4x - 6 - x < 9 + x - x$

   $3 < \quad 3x - 6 \quad < 9$

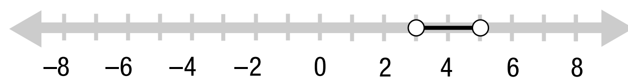
2. Add 6 to all sides. $\quad 3 + 6 < 3x - 6 + 6 < 9 + 6$

   $9 < \quad 3x \quad < 15$

3. Divide all sides by 3. $\quad \frac{9}{3} < \quad \frac{3x}{3} \quad < \frac{15}{3}$

   $3 < \quad x \quad < 5$

The graph of this solution is shown below.



Sample Problems

1. Solve for x: $10 - \frac{3}{7}x = 4 + \frac{1}{3}x$

   ☐ a. Multiply by the LCD to clear the fractions. $\quad$ ____$(10 - \frac{3}{7}x) =$ ____$(4 + \frac{1}{3}x)$

   $210 -$ ____ $= 84 +$ ____

   ☑ b. Subtract 210 from both sides. $\quad 210 - 9x - 210 = 84 + 7x - 210$

   $-9x = 7x - 126$

   ☐ c. Subtract $7x$ from both sides. $\quad -9x - 7x = 7x - 126 - 7x$

   ____ $= -126$

   ☐ d. Divide both sides by –16. $\quad \frac{-16x}{-16} =$ ____

   $x =$ ____

2. Solve the formula $E = K + mgh$ for h.

   ☑ a. Subtract K from both sides. $\quad E - K = K + mgh - K$

   $E - K = mgh$

   ☐ b. Divide both sides by m. $\quad$ ____ $= \frac{mgh}{m}$

   ____ $=$ ____

   ☐ c. Divide both sides by g. $\quad$ ____ $= \frac{gh}{g}$

   ____ $= h$

Answers to Sample Problems

a. 21, 21

9x, 7x

c. –16x

d. $\frac{-126}{-16}$

$\frac{63}{8}$

b. $\frac{E-K}{m}$

$\frac{E-K}{m}$, *gh*

c. $\frac{\frac{E-K}{m}}{g}$

$\frac{E-K}{mg}$

3. Solve for x: $-12\left(x + \frac{1}{2}\right) < 30$

☐ a. Distribute the –12.

$$-12 \cdot x + -12 \cdot \frac{1}{2} \_\_\_\_ 30$$
$$-12x - 6 \_\_\_\_ 30$$

☐ b. Add 6 to both sides.

$$-12x - 6 + 6 \_\_\_\_ 30 + 6$$
$$-12x \_\_\_\_ 36$$

☐ c. Divide both sides by –12.

$$\frac{-12x}{-12} \_\_\_\_ \frac{36}{-12}$$
$$x \_\_\_\_ -3$$

4. Solve for y: $1 \le \frac{2}{5}y + \frac{9}{5} < 2$

☐ a. Multiply by the LCD to clear the fractions.

$$\_\_\_ \cdot 1 \le \_\_\_ \cdot \left(\frac{2}{5}y + \frac{9}{5}\right) < \_\_\_ \cdot 2$$
$$\_\_\_ \le \_\_\_ \cdot \left(\frac{2}{5}y + \frac{9}{5}\right) < \_\_\_$$

☐ b. Distribute and simplify.

$$5 \le 5 \cdot \frac{2}{5}y + 5 \cdot \frac{9}{5} < 10$$
$$5 \le \_\_\_ + \_\_\_ < 10$$

✔ c. Subtract 9 from all sides.

$$5 - 9 \le 2y + 9 - 9 < 10 - 9$$
$$-4 \le 2y < 1$$

☐ d. Divide all sides by 2.

$$\frac{-4}{2} \_\_\_ \frac{2y}{2} \_\_\_ \frac{1}{2}$$
$$-2 \_\_\_ y \_\_\_ \frac{1}{2}$$

Answers to Sample Problems

a. $<$

$<$

b. $<$

$<$

c. $>$

$>$

a. 5, 5, 5

5, 5, 10

b. 2y, 9

d. $\le, <$

$\le, <$

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Linear Equations and Inequalities

