LESSON 8.1 – RATIONAL EXPRESSIONS I

OVERVIEW

Here is what you'll learn in this lesson:

Multiplying and Dividing

a. Determining when a rational expression is undefined

b. Writing a rational expression in lowest terms

c. Multiplying rational expressions

d. Dividing rational expressions

e. Simplifying a complex fraction

Adding and Subtracting

a. Adding rational expressions with the same denominator

b. Subtracting rational expressions with the same denominator

Almost 2000 years ago an Alexandrian mathematician named Eratosthenes figured out the circumference of the earth. He did this by setting up an equation that involved rational expressions, fractions in which the numerator and denominator are polynomials.

In this lesson, you will learn how to multiply, divide, add, and subtract rational expressions.

EXPLAIN

MULTIPLYING AND DIVIDING

Summary

Determining When a Rational Expression is Undefined

You have divided integers to form fractions, or rational numbers. Similarly, you can divide one polynomial by another to form an algebraic fraction. Algebraic fractions are also called rational expressions.

A rational number can be written as a ratio of two integers, $\frac{a}{b}$, where $b \neq 0$.

For example, here are some rational expressions:

$$\frac{5x^2 - 7x + 2}{x + 5} \qquad \frac{1}{7x^2 + 3} \qquad \frac{8x^2 - 9}{3x^5 + 4x - 1}$$

In general, a rational expression is written in the form $\frac{P}{Q}$ where P and Q are polynomials and $Q \neq 0$.

A rational number such as $\frac{5}{19}$ is a special case of a rational expression. Here, the polynomials in the numerator and the denominator each consist of one term, an integer.

A rational expression is undefined when the value of its denominator is zero. To find the values of the variable for which a rational expression is undefined:

1. Set the polynomial in the denominator equal to 0.
2. Solve the equation in step 1.

For example, to determine when $\frac{2x^2 + 5x - 6}{x - 4}$ is undefined:

1. Set the denominator equal to zero. $\quad x - 4 = 0$
2. Solve the equation $x - 4 = 0$ for x. $\quad x = 4$

So, $\frac{2x^2 + 5x - 6}{x - 4}$ is undefined when $x = 4$.

It's okay if the numerator of a rational expression is zero. But the denominator can't be zero since division by 0 is undefined.

Writing a Rational Expression in Lowest Terms

You have learned how to reduce fractions to lowest terms. In a similar way, you can reduce rational expressions to lowest terms.

To reduce a rational expression to lowest terms:

1. Factor the numerator.
2. Factor the denominator.
3. Cancel pairs of factors that are common to both the numerator and the denominator.

Remember, to reduce a fraction to lowest terms:

1. *Find the prime factorization of the numerator.* $\quad \frac{6}{9} = \frac{2 \cdot 3}{9}$
2. *Find the prime factorization of the denominator.* $\quad = \frac{2 \cdot 3}{3 \cdot 3}$
3. *Cancel pairs of prime factors common to both the numerator and the denominator.* $\quad = \frac{2 \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \cdot 3}$

So, $\frac{6}{9} = \frac{2}{3}$.

For example, to reduce $\frac{21a^4b^3c^2}{3a^3cd^2}$ to lowest terms:

1. Factor the numerator. $= \frac{3 \cdot 7 \cdot a \cdot a \cdot a \cdot a \cdot b \; b \cdot b \cdot c \cdot c}{3a^3cd^2}$

2. Factor the denominator. $= \frac{3 \cdot 7 \cdot a \cdot a \cdot a \cdot a \cdot b \; b \cdot b \cdot c \cdot c}{3 \cdot a \cdot a \cdot a \cdot c \cdot d \cdot d}$

3. Cancel common factors. $= \frac{\overset{1}{\cancel{3}} \cdot 7 \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot a \cdot b \; b \cdot b \cdot \overset{1}{\cancel{c}} \cdot c}{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{c}} \cdot d \cdot d}$

$= \frac{7ab^3c}{d^2}$

As another example, to reduce $\frac{x^2 - 3x - 10}{x^2 - 2x - 15}$ to lowest terms:

