LESSON 7.1 – FACTORING POLYNOMIALS I

OVERVIEW

Here's what you'll learn in this lesson:

Greatest Common Factor

a. Finding the greatest common factor (GCF) of a set of monomials

b. Factoring a polynomial by finding the GCF when the GCF is a monomial

Grouping

a. Factoring a polynomial by finding the GCF when the GCF is a binomial

b. Factoring a polynomial with four terms by grouping

You have learned how to multiply polynomials. Now you will learn how to factor them. When you factor a polynomial, you write it as the product of other polynomials.

In this lesson you will learn several different techniques for factoring polynomials.

EXPLAIN

GREATEST COMMON FACTOR

Summary

Factoring Polynomials

You already know how to factor numbers by writing them as the product of other numbers. Now you will learn how to factor polynomials by writing them as the product of other polynomials.

Finding the GCF of a Collection of Monomials

To find the GCF of a collection of monomials:

1. Factor each monomial into its prime factors.
2. List each common prime factor the **smallest** number of times it appears in any factorization.
3. Multiply all the prime factors in the list.

For example, to find the GCF of the monomials $16x^2y^2$, $4x^3y^2$, and $12xy^4$:

1. Factor each monomial into its prime factors.

$$16x^2y^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y$$

$$4x^3y^2 = 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot y$$

$$12xy^4 = 2 \cdot 2 \cdot 3 \cdot x \cdot y \cdot y \cdot y \cdot y$$

2. List each common prime factor the **smallest** number of times it appears in any factorization. 2, 2, x, y, y
3. Multiply all the prime factors in the list. GCF $= 2 \cdot 2 \cdot x \cdot y \cdot y = 4xy^2$

Factoring a Polynomial By Finding The Greatest Common Factor

One way to factor a polynomial is to find the greatest common factor of its monomial terms. Here are the steps:

1. Identify the monomial terms of the polynomial.
2. Factor each monomial term.
3. Find the GCF of the monomial terms.
4. Rewrite each term of the polynomial using the GCF.
5. Factor out the GCF.
6. Use the distributive property to check your factoring.

Remember that a monomial is a polynomial with only one term. For example: $14x^5y^3$, 32, 6x, and 9xyz are monomials; but $12x^5y + 1$ and $14y + 3x$ are not monomials.

Before deciding if a polynomial is a monomial, binomial, etc., be sure you first combine any like terms and apply the distributive property, if possible.

The GCF of a collection of monomials is the GCF of the coefficients of all the monomials multiplied by the smallest power of each variable in all the monomials.

The GCF of a collection of monomials evenly divides each monomial in the collection.

$$\frac{16x^2y^2}{4xy^2} = 4x$$

$$\frac{4x^3y^2}{4xy^2} = x^2$$

$$\frac{12xy^4}{4xy^2} = 3y^2$$

For example, to factor the polynomial $6x^2y^2 + 8y^2$:

1. Identify the terms of the polynomial. $6x^2y^2,\ 8y^2$
2. Factor each monomial term. $6x^2y^2 = 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y$
   $8y^2 = 2 \cdot 2 \cdot 2 \cdot y \cdot y$
3. Find the GCF of the monomial terms. $\text{GCF} = 2 \cdot y \cdot y = 2y^2$
4. Rewrite each term of the polynomial using the GCF $6x^2y^2 = 2y^2 \cdot 3x^2$
   $8y^2 = 2y^2 \cdot 4$
5. Factor out the GCF. $6x^2y^2 + 8y^2 = (2y^2)(3x^2 + 4)$
6. Use the distributive property to check your factoring. Is $6x^2y^2 + 8y^2 = (2y^2)(3x^2 + 4)$?
   Is $6x^2y^2 + 8y^2 = (2y^2)(3x^2) + (2y^2)(4)$?
   Is $6x^2y^2 + 8y^2 = 6x^2y^2 + 8y^2$? Yes.

