LESSON 7.3 – FACTORING BY PATTERNS

OVERVIEW

Here's what you'll learn in this lesson:

Recognizing Patterns

a. Factoring a perfect square trinomial

b. Factoring a difference of two squares

c. Factoring a sum and difference of two cubes

d. Factoring using a combination of methods

A shortcut can make you more efficient by reducing the amount of time it takes to accomplish a task. It's always nice when you discover a shortcut: for taking notes in class, for programming a VCR, or for getting to a friend's house.

There are shortcuts that you can use in algebra to help you solve problems. For instance, when factoring polynomials, there are patterns you can look for that will help you factor the polynomials more quickly and accurately.

In this lesson you will learn how to recognize patterns for factoring polynomials.

EXPLAIN

RECOGNIZING PATTERNS

Summary

Factoring by patterns means recognizing that polynomials having a certain form will always factor in a specific way. Perfect square trinomials, differences of two squares, and differences and sums of two cubes can all be factored using patterns. These patterns are described below.

The patterns are just a shortcut. You can always factor using algebra tiles or trial and error.

Perfect Square Trinomials

One type of polynomial that's easy to factor using a pattern is a perfect square trinomial.

A perfect square trinomial is a polynomial that can be written so that it:

- has three terms
- has a first term that is a perfect square: a^2
- has a third term that is a perfect square: b^2
- has a second term that is twice the product of a and b: $2ba$

For example, the polynomials below are perfect square trinomials:

$x^2 + 6x + 9$ $\quad$ $w^2 - 12w + 36$

$x^2 - 2xy + y^2$ $\quad$ $4y^4 + 24xy^2 + 36x^2$

The patterns for factoring perfect square trinomials are:

Notice that the only difference between these two patterns is the sign of the middle term.

$$\triangle^2 + 2\square\triangle + \square^2 = (\triangle + \square)^2$$

$$a^2 + 2ba + b^2 = (a + b)^2$$

$$\triangle^2 - 2\square\triangle + \square^2 = (\triangle - \square)^2$$

$$a^2 - 2ba + b^2 = (a - b)^2$$

For example, to factor $x^2 + 2x + 1$:

1. Decide which pattern to use. $\quad a^2 + 2ba + b^2 = (a + b)^2$
2 Substitute x for a and 1 for b. $\quad x^2 + 2(1)(x) + 1^2 = (x + 1)^2$

So, $x^2 + 2x + 1 = (x + 1)^2$.

As another example, to factor $x^2 - 4x + 4$:

1. Decide which pattern to use. $a^2 - 2ba + b^2 = (a - b)^2$
2. Substitute x for a and 2 for b. $x^2 - 2(2)(x) + 2^2 = (x - 2)^2$

So, $x^2 - 4x + 4 = (x - 2)^2$.

Difference of Two Squares

Another type of polynomial that can be factored using a pattern is a difference of two squares.

A difference of two squares is a polynomial that can be written so that it:

- has two terms
- has a first term that is a perfect square: a^2
- has a second term that is a perfect square: b^2
- has a minus sign between the terms

Why don't these polynomials have a middle term? Try multiplying (a + b)(a – b) using the FOIL method and see what happens.

*You can only use this pattern for factoring a **difference** of two squares. A **sum** of two squares can't be factored using integers.*

For example, the polynomials below are differences of two squares.

$x^2 - 25$ $y^2 - 100$

$w^4 - x^2$ $9x^2 - 81z^6$

The pattern for factoring a difference of two squares is:

$$\triangle^2 - \square^2 = (\triangle + \square)(\triangle - \square)$$
$$a^2 - b^2 = (a + b)(a - b)$$

For example, to factor $9y^2 - 25$:

1. Use this pattern. $a^2 - b^2 = (a + b)(a - b)$
2. Substitute $3y$ for a and 5 for b. $(3y)^2 - 5^2 = (3y + 5)(3y - 5)$

So, $9y^2 - 25 = (3y + 5)(3y - 5)$.

Differences or Sums of Two Cubes

You can also use patterns to factor a difference of two cubes or a sum of two cubes.These are polynomials that:

- have two terms
- have a first term that is a perfect cube: a^3
- have a second term that is a perfect cube: b^3

For example, the polynomials below are differences or sums of two cubes.