1. Factor the numerator. $= \frac{(x + 2)(x - 5)}{x^2 - 2x - 15}$

2. Factor the denominator. $= \frac{(x + 2)(x - 5)}{(x + 3)(x - 5)}$

3. Cancel common factors. $= \frac{(x + 2)\overset{1}{\cancel{(x - 5)}}}{(x + 3)\underset{1}{\cancel{(x - 5)}}}$

$= \frac{x + 2}{x + 3}$

So, $\frac{x^2 - 3x - 10}{x^2 - 2x - 15} = \frac{x + 2}{x + 3}$.

Remember, to multiply fractions:

1. *Factor the numerators and denominators into prime factors.*
   $\frac{7}{18} \cdot \frac{9}{14} = \frac{7}{2 \cdot 3 \cdot 3} \cdot \frac{3 \cdot 3}{2 \cdot 7}$
2. *Cancel pairs of factors common to the numerators and denominators.*
   $= \frac{\overset{1}{\cancel{7}}}{2 \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}}} \cdot \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}}}{2 \cdot \underset{1}{\cancel{7}}}$
3. *Multiply the numerators and the denominators.* $= \frac{1}{4}$

Multiplying Rational Expressions

You have learned how to multiply fractions. You can multiply rational expressions in the same way.

To multiply rational expressions:

1. Factor the numerators and denominators.
2. Cancel all pairs of factors common to the numerators and denominators.
3. Multiply the numerators. Multiply the denominators.

For example, to find $\frac{9x^2y^2}{4wz^2} \cdot \frac{8wz}{3x^2y}$:

1. Factor the numerators and denominators. $= \frac{3 \cdot 3 \cdot x \cdot x \cdot y \cdot y}{2 \cdot 2 \cdot w \cdot z \cdot z} \cdot \frac{2 \cdot 2 \cdot 2 \cdot w \cdot z}{3 \cdot x \cdot x \cdot y}$

2. Cancel pairs of factors common to the numerators and denominators. $= \frac{\overset{1}{\cancel{3}} \cdot 3 \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{y}} \cdot y}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{w}} \cdot \underset{1}{\cancel{z}} \cdot z} \cdot \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{w}} \cdot \overset{1}{\cancel{z}}}{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{y}}}$

3. Multiply the numerators. Multiply the denominators. $= \frac{3 \cdot y}{z} \cdot \frac{2}{1}$

$= \frac{6y}{z}$

So, $\frac{9x^2y^2}{4wz^2} \cdot \frac{8wz}{3x^2y} = \frac{6y}{z}$.

Dividing Rational Expressions

You have learned how to divide fractions. You can divide rational expressions in the same way.

To divide rational expressions:

1. Invert the second fraction and change "÷" to "·". Then multiply.
2. Factor the numerators and denominators.
3. Cancel pairs of factors common to the numerators and denominators.
4. Multiply the numerators. Multiply the denominators.

For example, to find $\frac{45xy^2}{2w^2} \div \frac{30x^2y^3}{w^3}$:

1. Invert the second fraction and change ÷ to ·. Then multiply. $= \frac{45xy^2}{2w^2} \cdot \frac{w^3}{30x^2y^3}$
2. Factor the numerators and denominators. $= \frac{3 \cdot 3 \cdot 5 \cdot x \cdot y \cdot y}{2 \cdot w \cdot w} \cdot \frac{w \cdot w \cdot w}{2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y}$
3. Cancel pairs of factors common to the numerator and denominator. $= \frac{\overset{1}{\cancel{3}} \cdot 3 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{y}}}{2 \cdot \underset{1}{\cancel{w}} \cdot \underset{1}{\cancel{w}}} \cdot \frac{\overset{1}{\cancel{w}} \cdot \overset{1}{\cancel{w}} \cdot w}{2 \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{x}} \cdot x \cdot \underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{y}} \cdot y}$
4. Multiply the numerators. Multiply the denominators. $= \frac{3}{2} \cdot \frac{w}{2xy}$

   $= \frac{3w}{4xy}$

So, $\frac{45xy^2}{2w^2} \div \frac{30x^2y^3}{w^3} = \frac{3w}{4xy}$.