Sample Problems

1. Find the GCF of $9x^3y$, $3xy^4$, and $6y^2$.

   ☐ a. Factor each monomial into its prime factors. $9x^3y = 3 \cdot 3 \cdot x \cdot x \cdot x \cdot y$
   $3xy^4 =$ _____
   $6y^2 =$ _____

   ☐ b. List each common factor the smallest number of times it appears in any factorization. _____

   ☐ c. Multiply all the prime factors in the list. GCF = _____

2. Factor: $6x^4y^4 + 30x^2y^3 + 10x^5y^2$

   ☑ a. Find the terms of the polynomial. $6x^4y^4$, $30x^2y^3$, and $10x^5y^2$

   ☐ b. Factor each monomial. $6x^4y^4 =$ _____
   $30x^2y^3 =$ _____
   $10x^5y^2 =$ _____

   ☐ c. Find the GCF of the monomial terms. GCF = _____

   ☐ d. Rewrite each term of the polynomial using the GCF. $6x^4y^4 =$ _____
   $30x^2y^3 =$ _____
   $10x^5y^2 =$ _____

   ☐ e. Factor out the GCF. = (_____)(_____)

   ☐ f. Use the distributive property to check your factoring.

Answers to Sample Problems

a. $3 \cdot x \cdot y \cdot y \cdot y \cdot y$
$2 \cdot 3 \cdot y \cdot y$

b. 3, y

c. $3y$

b. $2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$
$2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$
$2 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y$

c. $2x^2y^2$

d. $(2x^2y^2)(3x^2y^2)$
$(2x^2y^2)(15y)$
$(2x^2y^2)(5x^3)$

e. $(2x^2y^2)(3x^2y^2 + 15y + 5x^3)$

f. $(2x^2y^2)(3x^2y^2 + 15y + 5x^3)$
$= (2x^2y^2)(3x^2y^2) +$
$(2x^2y^2)(15y) + (2x^2y^2)(5x^3)$
$= 6x^4y^4 + 30x^2y^3 + 10x^5y^2$

GROUPING

Summary

Factoring a Polynomial by Finding the Binomial GCF

You have already learned how to factor a polynomial when the GCF of the terms of the polynomial is a monomial. You can use the same steps to factor a polynomial when the GCF of the terms is a binomial.

There are six steps in this procedure:

1. Identify the terms of the polynomial.
2. Factor each term.
3. Find the GCF of the terms.
4. Rewrite each term of the polynomial using the GCF.
5. Factor out the GCF.
6. Check your answer.

For example, to factor the polynomial $5(3x + 2) + x^2(3x + 2)$:

1. Identify the terms of the polynomial. $5(3x + 2)$ and $x^2(3x + 2)$
2. Factor each term. (Here each term is already factored.)
$$5(3x + 2) = 5 \cdot (3x + 2)$$
$$x^2(3x + 2) = x^2 \cdot (3x + 2)$$
3. Find the GCF of the terms. $\text{GCF} = 3x + 2$
4. Rewrite each term of the polynomial using the GCF.
$$5(3x + 2) = 5 \cdot (3x + 2)$$
$$x^2(3x + 2) = x^2 \cdot (3x + 2)$$
5. Factor out the GCF.
$$5(3x + 2) + x^2(3x + 2) = 5 \cdot (3x + 2) + x^2 \cdot (3x + 2)$$
$$= (3x + 2)(5 + x^2)$$
6. Check your answer.
$$\text{Is } (3x + 2)(5 + x^2) = 5(3x + 2) + x^2(3x + 2)?$$
$$\text{Is } (3x + 2)(5) + (3x + 2)(x^2) = 5(3x + 2) + x^2(3x + 2) \text{ ? Yes.}$$

Factoring By Grouping

Sometimes the GCF of the terms of a polynomial is 1.

