

LESSON 5.2 – PROBLEM SOLVING





OVERVIEW

Here's what you'll learn in this lesson:

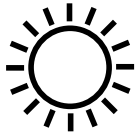
Word Problems

- a. *Number problems*
- b. *Interest problems*
- c. *Coin problems*
- d. *Mixture problems*

Suppose your dog, Rex, is chasing a rabbit that has a head start. Also suppose you know how fast Rex is running, and how fast the rabbit is running. Then you can figure out when Rex will catch up with the rabbit.

You have probably seen word problems like this before. However, you may not realize that word problems have been around for a very long time. In fact, the above dog and rabbit scenario is based on a word problem written approximately 1500 years ago!

In this lesson, you will use what you've learned about systems of linear equations to continue the long-standing tradition of solving word problems.



WORD PROBLEMS

Summary

Solving Word Problems Using Systems of Equations



So far, you have seen how to solve word problems using one equation. Now you'll learn how to use systems of equations to solve them. Using a system of equations allows you to solve a greater variety of problems.

To solve word problems using systems of equations, you can often follow these steps:

1. Make a sketch (when you can).
2. List the quantities to be found. Use English phrases.
3. Represent these quantities algebraically.
4. Write a system that describes the problem.
5. Solve the system of equations.
6. Check that the numbers work in the original problem.

Below are some examples.

Example 1 The sum of two numbers is 47. The sum of the larger number and twice the smaller number is 64. Find the two numbers.

- | | | |
|--|---|---|
| 1. Make a sketch. |  |  |
| 2. List the quantities to be found. | the larger number | the smaller number |
| 3. Represent these quantities algebraically. | x | y |
| 4. Write a system of equations that describes the problem. | $\textcircled{x} + \textcircled{y} = 47$ | |
| | $\textcircled{x} + 2\textcircled{y} = 64$ | |
| 5. Solve using the elimination method. | | |

Multiply the first equation by -1 and add it to the second equation.

$$\begin{array}{r}
 (-1)(x + y) = (-1)(47) \longrightarrow -x - y = -47 \\
 x + 2y = 64 \\
 \hline
 y = 17
 \end{array}$$

The sum of the two numbers is 47.

The sum of the larger number and twice the smaller number is 64.

The smaller number is 17.

The larger number is 30.

Substitute $y = 17$ into one of the original equations. Then solve for x .

$$x + y = 47$$

$$x + 17 = 47$$

$$x = 30$$

6. Check that the numbers work in the original problem.

Do the two numbers add up to 47?

$$\text{Is } 30 + 17 = 47?$$

$$\text{Is } 47 = 47? \text{ Yes.}$$

Is the sum of the larger number and twice the smaller number equal to 64?

$$\text{Is } 30 + 2(17) = 64?$$

$$\text{Is } 30 + 34 = 64?$$

$$\text{Is } 64 = 64? \text{ Yes.}$$

Example 2 Debbie owed a total of \$4200 on her two credit cards for one year. She paid \$706 in interest. If the rate charged by one card was 14% and the rate charged by the other card was 18%, how much did she owe on each card?

1. Make a sketch.



2. List the quantities to be found.

the amount owed
on the 14% card

the amount owed
on the 18% card

3. Represent these quantities algebraically.

x

y

4. Write a system of equations that describes the problem.

$$\textcircled{x} + \textcircled{y} = 4200$$

$$0.14 \textcircled{x} + 0.18 \textcircled{y} = 706$$

5. Solve using the substitution method.

Solve the first equation for x in terms of y .

$$x + y = 4200$$

$$x = 4200 - y$$

Substitute $x = 4200 - y$ into the other equation. Then solve for y .

$$0.14x + 0.18y = 706$$

$$0.14(4200 - y) + 0.18y = 706$$

$$588 - 0.14y + 0.18y = 706$$

$$588 + 0.04y = 706$$

$$0.04y = 118$$

$$y = 2950$$

Substitute $y = 2950$ into the first equation. Then solve for x .

$$x + y = 4200$$

$$x + 2950 = 4200$$

$$x = 1250$$

The total amount owed was \$4200.

The total amount of interest owed was \$706.

To find interest, use the equation $I = P \cdot r \cdot t$, where I is interest, P is principal (the amount of money), r is rate of interest, and t is time.

6. Check that the numbers work in the original problem.

Do the two numbers add up to 4200?

$$\text{Is } 1250 + 2950 = 4200?$$

$$\text{Is } \quad \quad 4200 = 4200? \text{ Yes.}$$

Does the amount of interest paid equal \$706?

$$\text{Is } 0.14(1250) + 0.18(2950) = 706?$$

$$\text{Is } \quad \quad 175 + 531 = 706?$$

$$\text{Is } \quad \quad 706 = 706? \text{ Yes.}$$

Sample Problems

1. Joyce counted the money she collected at the snack bar where she worked and found she had \$162 in one dollar and five dollar bills. If she had a total of 46 bills, how many one dollar bills did she have?