Simplifying a Complex Fraction

When a fraction contains other fractions or rational expressions it is called a complex fraction. One way to simplify a complex fraction is to use division.

To simplify a complex fraction using division:

1. Rewrite the complex fraction as a division problem.
2. To divide, invert the second fraction and multiply.
3. Factor the numerators and denominators.
4. Cancel pairs of factors common to the numerator and denominator.
5. Multiply the numerators. Multiply the denominators.

Remember, to divide fractions:

1. *Invert the second fraction and change ÷ to ·. Then multiply.*

   $\frac{4}{27} \div \frac{8}{9} = \frac{4}{27} \cdot \frac{9}{8}$
2. *Factor the numerators and denominators.*

   $= \frac{2 \cdot 2}{3 \cdot 3 \cdot 3} \cdot \frac{3 \cdot 3}{2 \cdot 2 \cdot 2}$
3. *Cancel pairs of common factors.*

   $= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}} \cdot 3} \cdot \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2}$
4. *Multiply the numerators. Multiply the denominators.*

   $= \frac{1}{3} \cdot \frac{1}{2}$

   $= \frac{1}{6}$

So, $\frac{4}{27} \div \frac{8}{9} = \frac{1}{6}$.

For example, to simplify this complex fraction $\dfrac{\frac{3m^3}{5n^5}}{\frac{6m^2}{10n^2}}$:

1. Rewrite the complex fraction as a division problem. $= \dfrac{3m^3}{5n^5} \div \dfrac{6m^2}{10n^2}$

2. Divide by inverting the second fraction and multiplying. $= \dfrac{3m^3}{5n^5} \cdot \dfrac{10n^2}{6m^2}$

3. Factor the numerators and denominators. $= \dfrac{3 \cdot m \cdot m \cdot m}{5 \cdot n \cdot n \cdot n \cdot n \cdot n} \cdot \dfrac{2 \cdot 5 \cdot n \cdot n}{2 \cdot 3 \cdot m \cdot m}$

4. Cancel pairs of factors common to the numerator and denominator. $= \dfrac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{m}} \cdot \overset{1}{\cancel{m}} \cdot m}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{n}} \cdot \underset{1}{\cancel{n}} \cdot n \cdot n \cdot n} \cdot \dfrac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{n}} \cdot \overset{1}{\cancel{n}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{m}} \cdot \underset{1}{\cancel{m}}}$

5. Multiply the numerators. Multiply the denominators. $= \dfrac{m}{n^3}$

So, $\dfrac{\frac{3m^3}{5n^5}}{\frac{6m^2}{10n^2}} = \dfrac{m}{n^3}$.

Sample Problems

Answers to Sample Problems

1. Find the values of x for which this rational expression is undefined: $\dfrac{x^2 - 4}{(x + 8)(x - 5)}$

   ☑ a. Set the polynomial in the denominator equal to 0. $(x + 8)(x - 5) = 0$

   ☐ b. Solve the equation $(x + 8)(x - 5) = 0$. $x =$ ____ or $x =$ ____

b. –8, 5 (in either order)

2. Reduce to lowest terms: $\dfrac{24x^3yz^2}{6x^2z^3w}$

   ☑ a. Factor the numerator. $= \dfrac{2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot z \cdot z}{6x^2z^3w}$

   ☐ b. Factor the denominator. $= \dfrac{2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot z \cdot z}{\_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_}$