For example find the GCF of the terms of $3x^2 + 9 + bx^2 + 3b$:

$$3x^2 = 1 \cdot 3 \cdot x \cdot x$$
$$9 = 1 \cdot 3 \cdot 3$$
$$bx^2 = 1 \cdot b \cdot x \cdot x$$
$$3b = 1 \cdot 3 \cdot b$$

You see that the GCF of the 4 terms $3x^2$, 9, bx^2, and $3b$ is 1. If you try to use the GCF to factor $3x^2 + 9 + bx^2 + 3b$ you get the following factorization:

$$3x^2 + 9 + bx^2 + 3b = 1 \cdot (3x^2 + 9 + bx^2 + 3b)$$

This isn't very interesting!

To factor the polynomial $3x^2 + 9 + bx^2 + 3b$ you need a technique other than finding the GCF of the terms. One such technique is called factoring by grouping. This procedure has 5 steps:

1. Factor each term.
2. Group terms with common factors.
3. Factor out the GCF in each grouping.
4. Factor out the binomial GCF of the polynomial.
5. Check your answer.

For example, use this technique to factor the polynomial $3x^2 + 9 + bx^2 + 3b$:

1. Factor each term.
$$3x^2 = 3 \cdot x \cdot x$$
$$9 = 3 \cdot 3$$
$$bx^2 = b \cdot x \cdot x$$
$$3b = 3 \cdot b$$

2. Group terms with common factors.
$$= 3x^2 + 9 + bx^2 + 3b$$
$$= (\mathbf{3} \cdot x \cdot x + \mathbf{3} \cdot 3) + (\boldsymbol{b} \cdot x \cdot x + 3 \cdot \boldsymbol{b})$$

3. Factor out the GCF in each grouping.
$$= \mathbf{3}(x \cdot x + 3) + \boldsymbol{b}(x \cdot x + 3)$$

4. Factor out the binomial GCF of the polynomial.
$$= \mathbf{3}(x^2 + 3) + \boldsymbol{b}(x^2 + 3)$$
$$= (\mathbf{3} + \boldsymbol{b})(x^2 + 3)$$

5. Check your answer.
$$\text{Is } (3 + b)(x^2 + 3) = 3x^2 + 9 + bx^2 + 3b?$$
$$\text{Is } 3(x^2 + 3) + b(x^2 + 3) = 3x^2 + 9 + bx^2 + 3b?$$
$$\text{Is } 3x^2 + 9 + bx^2 + 3b = 3x^2 + 9 + bx^2 + 3b? \text{ Yes.}$$

This isn't the only way to group the terms. For example, you could also have grouped the terms like this:

$$(3x^2 + bx^2) + (9 + 3b)$$

Try it; you'll get the same answer.

Notice that in steps (1) – (3) you have written the polynomial so that we can ***see*** *its binomial GCF. In step (4) we are really doing all of steps (1) – (5) from before.*

Sample Problems

1. Factor: $x(x^2 + y) + (-3)(x^2 + y)$

   ☐ a. Identify the terms of the polynomial. $x(x^2 + y)$ and _____

   ☐ b. Factor each term. $x(x^2 + y) = x \cdot (x^2 + y)$
   $(-3)(x^2 + y) =$ _____ $\cdot$ _____

   ☐ c. Find the GCF of the terms. GCF = _____

   ☐ d. Rewrite each term of the polynomial using the GCF. $x(x^2 + y) =$ _____ $\cdot (x^2 + y)$
   $(-3)(x^2 + y) =$ _____ $\cdot (x^2 + y)$

   ☐ e. Factor out the GCF. = (_____)(_____)

   ☐ f. Check your answer.

2. Factor: $x^2 + xy + 3x + 3y$

   ☐ a. Factor each term. $x^2 = x \cdot x$
   $xy = x \cdot y$
   $3x =$ ___ $\cdot$ ___
   $3y =$ ___ $\cdot$ ___

   ☐ b. Group terms with common factors. $x^2 + xy + 3x + 3y$
   $= (x \cdot x + x \cdot$ ___$) + (3 \cdot$ ___ $+ 3 \cdot y)$

   ☐ c. Factor out the GCF in each grouping. = ___$(x + y)$ + ___$(x + y)$

   ☐ d. Factor out the binomial GCF of the polynomial. $= (x + y)$(_____)

   ☐ e. Check your answer.