1. Solve for x: $5(x + 3) = 40$
2. Solve for m: $F = ma$
3. Solve for x: $\frac{2}{3}x + 1 \geq 7$
4. Solve for y: $2y + 3 = 2y + 5$
5. Solve for x: $y(2 - x) = 11$
6. Solve for x: $-5(x + 4) > 3$
7. Solve for x: $3(3 - x) = 5 + x$
8. Solve for y: $0 < 3y + 2 - 4y < 7$
9. Solve for z: $-2 < 2(z + 2) \leq 6$
10. Solve for x: $\frac{1}{4}(x + 3) = \frac{1}{6}x + 1$
11. Solve for x: $\frac{3x + 5}{10} - \frac{1}{4}x = \frac{1}{3}(x + 1)$
12. Solve for y: $\frac{18 - 2y}{4} > \frac{3}{2}\left(y + \frac{1}{3}\right) \geq \frac{3}{2} - \frac{1}{2}y$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Linear Equations and Inequalities

1. Solve for r: $C = 2\pi r$
2. Solve for m: $Fd = \frac{1}{2}mv^2$
3. Solve for y: $3x - 5y = 15$
4. Solve for x: $ax + b = c$
5. Solve for m: $y = mx + b$
6. Solve for x: $7x + 3 < 24$
7. Solve for x: $-\frac{3}{5}x - 2 \leq 10$
8. Solve for y: $\frac{4}{3}y + 1 \geq -7$
9. Solve for y: $3x(y - 5) = -24$
10. Solve for y: $7x(4 - y) = 56$
11. Solve for y: $12(y - 7) < 36$
12. Solve for x: $-5(x + 6) > 14$
13. Solve for y: $-3(y + 2) + 2y \geq 17$
14. Solve for y: $3(4 - 2y) = 4 + 2y$
15. Solve for y: $2(6 + y) = 9 - y$
16. Solve for y: $17 - 3(5 - 2y) = 8 + 4y$
17. Solve for x: $-6 < 5x - 12 - 2x < 9$
18. Solve for x: $-2 \leq 2x - 6 - 4x \leq 8$
19. Solve for x: $16 - x \leq 5x + 12 - 2x \leq 24 - x$
20. Solve for y: $\frac{1}{5}(y - 5) = \frac{2}{5} - \frac{1}{4}y$
21. Solve for x: $\frac{1}{4}\left(x - \frac{5}{3}\right) = \frac{2}{3}(x + 5)$
22. Solve for x: $\frac{2}{3}\left(x + \frac{1}{5}\right) = \frac{1}{5}(x - 5)$
23. Solve for x: $\frac{7}{3}(y + 2) = \frac{3}{4}(x + 3)$
24. Solve for y: $\frac{3y - 1}{2} = \frac{4}{5}(x - 10)$
25. Solve for y: $\frac{4y - 8}{3} = \frac{2}{5}(x + 10)$
26. Solve for x: $\frac{3}{5}(x + 9) = \frac{1}{10}(x - 14)$
27. Solve for y: $\frac{2}{3}(y + 8) = \frac{4}{9}(y + 6)$
28. Solve for x: $\frac{3}{4}(x + 6) = \frac{5}{12}\left(x - \frac{2}{5}\right)$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Solve for x: $2(x + 4) - 3x = 0$

2. Solve for y: $15y - 20 = 14y - 9$

3. Circle the value of z that is the solution of the equation $12\left(z + \frac{1}{3}\right) = -20$.

    $z = 3$

    $z = 0$

    $z = -2$

4. Solve for x: $-4 - 4(2x + 3) = 16$

5. Solve for x: $\frac{1}{2}x + 4 = \frac{1}{5}x + 7$

6. Solve for x: $z = 3x - 5y$

7. Circle the values of y that are solutions of the inequality $3y + 5 < 14$.

    $y = -1$

    $y = 1$

    $y = 3$

    $y = 5$

8. Solve the following inequality, then graph the solution on the number line.

    $0 \leq 5(x + 2) < 25$