   ☐ c. Cancel common factors. $=$ ________

b. 2, 3, x, x, z, z, z, w

c. $\dfrac{4xy}{zw}$

3. Reduce to lowest terms: $\dfrac{x^2 + 3x - 4}{x^2 - 3x + 2}$

   ☑ a. Factor the numerator. $= \dfrac{(x - 1)(x + 4)}{x^2 - 3x + 2}$

   ☐ b. Factor the denominator. $= \dfrac{(x - 1)(x + 4)}{(\_\_\_)(\_\_\_)}$

   ☐ c. Cancel common factors. $= \dfrac{x + 4}{\_\_\_}$

b. x – 1, x – 2 (in either order)

c. x – 2

4. Find: $\frac{3a^2b}{2cd^3} \cdot \frac{cd^2}{6a^3b^2}$

☑ a. Factor the numerators and denominators. $= \frac{3 \cdot a \cdot a \cdot b}{2 \cdot c \cdot d \cdot d \cdot d} \cdot \frac{c \cdot d \cdot d}{2 \cdot 3 \cdot a \cdot a \cdot a \cdot b \cdot b}$

☐ b. Cancel pairs of factors common to the numerators and denominators. $= \frac{1}{\_\_ \cdot \_\_} \cdot \frac{1}{\_\_ \cdot \_\_ \cdot \_\_}$

☐ c. Multiply the numerators. Multiply the denominators. $= \_\_\_\_\_\_$

5. Find: $\frac{6a^2b^2}{5c^3} \div \frac{3ab}{10c^2d}$

☑ a. Invert the second fraction and change ÷ to ·. Then multiply. $= \frac{6a^2b^2}{5c^3} \cdot \frac{10c^2d}{3ab}$

☐ b. Factor the numerators and denominators. $= \frac{2 \cdot 3 \cdot a \cdot a \cdot b \cdot b}{\_\_ \cdot \_\_ \cdot \_\_ \cdot \_\_} \cdot \frac{2 \cdot 5 \cdot c \cdot c \cdot d}{\_\_ \cdot \_\_ \cdot \_\_}$

☐ c. Cancel pairs of factors common to the numerator and denominator. $= \_\_\_\_ \cdot \_\_\_\_$

☐ d. Multiply the numerators. Multiply the denominators. $= \_\_\_\_\_\_$

6. Simplify this complex fraction: $\frac{\frac{a^3}{3b^4}}{\frac{7a^2}{6b^2}}$

☑ a. Rewrite as a division problem. $= \frac{a^3}{3b^4} \div \frac{7a^2}{6b^2}$

☐ b. Divide. (Invert the second fraction and multiply.) $= \frac{a^3}{3b^4} \cdot \_\_\_\_$

☐ c. Factor the numerators and denominators. $= \_\_\_\_\_\_\_\_\_\_$

☐ d. Cancel pairs of factors common to the numerator and denominator. $= \_\_\_\_\_\_\_\_\_\_$

☐ e. Multiply the numerators. Multiply the denominators. $= \_\_\_\_\_\_\_\_\_\_$

Answers to Sample Problems

4. b. 2, d; 2, a, b

c. $\frac{1}{4abd}$

5. b. 5, c, c, c; 3, a, b

c. $\frac{2ab}{c}$, $\frac{2d}{1}$ or 2d

d. $\frac{4abd}{c}$

6. b. $\frac{6b^2}{7a^2}$

c. $\frac{a \cdot a \cdot a}{3 \cdot b \cdot b \cdot b \cdot b} \cdot \frac{2 \cdot 3 \cdot b \cdot b}{7 \cdot a \cdot a}$

d. $\frac{a}{b \cdot b} \cdot \frac{2}{7}$

e. $\frac{2a}{7b^2}$

ADDING AND SUBTRACTING

Summary

Adding Rational Expressions with the Same Denominator

Remember, to add fractions with the same denominator:

1. *Add the numerators. The denominator stays the same.*
$$\frac{3}{4} + \frac{7}{4} = \frac{3+7}{4} = \frac{10}{4}$$
2. *Factor the numerator and denominator.*
$$= \frac{\overset{1}{\cancel{2}} \cdot 5}{\underset{1}{\cancel{2}} \cdot 2}$$
3. *Reduce to lowest terms.*
$$= \frac{5}{2}$$

You have learned how to add fractions with the same denominator.
You can add rational expressions with the same denominator in a similar way.

To add rational expressions with the same denominator, add the numerators. The denominator stays the same.