Answers to Sample Problems

a. $(-3)(x^2 + y)$

b. -3, $(x^2 + y)$ *(in either order)*

c. $x^2 + y$

d. x
-3

e. $x^2 + y$; $x + (-3)$ or $x - 3$ *(in either order)*

f. $(x^2 + y)[x + (-3)] =$
$(x^2 + y)(x) + (x^2 + y)(-3)$

a. 3, x *(in either order)*
3, y *(in either order)*

b. y, x

c. x, 3

d. $x + 3$

e. $(x + y)(x + 3)$

$= x(x + 3) + y(x + 3)$

$= x^2 + 3x + xy + 3y$

$= x^2 + xy + 3x + 3y$

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Greatest Common Factor

1. Circle the expressions below that are monomials.

   $x^2 + 2$ $\quad$ $xy^2 + y^2x$

   x^3yz^2 $\quad$ x

2. Circle the expressions below that are **not** monomials.

   xzy^8 $\quad$ $\frac{4}{x}$

   $\frac{13x}{12}$ $\quad$ $x^2z + zy^2$

3. Find the GCF of $12x^3y$ and $6xy^2$.

4. Find the GCF of $3xyz^3$, z, and $16yz$.

5. Factor: $x^2y + 6y^2$

6. Factor: $3x^2 + 9xy^3 - 12xy$

7. Factor: $4a^2b - 4ab^2$

8. Factor: $3x^4yz + 3xyz + 9yz$

9. Factor: $6xy^3 - 4x^2y^2 + 2xy$

10. Factor: $16a^3b^2 + 20a^2b^4 - 8a^3b^3$

11. Factor: $17x^2y^2z^2 + 68x^{10}y^{32}z + 153x^9y^4z^{12}$

12. Factor: $x^2 + xy + xz$

Factoring by Grouping

13. Find the binomial GCF: $(x^5 + y) + 6x^2(x^5 + y)$

14. Factor: $(x^5 + y) + 6x^2(x^5 + y)$

15. Find the binomial GCF:
    $(3x + y)(xy + yz) + x^2y(xy + yz) + z^3(xy + yz)$

16. Factor: $(3x + y)(xy + yz) + x^2y(xy + yz) + z^3(xy + yz)$

17. Factor: $a^3 - a^2b + ab^2 - b^3$

18. Factor: $3x^2 - 3xy + 3xy^3z^4 - 3y^4z^4$

19. Factor: $x^5y + zx + x^4y^2 + yz + x^4yz + z^2$

20. Factor: $15m^3 + 21m^2n + 10mn + 14n^2$

21. Factor: $x^2z + 3x^2 + y^2z + 3y^2$

22. Factor: $x^3 + x^2y + x^2z + 3x + 3z + 3y$

23. Factor: $3x + yz + xz + 3y$

24. Factor: $x^2 - 3x + 2$
    (Hint: rewrite the polynomial as $x^2 - x - 2x + 2$)

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Greatest Common Factor

1. Circle the expressions below that are monomials.

   $8m^3n$ $\quad 7y - 2y^2 + 14$ $\quad x - y$

   23 $\quad \frac{3}{z}$

2. Circle the expressions below that are monomials.

   $3x + 4x^2 - 7$ $\quad 17$ $\quad 5xyz^3$

   $y + z$ $\quad \frac{1}{x}$

3. Find the GCF of $12a^3b$ and $16ab^4$.

4. Find the GCF of $18m^3n^5$ and $24m^4n^3$.

5. Find the GCF of $10xy^4$, and $15x^3y^2$.

6. Find the GCF of $9xy^2z^3$, $24x^5y^3z^6$, and $18x^3yz^4$.