- a. List the quantities to be found. the number the number
Use English phrases. of \$1 bills of \$5 bills
- b. Represent these quantities x y
algebraically.
- c. Write a system that describes $x + y = 46$
the problem. $1x + 5y = 162$

d. Solve the system of equations.

$$\begin{array}{r} x + y = 46 \\ -x - 5y = -162 \\ \hline -4y = -116 \\ y = \underline{\quad} \end{array}$$

$$\begin{array}{r} x + \underline{29} = 46 \\ x = \underline{17} \end{array}$$

- e. Check that the numbers work Do the numbers add up to 46?
in the original problem. Is $\underline{\quad} + \underline{\quad} = 46?$

$$\text{Is } \quad \quad 46 = 46? \text{ Yes.}$$

Is the money worth \$162?

$$\text{Is } 1(\underline{17}) + 5(\underline{29}) = 162?$$

$$\text{Is } \underline{17} + \underline{145} = 162?$$

$$\text{Is } \quad \quad 162 = 162? \text{ Yes.}$$

Answers to Sample Problems

d. 29

29

17

e. 17, 29

17, 29

17, 145



Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.



Explain

Word Problems

1. The difference between two numbers is 9. The sum of twice the larger number and three times the smaller number is 33. Find the two numbers.
2. Last year Wanda divided \$12,400 between a savings account and a mutual fund. The savings account paid 4% in interest and the mutual fund paid 8% in interest. If she earned a total of \$896 in interest, how was her money split between the two accounts?
3. Jae-Hun is the manager of a movie theater. For one show he sold 540 tickets and collected a total of \$2935. If he sold adult tickets for \$6.50 and student tickets for \$4.00, how many of each did he sell?
4. Midori emptied out a token machine in the arcade in which she works and collected \$278 in one dollar and five dollar bills. If she had a total of 114 bills, how many five dollar bills did she collect?
5. Josef wants to earn \$63.75 in interest this year. He has \$1400 to split between his checking account, which pays $2\frac{1}{2}\%$ interest, and his savings account, which pays 5% interest. How should Josef divide his money?
6. The sum of two numbers is 15. The sum of the larger number and twice the smaller number is -1 . What are the two numbers?
7. Vladimir collected \$16.35 in dimes and quarters from a vending machine. If he had a total of 87 coins, how many quarters did he get?
8. Cara is making punch for a party. She wants to combine a 20% real fruit juice drink with a 100% real fruit juice drink to get 16 cups of a mixture that is 80% real fruit juice. How much of each type of drink should she use?
9. The difference between two numbers is 7. The sum of half the larger number and twice the smaller number is 51. What are the two numbers?
10. Harry owed \$5568 on his two credit cards last year on which he paid a total of \$1015.68 in interest. If the rate charged by one card was 21% and the rate charged by the other card was 15%, how much did he owe on each card?
11. Marina mixed almonds and walnuts to take on her camping trip. Almonds cost \$3.00 per pound and walnuts cost \$4.20 per pound. If she ended up with three pounds of a mixture that cost \$3.36 per pound, how many pounds of each kind of nut did she use?
12. Abebe has \$6.65 in nickels and dimes. If he has a total of 99 coins, how many of each does he have?



Practice Problems

Here are some additional practice problems for you to try.

Word Problems

1. The sum of two numbers is 367. The difference between the two numbers is 29. What are the numbers?
2. The sum of two numbers is 245. The difference between the two numbers is 19. What are the numbers?
3. The sum of two numbers is 135. One-fourth the larger number is equal to two times the smaller number. What are the numbers?
4. The difference between two numbers is 23. One-third the larger number plus three times the smaller number is 81. What are the numbers?
5. Sophia has a total of \$2475 in two different savings accounts. Last year, one savings account paid 3% interest and the other account paid 6.5% interest. If she earned \$116.25 in interest for the year, how was her money divided between the two accounts?
6. Samuel has a total of \$1639 in two different savings accounts. Last year, one savings account paid 2% interest and the other account paid 5.5% interest. If he earned \$59.59 in interest for the year, how was his money divided between the two accounts?
7. Roderick has invested a total of \$3500 in two mutual funds. Last year, one of the mutual funds earned a 12% dividend and the other fund earned a 9.5% dividend. If he earned \$377.20 in dividends for the year, how was his money divided between the two funds?
8. The Heaths are paying off two car loans. One loan charges 5% interest per year. The other loan charges 8% interest per year. They owe \$2400 more on the 5% loan than they do on the other. Last year they paid a total of \$710.20 interest. How much do they owe on each loan?
9. Hollis is paying off two student loans. One loan charges 7% interest per year. The other loan charges 9% interest per year. He owes \$1500 more on the 7% loan than he does on the other. Last year he paid a total of \$617 interest. How much does he owe on each loan?
10. Stacey is paying off two student loans. One loan charges 6% interest per year. The other loan charges 10% interest per year. She owes \$2000 more on the 6% loan than she does on the other. Last year she paid a total of \$815 interest. How much does she owe on each loan?
11. Sally has a total of 40 quarters and dimes worth \$6.55. How many of each does she have?
12. Phylicia has a total of 60 five dollar bills and twenty dollar bills worth \$525. How many of each does she have?
13. Saul has a total of 65 dimes and nickels worth \$4.15. How many of each does he have?
14. Reese has \$225 in ten dollar bills and five dollar bills. The number of five dollar bills is 15 more than the number of ten dollar bills. How many of each does he have?
15. Lena has \$31.05 in dimes and quarters. The number of dimes is one-fifth the number of quarters. How many of each does she have?
16. Zack has \$20.25 in nickels and quarters. The number of nickels is 33 less than the number of quarters. How many of each does he have?
17. Zoe has a solution that is 75% sulfuric acid and a solution that is 25% sulfuric acid. How much of each should she use to obtain 400 ml of a solution that is 45% sulfuric acid?

