LESSON 5.1 – SOLVING LINEAR SYSTEMS

OVERVIEW

Here's what you'll learn in this lesson:

Solution by Graphing

a. The solution of a linear system

b. Graphing linear systems

c. Systems with a unique solution

d. Systems with no solutions

e. Systems with an infinite number of solutions

Solution by Algebra

a. Solving linear systems by the substitution method: one solution, no solution, and an infinite number of solutions

b. Solving linear systems by the elimination method: one solution, no solution, and an infinite number of solutions

A customer wants to buy a blend of coffee for a specific price, and as the clerk you want to know how much of each type of bean to put in the mixture. The owner of the club where you sing sold some cheap tickets and some expensive tickets to the show last night, and you want to know how many tickets of each type were sold.

In both of these situations, you can find the answer by setting up and solving a system of two linear equations.

In this lesson, you will learn about systems of two linear equations. First, you will learn how to find the solution of such systems by graphing. Then, you will learn some algebraic methods for finding these solutions.

EXPLAIN

SOLUTION BY GRAPHING

Summary

Systems of Two Linear Equations

A system of equations consists of two or more equations, each of which contains at least one variable. Some examples of systems of linear equations in two variables are:

$$\begin{aligned} 3x + y &= -5 \\ 4x - 2y &= 7 \end{aligned}$$

$$\begin{aligned} -7x + 9y &= 0 \\ 3y &= 8 \end{aligned}$$

$$\begin{aligned} 5x - 4y &= 11 \\ 10x + 7y &= -6 \end{aligned}$$

Systems of equations are sometimes used to describe problems that are too complicated to be solved using only one equation.

One way to find the solution of a system of equations is to look at the graphs of the equations. If you graph a system of two linear equations in two variables, the result is two straight lines. You can figure out how many solutions a system has by looking at these lines.

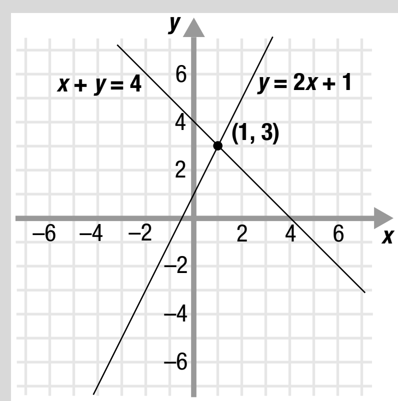
- If the lines intersect, the system has one solution.
- If the lines are parallel, the system has no solutions.
- If the lines are the same, the system has infinitely many solutions.

Systems of Two Linear Equations with One Solution

A system has one solution when the graphs of the two linear equations intersect. This is because the point of intersection is the only point whose coordinates satisfy both equations.

For example, find the solution of this system:

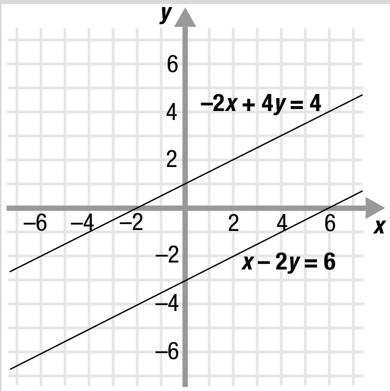
$$\begin{aligned} x + y &= 4 \\ y &= 2x + 1 \end{aligned}$$



The lines intersect at the point (1, 3). This point is called the solution of the system since $x = 1$ and $y = 3$ satisfy both equations:

$x + y = 4$	$y = 2x + 1$
Is $1 + 3 = 4$?	Is $3 = 2(1) + 1$?
Is $4 = 4$? Yes	Is $3 = 2 + 1$?
	Is $3 = 3$? Yes.

Systems of Two Linear Equations with No Solution



A system has no solutions when the graphs of the two linear equations are parallel. This is because parallel lines never intersect, so there is no point which lies on both lines. Since the lines have no points in common, there is no point whose coordinates satisfy both equations.