$x^3 - 27$ $64y^3 + 1$

$x^3 + y^3$ $8w^3 - 125x^6$

The patterns that can be used to factor a difference or sum of two cubes are:

$$\triangle^3 - \square^3 = (\triangle - \square)(\triangle^2 + \triangle\square + \square^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\triangle^3 + \square^3 = (\triangle + \square)(\triangle^2 - \triangle\square + \square^2)$$
$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

For example, to factor $x^3 - 8$:

1. Decide which pattern to use. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
2. Substitute x for a and 2 for b. $x^3 - 2^3 = (x - 2)(x^2 + 2x + 2^2)$

   $= (x - 2)(x^2 + 2x + 4)$

So, $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$.

As another example, to factor $27y^3 + z^6$:

1. Decide which pattern to use. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
2. Substitute $3y$ for a and z^2 for b. $(3y)^3 + (z^2)^3 = (3y + z^2)[(3y)^2 - 3yz^2 + (z^2)^2]$

   $= (3y + z^2)(9y^2 - 3yz^2 + z^4)$

So, $27y^3 + z^6 = (3y + z^2)(9y^2 - 3yz^2 + z^4)$.

Combining Patterns

When a polynomial doesn't look like it fits into one of the patterns you've seen, don't give up. Try to factor using another method first, then see if one of the polynomials in the factorization fits a pattern you have learned. For example, sometimes you need to factor out the greatest common factor of the terms before the polynomial will fit one of the patterns.

For example, to factor $2x^2y - 28xy + 98y$:

1. Factor out the GCF of the terms. $2y(x^2 - 14x + 49)$
2. Decide which pattern to use. $a^2 - 2ba + b^2 = (a - b)^2$
3. Substitute x for a and 7 for b. $x^2 - 2(7)(x) + 7^2 = (x - 7)^2$

So, $2x^2y - 28xy + 98y = 2y(x - 7)^2$.

Whatever factoring technique you use, look at the final product to be sure it cannot be factored any further.

For example, to factor $x^4 - y^4$:

1. Decide which pattern to use. $a^2 - b^2 = (a + b)(a - b)$
2. Substitute x^2 for a and y^2 for b. $(x^2)^2 - (y^2)^2 = (x^2 + y^2)(x^2 - y^2)$

3. Determine if any factor can be factored further. — $x^2 - y^2$ can be factored further

4. Decide which pattern to use. — $a^2 - b^2 = (a + b)(a - b)$

5. Substitute x for a and y for b. — $x^2 - y^2 = (x + y)(x - y)$

So, $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x + y)(x - y)$.

Sample Problems

1. Factor: $w^2 - 10w + 25$

☑ a. Decide which pattern to use.

__ $a^2 + 2ba + b^2 = (a + b)^2$

✓ $a^2 - 2ba + b^2 = (a - b)^2$

__ $a^2 - b^2 = (a + b)(a - b)$

__ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

__ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

☑ b. Substitute w for a and 5 for b.

$a^2 - 2ba + b^2 = (a - b)^2$

$w^2 - 2(5)(w) + 5^2 = (w - 5)^2$

2. Factor: $y^2 - 9$

☑ a. Decide which pattern to use. — $a^2 - b^2 = (a + b)(a - b)$

☐ b. Substitute y for a and 3 for b. — $y^2 - 9 = (\_\_\_\_\_\_)(\_\_\_\_\_\_)$

3. Factor: $27x^3 - 8$

☐ a. Decide which pattern to use. — $\_\_\_\_\_\_ = (\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_\_\_\_\_)$

☐ b. Substitute $3x$ for a and 2 for b. — $27x^3 - 8 = (\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_\_\_\_\_)$

4. Factor: $12wx^2 - 27wy^2$

☐ a. Factor out the GCF of the terms. — $12wx^2 - 27wy^2 = 3w(\_\_\_\_\_\_\_\_)$

☐ b. Decide which pattern to use. — $\_\_\_\_\_\_\_\_ = (\_\_\_\_\_\_)(\_\_\_\_\_\_)$

☐ c. Substitute for a and b. — $3w(\_\_\_\_\_\_) = 3w(\_\_\_\_\_)(\_\_\_\_\_)$

Answers to Sample Problems

2. b. $(y + 3)(y - 3)$

3. a. $a^3 - b^3$, $(a - b)(a^2 + ab + b^2)$

3. b. $(3x - 2)(9x^2 + 6x + 4)$

4. a. $4x^2 - 9y^2$

4. b. $a^2 - b^2$, $(a + b)(a - b)$

4. c. $4x^2 - 9y^2$, $2x + 3y$, $2x - 3y$

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Recognizing Patterns

Factor the polynomials in problems 1 through 12.