7. Find the GCF of $6abc^4$, $12ac^3$, and $9a^5b^4c^2$.

8. Factor: $5a^3b + 10b$

9. Factor: $16mn^4 + 8m$

10. Factor: $6xy^2 + 12x$

11. Factor: $6x^4y^3 + 14xy$

12. Factor: $24mn - 16m^6n^2$

13. Factor: $8a^3b^2 - 10ab$

14. Factor: $24a^3b^4 + 42a^6b^5$

15. Factor: $36y^7z^8 - 45y^3z^5$

16. Factor: $25x^5y^7 + 35x^2y^4$

17. Factor: $4mn + 10mn^3 - 18m^4n$

18. Factor: $6xy + 9x^3y - 15xy^2$

19. Factor: $8a^3b^4 - 12ab + 20a^3b$

20. Factor: $15a^3b^4c^7 + 25a^5b^3c^2$

21. Factor: $32p^7q^3r^4 - 40p^5q^5r$

22. Factor: $24x^2y^5z^8 - 32x^4y^6z^4$

23. Factor: $9xy^2z^3 - 15x^3y^5z^4 + 21x^4y^2z^5$

24. Factor: $10h^4j^3k^6 + 25h^3j^2k - 40hj^5k^2$

25. Factor: $20a^3b^5c^2 + 12a^4b^2c^3 - 8a^2bc^3$

26. Factor: $20x^2y^4 + 10x^5y^3 - 18x^3y^4 + 12xy^3$

27. Factor: $6a^3b^5c^2 - 9a^4b^4c^3 + 18a^2b^3c^2 - 21a^6b^2c^3$

28. Factor: $18x^2y^4z^3 - 16x^5y^3z + 6x^4y^2z^3 - 10x^3y^4z^2$

Factoring by Grouping

29. Factor: $x(z + 3) + y(z + 3)$

30. Factor: $a(b - 2) + c(b - 2)$

31. Factor: $a(3b - 4) + 9(3b - 4)$

32. Factor: $z(2w + 3) - 12(2w + 3)$

33. Factor: $8m(3n^3 - 4) + 17(3n^3 - 4)$

34. Factor: $12b(2c^4 + 5) - 23(2c^4 + 5)$

35. Factor: $7x(2x^2 + 3) - 11(2x^2 + 3)$

36. Factor: $a(3a - b) - b(3a - b)$

37. Factor: $m(5m + 2n) - 3n(5m + 2n)$

38. Factor: $y(2x + y) + x(2x + y)$

39. Factor: $xw + xz + yw + yz$

40. Factor: $mp - mq + np - nq$

41. Factor: $ac + ad - bc - bd$

42. Factor: $8a^2 + 4a + 10a + 5$

43. Factor: $4a^2 + 2a - 14a - 7$

44. Factor: $6x^2 - 2x + 12x - 4$

45. Factor: $12a^2 + 18a + 10ab + 15b$

46. Factor: $21m^2 - 14m + 24mn - 16n$

47. Factor: $15x^2 + 35x + 6xy + 14y$

48. Factor: $3u^2 + 6u + uv + 2v$

49. Factor: $8z^2 - 2z + 4zw - w$

50. Factor: $2x^2 + 4x - xy - 2y$

51. Factor: $12a^2 - 10b - 15ab + 8a$

52. Factor: $8m^2 + 21n + 12m + 14mn$

53. Factor: $18x^2 - 10y - 15xy + 12x$

54. Factor: $16uv^2 + 10vw + 25w + 40uv$

55. Factor: $12pr^2 - 16rs - 20s + 15pr$

56. Factor: $20ab^2 + 15bc - 6c - 8ab$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Find the GCF of $6xz$, $3xy$, and $2x$.
2. Find the GCF of $16xyz$, $x^2y^2z^2$, and $4x^3y^2z$.
3. Factor: $3x^2y - 3xy^2$
4. Factor: $3xy^3 - 6xy^2 + 3x^3y^4$
5. Factor: $13(x^2 + 4) + 6y(x^2 + 4)$
6. Factor: $17x^2(3xyz + 4z) - 3yz(3xyz + 4z)$
7. Factor: $39rs - 13s + 9r - 3$
8. Factor: $12wz - 44z + 18w - 66$