For example, find the solution of this system:

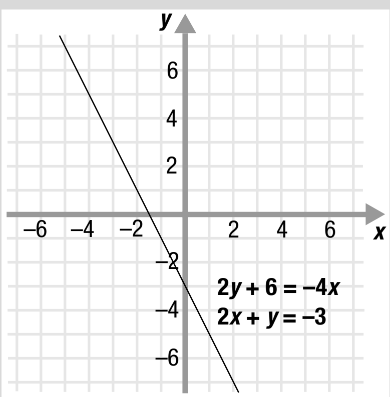
$$x - 2y = 6$$
$$-2x + 4y = 4$$

Notice that the lines appear to be parallel. You can check that they are in fact parallel by rewriting both equations in slope-intercept form:

$$x - 2y = 6 \qquad\qquad -2x + 4y = 4$$
$$-2y = -x + 6 \qquad\qquad 4y = 2x + 4$$
$$y = \frac{1}{2}x - 3 \qquad\qquad y = \frac{1}{2}x + 1$$

Since the lines have the same slope ($m = \frac{1}{2}$), but different y-intercepts ($b = -3$ and $b = 1$), they are parallel. Parallel lines never meet, so this system has no solutions.

Systems of Two Linear Equations with Infinitely Many Solutions



A system has infinitely many solutions when the graphs of the two linear equations are the same. This is because all of the points on one line also lie on the other line, so the coordinates of any point on either line satisfy both equations.

For example, find the solution of this system:

$$2x + y = -3$$
$$2y + 6 = -4x$$

The two lines are the same. You can check that the lines are identical by writing each equation in slope-intercept form:

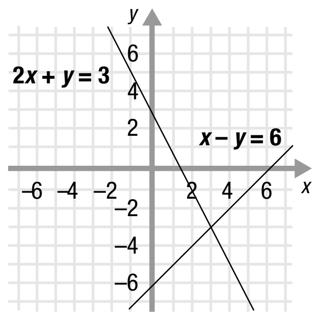
$$2x + y = -3 \qquad\qquad 2y + 6 = -4x$$
$$y = -2x - 3 \qquad\qquad 2y = -4x - 6$$
$$y = -2x - 3$$

Since the lines have the same slope ($m = -2$) and the same y-intercept ($b = -3$), they are identical. So, if the coordinates of a point satisfy one equation, they will also satisfy the other equation.

Sample Problems

1. Find the solution of this system of linear equations:

$$x - y = 6$$
$$2x + y = 3$$



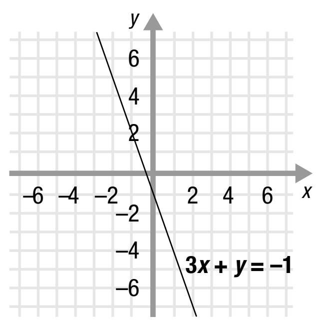
☐ a. Plot the point where the two lines intersect.

☐ b. Write the coordinates of their point of intersection.

The solution is (____, ____).

2. Graph each equation to find the solution of this linear system:

$$3x + y = -1$$
$$x - 2y = -5$$



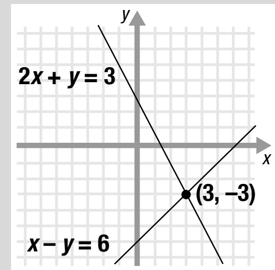
☑ a. Graph the line $3x + y = -1$.

☐ b. Graph the line $x - 2y = -5$.

☐ c. Find their point of intersection.
The solution is (____, ____).

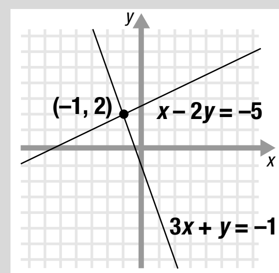
Answers to Sample Problems

a.



b. (3, –3)

b.



c. (–1, 2)

SOLUTION BY ALGEGRA

Summary

Substitution Method

Graphing is not always the best method for finding the solution of a system of two linear equations. This is especially true when the coordinates of the point of intersection are not whole numbers. For this reason, it is often easier to use algebraic methods for finding solutions of linear systems. One algebraic method for finding the solution of a system is the substitution method.

To use the substitution method for finding the solution of a system of two linear equations:

1. Solve one of the equations for one variable in terms of the other variable. For example, solve one of the equations for x in terms of y.
2. Substitute this value for x into the other equation. Then solve for y.
3. Substitute this value for y into either of the original equations. Then solve for x.
4. Check the solution by substituting it in both of the original equations.