For example, to find $\frac{3x}{x+5} + \frac{11}{x+5}$:

1. Add the numerators. The denominator stays the same. $= \frac{3x+11}{x+5}$

So, $\frac{3x}{x+5} + \frac{11}{x+5} = \frac{3x+11}{x+5}$.

After you add rational expressions, you often simplify the resulting rational expression by reducing it to lowest terms.

For example, to find $\frac{x+1}{x^2-3x-4} + \frac{2x+2}{x^2-3x-4}$:

1. Add the numerators. The denominator stays the same. $= \frac{x+1+2x+2}{x^2-3x-4}$
$= \frac{3x+3}{x^2-3x-4}$
2. Factor the numerator and denominator. $= \frac{3\overset{1}{\cancel{(x+1)}}}{\underset{1}{\cancel{(x+1)}}(x-4)}$
3. Reduce to lowest terms. $= \frac{3}{x-4}$

So, $\frac{x+1}{x^2-3x-4} + \frac{2x+2}{x^2-3x-4} = \frac{3}{x-4}$.

Subtracting Rational Expressions with the Same Denominator

Remember, to subtract fractions with the same denominator:

1. *Subtract the numerators. The denominator stays the same.*
$$\frac{5}{6} - \frac{1}{6} = \frac{5-1}{6} = \frac{4}{6}$$
2. *Factor the numerator and denominator.*
$$= \frac{\overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot 3}$$
3. *Reduce to lowest terms.*
$$= \frac{2}{3}$$

You have learned how to subtract fractions with the same denominator.
You can subtract rational expressions with the same denominator in a similar way.

To subtract rational expressions with the same denominator, subtract the numerators. The denominator stays the same.

For example, to find $\frac{4y}{17w} - \frac{5}{17w}$:

1. Subtract the numerators. The denominator stays the same. $= \frac{4y-5}{17w}$

So, $\frac{4y}{17w} - \frac{5}{17w} = \frac{4y-5}{17w}$.

After you subtract rational expressions, you often simplify the resulting rational expression by reducing it to lowest terms.

For example, to find $\frac{5x-6}{x^2+6x+5} - \frac{4x-7}{x^2+6x+5}$:

1. Subtract the numerators. The denominator stays the same. $= \frac{5x-6-(4x-7)}{x^2+6x+5}$

2. Distribute. Be careful with the signs. $= \frac{5x-6-4x+7}{x^2+6x+5}$

$= \frac{x+1}{x^2+6x+5}$

3. Factor the numerator and denominator. $= \frac{\overset{1}{\cancel{x+1}}}{\underset{1}{\cancel{(x+1)}}(x+5)}$

4. Reduce to lowest terms. $= \frac{1}{x+5}$

So, $\frac{5x-6}{x^2+6x+5} - \frac{4x-7}{x^2+6x+5} = \frac{1}{x+5}$.

Sample Problems

1. Find: $\frac{2a}{5b} + \frac{6a}{5b}$

   ☐ a. Add the numerators. The denominator stays the same. $= \frac{\_\_\_\_}{5b}$

   $= \_\_\_\_$

2. Find: $\frac{2x}{x-1} + \frac{13}{x-1}$:

   ☐ a. Add the numerators. The denominator stays the same. $= \frac{\_\_\_\_}{x-1}$

3. Find: $\frac{x-12}{x^2-2x-15} + \frac{3x-8}{x^2-2x-15}$

   ☐ a. Add the numerators. The denominator stays the same. $= \frac{(x-12)+(3x-8)}{x^2-2x-15}$