1. $x^2 + 14x + 49$
2. $w^2 - 16$
3. $x^3 + 125$
4. $25y^2 - 30y + 9$
5. $9xy^2 - x$
6. $64y^3 - 27w^9$
7. $x^2 + 8w^2x + 16w^4$
8. $2x^6 - 72y^2$
9. $49y^2 - 28xy + 4x^2$
10. $x^3y^2 + 8y^2$
11. $2x^3 + 12x^2 + 18x$
12. $y^6 - 16y^2$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Recognizing Patterns

1. Factor: $a^2 + 18a + 81$
2. Factor: $y^2 + 14y + 49$
3. Factor: $9x^2 + 42x + 49$
4. Factor: $25m^2 + 30m + 9$
5. Factor: $4a^2 + 20a + 25$
6. Factor: $b^2 - 16b + 64$
7. Factor: $z^2 - 22z + 121$
8. Factor: $y^2 - 18y + 81$
9. Factor: $16a^2 - 40a + 25$
10. Factor: $4c^2 + 28c + 49$
11. Factor: $9x^2 - 12x + 4$
12. Factor: $m^2 - 144$
13. Factor: $x^2 - 36$
14. Factor: $9m^2 - 81n^2$
15. Factor: $25a^2 - 625b^2$
16. Factor: $16x^2 - 64y^2$
17. Factor: $a^3 - 216$
18. Factor: $m^3 - 1000$
19. Factor: $x^3 - 125$
20. Factor: $8b^3 - 125$
21. Factor: $27z^3 - 343$
22. Factor: $64a^3 - 216$

23 Factor: $c^3 + 64$

24. Factor: $p^3 + 512$
25. Factor: $y^3 + 27$
26. Factor: $3a^3 + 42a^2b + 147b^2$
27. Factor: $50m^3n - 128mn^3$
28. Factor: $5x^3 - 20xy^2$

EVALUATE

Practice Test

1. Circle the expressions below that are perfect square trinomials.

   $9x^2 + 12x + 4$

   $0.25x^2 + 8x + 64$

   $25x^2 - 9$

   $9x^2 + 20x + 4$

   $x^2 - 2x + 1$

   $x^2 - 7x + 6$

2. Factor the polynomials below.

   a. $x^2 - 10x + 25$

   b. $49y^2 + 28y + 4$

   c. $16x^2 - 1$

   d. $9y^2 - 36$

3. Circle the polynomials below that **cannot** be factored any further using integers.

   $x^2 - 1000$

   $4y^2 - 4y + 1$

   $3x^2 - 27x + 9$

   $9m^2 - 24mn - 16n^2$

   $12x^3 - 8xy + 2y$

4. Factor: $12x^3 - 60x^2 + 75x$

5. Circle the expressions below that are perfect square trinomials.

   $36x^2 - 1$

   $4x^2 - 2x - 56$

   $x^2 + 8x + 16$

   $4x^2 - 12x + 9$

   $x^2 - 16x + 4$

6. Factor the polynomials below.

   a. $4x^2 - 24x + 36$

   b. $64z^2 + 16z + 1$

   c. $4w^2 - 49$

   d. $9m^2 - n^2$

7. Factor the polynomials below.

   a. $x^3 + 1000$

   b. $216y^3 - 1$

   c. $343x^3 + 8y^3$

8. Factor: $27w^3 + 90w^2 + 75w$

TOPIC 7 CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic. Or you may wish to do these problems to review for a test.

1. Find: $(a^5 - 9a^3 + 5a^2 + 14a - 35) \div (a^2 - 7)$

2. Find the slope of the line perpendicular to the line through the points (–3, 7) and (9, –5).

3. Simplify this expression: $11x^2 + 6y + 2 - 4x^2 - y$

4. Graph the inequality $2x + 3y \leq 6$.

5. Alfredo needs to make 250 ml of a 27% alcohol solution using a 15% solution and a 40% solution. How much of each should he use?

6. The point (–2, –3) lies on a line with slope 2. Graph this line by finding another point that lies on the line.

7. Factor: $x^2 - 6x + 9$

8. Find the GCF of $6x^2y^2$ and $8xy^4$.

9. Circle the true statements below.

   $5(7 + 3) = 5(10)$

   $|9 - 20| = -11$

   The fraction $\frac{4}{6}$ is in lowest terms.

   The LCM of 45 and 75 is 225.