You could also solve for y in terms of x. If you do this, switch the letters x and y in each of the steps.

For example, use substitution to solve this system:

$$2x + y = 4$$
$$3x + y = 7$$

1. Solve one of the equations, say $2x + y = 4$, for y.

$$2x + y = 4$$
$$y = 4 - 2x$$

2. Substitute $y = 4 - 2x$ into the second equation. Then solve for x.

$$3x + y = 7$$
$$3x + (4 - 2x) = 7$$
$$3x + 4 - 2x = 7$$
$$x + 4 = 7$$
$$x = 3$$

3. Substitute $x = 3$ into either of the original equations. Then solve for y.

$$2x + y = 4$$
$$2(3) + y = 4$$
$$6 + y = 4$$
$$y = -2$$

The solution of the system of equations is $(x, y) = (3, -2)$.

4. Check the answer by substituting $x = 3$ and $y = -2$ into both equations:

$$2x + y = 4$$

Is $2(3) + (-2) = 4$?

Is $6 - 2 = 4$?

Is $4 = 4$? Yes.

$$3x + y = 7$$

Is $3(3) + (-2) = 7$?

Is $9 - 2 = 7$?

Is $7 = 7$? Yes.

Elimination Method

Another algebraic method for finding the solution of a system of linear equations is the elimination method.

To use the elimination method to find the solution of a linear system:

If the substitution method gives you a fraction when you solve for one of the variables in terms of the other variable, it's usually easier to use the elimination method.

1. Multiply one or both of the equations by an appropriate number so that when you add the two resulting equations, one of the variables disappears. Solve this new equation for the remaining variable.
2. Substitute this value into either of the original equations and solve for the other variable.
3. Check your answer by substituting your solution in both of the original equations.

For example, use the elimination method to solve this system:

$$\begin{aligned} 4x + 3y &= -1 \\ 2x - y &= -13 \end{aligned}$$

1. Multiply the second equation by 3 and add it to the first equation. Then solve for x.

$$3(2x - y) = 3(-13) \longrightarrow \quad \begin{aligned} 4x + 3y &= -1 \\ \underline{6x - 3y} &\underline{= -39} \\ 10x \qquad &= -40 \\ x \qquad &= -4 \end{aligned}$$

2. Substitute $x = -4$ into either of the original equations. Then solve for y.

$$\begin{aligned} 4x + 3y &= -1 \\ 4(-4) + 3y &= -1 \\ -16 + 3y &= -1 \\ 3y &= 15 \\ y &= 5 \end{aligned}$$

The solution of the system of equations is $(x, y) = (-4, 5)$.

3. Check the answer by substituting $x = -4$ and $y = 5$ into both equations.

$$4x + 3y = -1$$

Is $4(-4) + 3(5) = -1$?

Is $-16 + 15 = -1$?

Is $-1 = -1$? Yes.

$$2x - y = -13$$

Is $2(-4) - 5 = -13$?

Is $-8 - 5 = -13$?

Is $-13 = -13$? Yes.

Finding the Multiplier

Instead you could multiply the first equation by the opposite of the coefficient of y in the second equation, and multiply the second equation by the coefficient of y in the first equation.

How do you know what to multiply each equation by so that one of the variables will disappear when you add the two equations? Suppose you want to eliminate y when you add the equations. One way is to multiply the first equation by the coefficient of y in the second equation and multiply the second equation by the opposite of the coefficient of y in the first equation.

For example, given this system:

$$\begin{aligned} 3x + 4y &= 7 \\ 17x + 6y &= 23 \end{aligned}$$

If you want to eliminate x, just follow the same process as you do to eliminate y.

You can eliminate y by multiplying the first equation by 6 (since 6 is the coefficient of y in the second equation) and multiplying the second equation by –4 (since –4 is the opposite of the coefficient of y in the first equation). This results in the rightmost equations below:

$$\begin{aligned} 3x + 4y = 7 &\longrightarrow 6(3x + 4y) = 6(7) \longrightarrow 18x + 24y = 42 \\ 17x + 6y = 23 &\longrightarrow -4(17x + 6y) = -4(23) \longrightarrow -68x - 24y = -92 \end{aligned}$$

Then you can add the equations to eliminate y.