   $= \frac{\_\_\_\_}{x^2-2x-15}$

   ☐ b. Factor the numerator and denominator. $= \_\_\_\_$

   ☐ c. Reduce to lowest terms. $= \_\_\_\_$

Answers to Sample Problems

1. a. $2a + 6a$

$\frac{8a}{5b}$

2. a. $2x + 13$

3. a. $4x - 20$

b. $\frac{4(x-5)}{(x+3)(x-5)}$

c. $\frac{4}{x+3}$

Answers to Sample Problems

a. $7 - 2$

$\frac{5}{11y}$

a. $13 - (2 + x)$

b. $13 - 2 - x$

$\frac{11 - x}{4x}$

a. $\frac{x - 5}{x^2 - 25}$

b. $\frac{x - 5}{(x + 5)(x - 5)}$

c. $\frac{1}{x + 5}$

4. Find: $\frac{7}{11y} - \frac{2}{11y}$

☐ a. Subtract the numerators. The denominator stays the same. $= \frac{\quad}{11y}$

$=$ ____

5. Find: $\frac{13}{4x} - \frac{2 + x}{4x}$

☐ a. Subtract the numerators. The denominator stays the same. $= \frac{\quad}{4x}$

☐ b. Distribute. Be careful with the signs. $= \frac{\quad}{4x}$

$=$ ____________

6. Find: $\frac{5x + 2}{x^2 - 25} - \frac{4x + 7}{x^2 - 25}$

☑ a. Subtract the numerators. The denominator stays the same. $= \frac{5x + 2 - (4x + 7)}{x^2 - 25}$

$=$ ____________

☐ b. Factor the numerator and denominator. $=$ ____________

☐ c. Reduce to lowest terms. $=$ ____________

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Multiplying and Dividing

1. For what values of x is the rational expression below undefined?

   $$\frac{(x+7)(x-8)}{(x-14)(x+2)}$$

2. For what values of x is the rational expression below undefined?

   $$\frac{x^2-9}{x^2-4}$$

3. Reduce to lowest terms: $\frac{3a^2b^5}{27ab^7}$
4. Reduce to lowest terms: $\frac{x^2-3x-28}{x^2+5x+4}$
5. Find: $\frac{3y^3}{z} \cdot \frac{yz}{7y^2}$
6. Find: $\frac{15a^2b^2}{2c^3d} \cdot \frac{2cd^3}{5ab^2}$
7. Find: $\frac{12xy}{w^2} \div \frac{4xy^3}{w^3}$
8. Find: $\frac{3ab^2}{13d^4} \div \frac{6a^2b}{11d^2}$
9. Simplify this complex fraction: $\frac{\frac{5}{a^3}}{\frac{9}{a^2}}$. Write your answer in lowest terms.
10. The ratio of the area of a circle to its circumference is given by $\frac{\pi r^2}{2\pi r}$.
    a. What value of r makes this ratio undefined?
    b. Simplify this expression and then determine what value of r will make the ratio equal to 2. That is, find the radius that will yield an area that is twice the circumference.
11. Simplify this complex fraction: $\frac{\frac{x}{x-1}}{\frac{1}{x+1}}$. Write your answer in lowest terms.
12. Simplify this complex fraction: $\frac{\frac{3y^4}{y+2}}{\frac{9y^2}{y-2}}$. Write your answer in lowest terms.

Adding and Subtracting

13. Find: $\frac{3x}{5y} + \frac{18x}{5y}$
14. Find: $\frac{3+5a}{8-a} + \frac{2a+1}{8-a}$
15. Find: $\frac{x}{x^2-4} + \frac{4}{x^2-4}$
16. Find: $\frac{3z+4}{z-11} + \frac{2z}{z-11}$
17. Find: $\frac{2x-5}{x^2-5x-14} + \frac{3x+15}{x^2-5x-14}$
18. Find: $\frac{2z+11}{z^2-3z-18} + \frac{3z+4}{z^2-3z-18}$
19. Find: $\frac{9}{15x} - \frac{2}{15x}$
20. Find: $\frac{17}{13y} - \frac{5+y}{13y}$
21. The volume, V, of a sphere of radius r is defined by the formula $V = \frac{4\pi r^3}{3}$. Find the volume of two identical spheres. That is, find $\frac{4\pi r^3}{3} + \frac{4\pi r^3}{3}$.
22. Find: $\frac{y}{y^2-81} - \frac{9}{y^2-81}$
23. Find: $\frac{4y+6}{3y+6} - \frac{3y+4}{3y+6}$
24. Find: $\frac{4x-4}{x^2-2x-15} - \frac{3x-7}{x^2-2x-15}$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Multiplying and Dividing