   $\frac{9}{11} \cdot \frac{22}{3} = 6$

10. Solve for x: $3(x + 1) = x + 2\left(x + \frac{3}{2}\right)$

11. Graph the system of inequalities below to find its solution.

$$\begin{aligned} 3x - 2y &\leq 7 \\ 4x + y &> 3 \end{aligned}$$

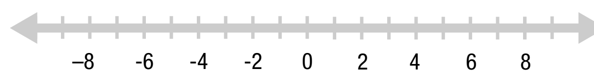
12. Find the slope of the line parallel to the line through the points (7, –1) and (2, 8).

13. Factor: $a^4 + 4a^2 + 4$

14. Solve this system:

$$\begin{aligned} 3x - y &= 23 \\ 2x + y &= 22 \end{aligned}$$

15. Solve $5 \leq 3x - 13 < 17$ for x, then graph the solution on the number line below.



16. Factor: $6x + 3ax + 2b + ab$

17. Find the equation of the line through the point (–7, 12) with slope m = –2:

    a. in point-slope form.

    b. in slope-intercept form.

    c. in standard form.

18. Find:

    a. $-2x^0 - \frac{4}{y^0}$

    b. $\left(\frac{x^7yz^4}{x^2z}\right)^2$

    c. $a^0 \cdot a^0 \cdot a^0$

19. Solve for x by factoring: $x^2 - 5x - 14 = 0$

20. Find:

    a. $11^3 \cdot 11^5$

    b. $\frac{x}{x^8}$

    c. $(ab^6)^3$

21. Find the equation of the line through the point (4, –3) with slope $-\frac{8}{5}$:

    a. in point-slope form. b. in slope-intercept form.

    c. in standard form.

22. Factor: $4y^2 - 28y + 49$

23. Find the slope of the line through the points (31, 16) and (–2, 8).

24. Find: $(5y - 3)^2$

25. Solve $5y + 5 = 5(2 + y)$ for y.

26. Find the slope and y-intercept of the line $\frac{9}{4}x - \frac{2}{3}y = 2$.

27. Factor: $5x^2 + 2x - 7$

28. Find: $(xy^3 - 5x^2y + 11xy - 1) - (4xy^3 - 7 - x^2y + 3xy)$

29. Evaluate the expression $2a^3 - 8ab + 5b^2$ when $a = -2$ and $b = 4$.

30. Factor: $2x^2 + 9x - 18$

31. Use the FOIL method to find: $(a + 4)(a - 2)$

32. Graph the inequality $3x + 5y > 8$.

33. Jerome owed a total of $1820 on his two credit cards last year for which he paid $278.60 in interest. If one card charged 14% in interest and the other card charged 16% in interest, how much did he owe on each card?

34. Solve $-6 \le 4y - 3 \le 5$ for y.

35. Find the equation of the line through the point (5, 3) with slope $m = \frac{5}{6}$:

    a. in point-slope form.

    b. in slope-intercept form.

    c. in standard form.

36. Factor: $x^2 - 7x + 12$

37. Graph the system of inequalities below to find its solution.

$$4x - 3y \le 6$$
$$y \le \frac{1}{2}x + 5$$

38. Circle the expressions below that are monomials.

15	$2x^4 + x$
$9y$	$a^3b^4c^2$

39. Graph the inequality $2x - 3y > 12$.

40. Hye was cleaning out her car and found a total of 44 nickels and quarters worth $4.80. How many of each did she find?

41. Factor: $36b^2 + 60b + 25$

42. Circle the true statements below.

$$\frac{17}{21} - \frac{7}{18} = \frac{10}{3}$$
$$\frac{15}{33} = \frac{5}{11}$$
$$|8| - |13| = |8 - 13|$$

The GCF of 120 and 252 is 12.

$11^2(15 - 2) = 121(15 - 2)$

43. Simplify this expression: $8x^3 + 5xy^2 + 7 - 4x^3 + 2xy^2$

44. Graph the system of inequalities below to find its solution.

$$\begin{aligned} x + 2y &< 6 \\ x + 2y &\ge -5 \end{aligned}$$

45. Solve this system:

$$\begin{aligned} 5x + 7y &= 25 \\ x - 3y &= -17 \end{aligned}$$

46. Find the slope of the line through the points (2, 11) and (6, –8).

47. Find: $(x + 5)(5x + xy - 3y)$

48. Factor: $3x^2 - 5x - 2$

49. The length of a rectangle is 3 times its width. If the perimeter of the rectangle is 136 feet, what are its dimensions?

50. Solve this system:

$$\begin{aligned} 6x + y &= 1 \\ 2x + 5y &= -9 \end{aligned}$$