This method will always work, but sometimes it gives you big numbers. To find the smallest numbers by which you can multiply each equation to eliminate one of the variables, for example y:

1. Find the LCM of the coefficients of the y's.
2. For each equation, find the number by which you need to multiply the y coefficient to get the LCM.
3. Multiply the first equation by the number you found for the first equation in step (2). Multiply the second equation by the opposite of the number you found for the second equation in step (2).

For example, suppose you want to eliminate y in this same system:

$$\begin{aligned} 3x + 4y &= 7 \\ 17x + 6y &= 23 \end{aligned}$$

1. First find the LCM of 4 and 6. 12
2. Find the number by which you multiply 4 to get 12. $4 \cdot 3 = 12$

   Find the number by which you multiply 6 to get 12. $6 \cdot 2 = 12$
3. So you can eliminate y by multiplying the first equation by 3 and multiplying the second equation by –2. This results in the rightmost equations below.

$$\begin{aligned} 3x + 4y = 7 &\longrightarrow 3(3x + 4y) = 3(7) \longrightarrow 9x + 12y = 21 \\ 17x + 6y = 23 &\longrightarrow -2(17x + 6y) = -2(23) \longrightarrow -34x - 12y = -46 \end{aligned}$$

You can add these equations to eliminate y.

Sample Problems

1. Use the substitution method to solve this system:

$$5x + 2y = -5$$
$$3x + y = -4$$

☑ a. Solve the second equation for y.

$$3x + y = -4$$
$$y = -4 - 3x$$

☑ b. Substitute this value for y into the first equation.

$$5x + 2(-4 - 3x) = -5$$
$$5x - 8 - 6x = -5$$
$$-x = 3$$
$$x = -3$$

☐ c. Substitute $x = -3$ into one of the original equations. Then solve for y.

$$5(\_\_\_) + 2y = -5$$
$$\_\_\_ + 2y = -5$$
$$2y = \_\_\_$$
$$y = \_\_\_$$

☐ d. Write the solution.

$$x = \_\_\_,\ y = \_\_\_$$

☐ e. Check your solution in the original equations.

Is $5(\_\_\_) + 2(\_\_\_) = -5$?
Is $\_\_\_ + \_\_\_ = -5$?
Is $\_\_\_ = -5$? _____

Is $3(\_\_\_) + \_\_\_ = -4$?
Is $\_\_\_ + \_\_\_ = -4$?
Is $\_\_\_ = -4$? _____

Answers to Sample Problems

c. −3
−15
10
5

d. −3, 5

e. −3, 5
−15, 10
−5, Yes

−3, 5
−9, 5
−4, Yes

Answers to Sample Problems

c. −6
−12
30
3

d. −6, 3

e. −6, 3
−12, 30
18, Yes

−6, 3
−18, 15
−33, Yes

2. Use the elimination method to solve this system:

$$\begin{aligned} 2x + 10y &= 18 \\ 3x - 5y &= -33 \end{aligned}$$

☑ a. Multiply the second equation by 2.

$2(3x - 5y) = 2(-33) \longrightarrow 6x - 10y = -66$

☑ b. Add the equations to eliminate *y*. Then solve for *x*.

$$\begin{aligned} 2x + 10y &= 18 \\ \underline{6x - 10y} &\underline{= -66} \\ 8x \quad\quad &= -48 \\ x &= -6 \end{aligned}$$

☐ c. Substitute $x = -6$ into one of the original equations. Then solve for *y*.

$$\begin{aligned} 2(\_\_\_) + 10y &= 18 \\ \_\_\_ + 10y &= 18 \\ 10y &= \_\_\_ \\ y &= \_\_\_ \end{aligned}$$

☐ d. Write the solution.

$x = \_\_\_,\ y = \_\_\_$

☐ e. Check your solution in the original equations.

Is $2(\_\_\_) + 10(\_\_\_) = 18$?
Is $\_\_\_\_ + \_\_\_\_ = 18$?
Is $\_\_\_\_ = 18$? ____

Is $3(\_\_\_) - 5(\_\_\_) = -33$?
Is $\_\_\_\_ - \_\_\_\_ = -33$?
Is $\_\_\_\_ = -33$? ____

EXPLORE

Sample Problems

On the computer you used the Grapher to analyze and solve systems of linear equations. Below are some additional exploration problems.