1. For what value(s) of x is the rational expression below undefined?
$\frac{1}{x+5}$
2. For what value(s) of x is the rational expression below undefined?
$\frac{2}{(x-3)(x+5)}$
3. For what value(s) of x is the rational expression below undefined?
$\frac{25}{3x^2-12}$
4. For what value(s) of x is the rational expression below undefined?
$\frac{17}{2x^2-18}$
5. Reduce to lowest terms: $\frac{36m^5n^3}{27mn^6}$
6. Reduce to lowest terms: $\frac{75xy^2z^7}{45x^4y^3z^6}$
7. Reduce to lowest terms: $\frac{44a^2b^4c}{77a^7bc}$
8. Reduce to lowest terms: $\frac{x^2+10x+21}{x^2+5x+6}$
9. Reduce to lowest terms: $\frac{x^2+4x-5}{x^2-3x+2}$
10. Reduce to lowest terms: $\frac{x^2-x-12}{x^2+5x+6}$
11. Find: $\frac{6a}{b^3c^2} \cdot \frac{7b}{3a^3}$
12. Find: $\frac{8x}{y^2z} \cdot \frac{5y}{4x^2}$
13. Find: $\frac{10m^3n^5}{9mn^3} \cdot \frac{21m^5}{15n^6}$
14. Find: $\frac{5ab^4}{3c^2} \cdot \frac{6c}{10b^3}$
15. Find: $\frac{8m^5n}{7p^2} \cdot \frac{14mp}{24m^2n^4}$
16. Find: $\frac{3xy^3}{4z} \cdot \frac{2z^2}{9xy^2}$
17. Find: $\frac{4a^2b}{5c^3} \div \frac{8ab^2}{15c}$
18. Find: $\frac{12m^3n^4}{7p} \div \frac{18mn^5}{21p^4}$
19. Find: $\frac{3xy^2}{7z} \div \frac{6x^2y}{14z^2}$
20. Find: $\frac{5x^2y}{12z^3w} \div \frac{xy}{4z}$
21. Find: $\frac{9m^3n^4}{11pq^2} \div \frac{12mn^3}{22p^2q}$
22. Find: $\frac{7a^2b}{9c^2d^2} \div \frac{ab}{3cd}$
23. Simplify the complex fraction below.
$$\frac{\frac{4m^2}{n^3}}{\frac{2m}{n}}$$
24. Simplify the complex fraction below.
$$\frac{\frac{6x^5}{5y^3}}{\frac{3x^3}{10y}}$$
25. Simplify the complex fraction below.
$$\frac{\frac{6a^2}{b^3}}{\frac{3a}{2b}}$$
26. Simplify the complex fraction below.
$$\frac{\frac{6a^3}{a+5}}{\frac{12a}{a-4}}$$
27. Simplify the complex fraction below.
$$\frac{\frac{5x^2}{x+7}}{\frac{10x}{x-3}}$$
28. Simplify the complex fraction below.
$$\frac{\frac{15y^5}{y-3}}{\frac{18y^3}{y+3}}$$