18. Coltin has a solution that is 55% isopropyl alcohol and a solution that is 20% isopropyl alcohol. How much of each should he use to obtain 250 ml of a solution that is 41% isopropyl alcohol?
19. Dory has a solution that is 65% boric acid and a solution that is 15% boric acid. How much of each should she use to obtain 300 ml of a solution that is 35% boric acid?
20. Phil has 30 ounces of a 60% sulfuric acid solution. How many ounces of 15% sulfuric acid solution should he add to obtain a solution that is 40% sulfuric acid?
21. Kayla has 25 ounces of a 75% boric acid solution. How many ounces of 35% boric acid solution should she add to obtain a solution that is 60% boric acid?
22. Dion has 20 ounces of a 70% salt solution. How many ounces of 45% salt solution should he add to obtain a solution that is 55% salt?
23. Tasha wants 25 pounds of a nut mix that she can sell for \$5.00 per pound. If she has cashews that sell for \$6.50 per pound and pistachio nuts that sell for \$4.00 per pound, how much of each should she use?
24. Aaron wants 30 pounds of a coffee blend that he can sell for \$7.00 per pound. If he has coffee that sells for \$6.00 per pound and coffee that sells for \$7.50 per pound, how much of each should he use?
25. Tanya wants to make 20 pounds of a snack mix that she can sell for \$2.44 per pound. If she has chocolate covered raisins that sell for \$2.80 per pound and peanuts that sell for \$2.00 per pound, how much of each should she use?
26. Casey has 15 pounds of cashews that sell for \$5.25 per pound. If peanuts sell for \$2.50 per pound, how many pounds of peanuts should he add to the cashews to obtain a mixture that will sell for \$3.75 per pound?
27. Lena has 16 pounds of coffee that sells for \$6.50 per pound. If she has a second coffee that sells for \$8.00 per pound, how many pounds of the second coffee should be added to the first coffee to obtain a blend that will sell for \$7.00 per pound?
28. Horatio has 12 pounds of carob coated peanuts that sell for \$3.60 per pound. If sunflower seeds sell for \$1.80 per pound, how many pounds of sunflower seeds should he add to the carob coated peanuts to obtain a snack mix that will sell for \$2.80 per pound?

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. The sum of two numbers is 25. When twice the larger number is subtracted from four times the smaller number, the result is 4. What are the two numbers?
2. Sunil has 19 five dollar and ten dollar bills worth a total of \$150. How many of each does she have?
3. Last year a small college accepted 363 students out of the 1340 people that applied. If the college accepted 30% of the female applicants and 25% of the male applicants, how many women applied?
4. The local coffee shop combines two kinds of beans to make a blended mixture of beans. If Ethiopian Harrar beans cost \$7.25 per pound, and Arabian Mocha beans cost \$13.70 per pound, how many pounds of each should be used to make 3 pounds of a blend that costs \$10.26 per pound?
5. In 7 years Deac will be twice as old as Irina. Four years ago he was three times as old as Irina was then. How old are each of them now?
6. Admission to a skating rink is \$4.25 for children and \$7.50 for adults. If one evening the skating rink collected \$930 and 150 people were admitted, how many children went skating?
7. Nobutaka needs 200 ml of 6% HCl. If he has solutions of 5% HCl and 9% HCl, how many milliliters of each should he mix together?
8. Alexis had \$2375 to invest last year. She put some of her money in a savings account that paid 4% interest and the rest of her money in a mutual fund that paid 11% interest. If she earned \$168.50 in interest, how did she divide her money between the two accounts?