Adding and Subtracting

29. Find: $\frac{3a}{7b} + \frac{2a}{7b}$

30. Find: $\frac{2x}{5y} + \frac{7x}{5y}$

31. Find: $\frac{3b}{2b+1} + \frac{5}{2b+1}$

32. Find: $\frac{9n}{4n-7} + \frac{2}{4n-7}$

33. Find: $\frac{7x}{5x-1} + \frac{2}{5x-1}$

34. Find: $\frac{5y+2}{3y-2} - \frac{4y-5}{3y-2}$

35. Find: $\frac{7b+1}{2b+9} - \frac{5b+5}{2b+9}$

36. Find: $\frac{6x+1}{2x+3} - \frac{5x-3}{2x+3}$

37. Find: $\frac{15}{7n} - \frac{4-5n}{7n}$

38. Find: $\frac{11}{9x} - \frac{7-5x}{9x}$

39. Add and reduce your answer to lowest terms: $\frac{3x+4}{x^2+5x+6} + \frac{5}{x^2+5x+6}$

40. Add and reduce your answer to lowest terms: $\frac{4x+7}{x^2+7x+10} + \frac{1}{x^2+7x+10}$

41. Add and reduce your answer to lowest terms: $\frac{2x+5}{x^2+x-12} + \frac{3}{x^2+x-12}$

42. Add and reduce your answer to lowest terms: $\frac{3x+12}{x^2+2x-3} + \frac{2x+3}{x^2+2x-3}$

43. Add and reduce your answer to lowest terms: $\frac{3x+19}{x^2+3x-10} + \frac{x+1}{x^2+3x-10}$

44. Add and reduce your answer to lowest terms: $\frac{2x+15}{x^2+5x-14} + \frac{x+6}{x^2+5x-14}$

45. Add and reduce your answer to lowest terms: $\frac{x^2-5x+2}{x^2+7x+12} + \frac{2(5x+1)}{x^2+7x+12}$

46. Add and reduce your answer to lowest terms: $\frac{x^2-3x+2}{x^2+3x+2} + \frac{4(2x+1)}{x^2+3x+2}$

47. Add and reduce your answer to lowest terms: $\frac{x^2-7x+12}{x^2+9x+18} + \frac{5(3x+1)-2}{x^2+9x+18}$

48. Subtract and reduce your answer to lowest terms: $\frac{4x+7}{3x-9} - \frac{3x+10}{3x-9}$

49. Subtract and reduce your answer to lowest terms: $\frac{5x+6}{2x+10} - \frac{2x-9}{2x+10}$

50. Subtract and reduce your answer to lowest terms: $\frac{3x+2}{2x+14} - \frac{2x-5}{2x+14}$

51. Subtract and reduce your answer to lowest terms: $\frac{2x+5}{x^2+3x-10} - \frac{x+7}{x^2+3x-10}$

52. Subtract and reduce your answer to lowest terms: $\frac{4x+2}{x^2-x-20} - \frac{3x-2}{x^2-x-20}$

53. Subtract and reduce your answer to lowest terms: $\frac{3x+5}{x^2-4x+3} - \frac{2x+8}{x^2-4x+3}$

54. Subtract and reduce your answer to lowest terms: $\frac{8x+5}{4x-4} - \frac{5x+8}{4x-4}$

55. Subtract and reduce your answer to lowest terms: $\frac{5x+9}{5x+20} - \frac{2x-3}{5x+20}$

56. Subtract and reduce your answer to lowest terms: $\frac{7x-2}{3x+3} - \frac{5x-4}{3x+3}$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. For what values of x is the following expression undefined? $\frac{x^2 - 16}{(x+3)(x-2)}$

2. Reduce to lowest terms: $\frac{x^2 - 3x - 28}{x^2 + 10x + 24}$

3. Find:

   a. $\frac{5y^2}{9z^2} \cdot \frac{z}{y^2}$

   b. $\frac{12x^2y^2}{z^3w} \cdot \frac{2zw}{3xy^2}$

4. a. Find: $\frac{3x^2}{yz} \div \frac{7x}{2yz}$

   b. Simplify this complex fraction: $\frac{\frac{2x^2y}{9w}}{\frac{10xy^2}{3w^2}}$. Write your answer in lowest terms.

5. Find:

   a. $\frac{5x}{13y} + \frac{2x}{13y}$

   b. $\frac{3w}{z-8} + \frac{14}{z-8}$

6. Find: $\frac{15y}{7x} - \frac{3+6y}{7x}$

7. Find the following. Reduce your answer to lowest terms.

   $$\frac{x+7}{x^2 - 3x - 18} + \frac{3x+5}{x^2 - 3x - 18}$$

8. Find the following. Reduce your answer to lowest terms.

   $$\frac{9y}{y-7} - \frac{3y-4}{y-7}$$