1. Find the slope of the line $2x - 3y = 5$.

☐ a. Write the equation $2x - 3y = 5$ in slope-intercept form.

$$2x - 3y = 5$$
$$-3y = -2x + 5$$
$$y = \_\_\_x - \_\_\_$$

☐ b. Find the slope of the line $2x - 3y = 5$.

slope = ___

2. Find the vertices of the triangle formed by the lines:

$y = x - 5 \qquad 3x + 13y = 31 \qquad 11x + 5y = -57$

✓ a. Find the point of intersection of these lines:
$y = x - 5$
$3x + 13y = 31$

Substitute for y:

$$3x + 13(x - 5) = 31$$
$$3x + 13x - 65 = 31$$
$$16x = 96$$
$$x = 6$$

Solve for y:

$$y = x - 5$$
$$= 6 - 5$$
$$= 1$$

☐ b. Find the point of intersection of these lines:
$y = x - 5$
$11x + 5y = -57$

Substitute for y:

$$11x + 5(\_\_\_\_\_) = -57$$
$$11x + \_\_\_x - \_\_\_ = -57$$
$$\_\_\_\_x = -57 + \_\_\_$$
$$\_\_\_\_x = \_\_\_\_$$
$$x = \_\_\_\_$$

Solve for y:

$$y = x - 5$$
$$= \_\_\_ - 5$$
$$= \_\_\_$$

Answers to Sample Problems

a. $\frac{2}{3}, \frac{5}{3}$

b. $\frac{2}{3}$

b. *x – 5*
5, 25
16x, 25
16x, –32
–2

–2
–7

Answers to Sample Problems

c. *4*

52

52

–21

–7

d. *(6, 1)*

(–2, –7)

(–7, 4)

☐ c. Find the point of intersection of these lines:

$$\begin{aligned} 3x + 13y &= 31 \\ 11x + 5y &= -57 \end{aligned}$$

Multiply the first equation by 11 and the second equation by –3, then add these equations and solve for y:

$$\begin{array}{r} 33x + 143y = 341 \\ \underline{-33x - 15y = 171} \\ 128y = 512 \\ y = \frac{512}{128} \\ = 4 \end{array}$$

Solve for x:

$3x + 13(\_\_\_) = 31$

$3x + \_\_\_\_\_ = 31$

$3x = 31 - \_\_\_\_$

$3x = \_\_\_\_$

$x = \_\_\_\_$

☐ d. Write the vertices of the triangle.

(___, ___)

(___, ___)

(___, ___)

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain
Solution By Graphing

Use Figure 5.1.1 to answer questions 1 through 3.

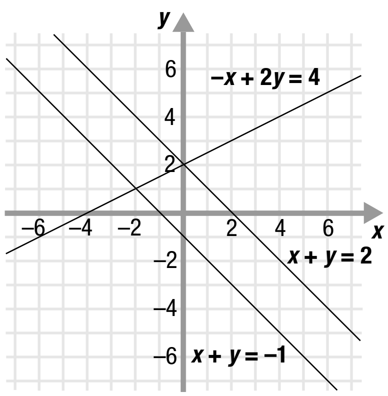


Figure 5.1.1

1. Which two lines form a system that has a solution of (–2, 1)?
2. Which two lines form a system that has no solution?
3. Which two lines form a system that has a solution of (0, 2)?
4. The ordered pair (–2, –5) is a solution of which system of equations?

$$\begin{aligned} x - y &= 7 \\ 3x + 2y &= -11 \end{aligned} \qquad \begin{aligned} 4x + y &= 6 \\ x - 2y &= 8 \end{aligned}$$

$$\begin{aligned} 2x - y &= 1 \\ x + y &= -7 \end{aligned} \qquad \begin{aligned} x - 3y &= 9 \\ 2x + y &= -1 \end{aligned}$$

Use Figure 5.1.2 to answer questions 5 through 8.

5. Which two lines form a system that has a solution of (1, 6)?
6. Which two lines form a system that has a solution of (–4, 1)?
7. Which two lines form a system that has no solution?
8. What is the solution of the system of equations $x + 2y = -2$ and $3x + y = 9$?

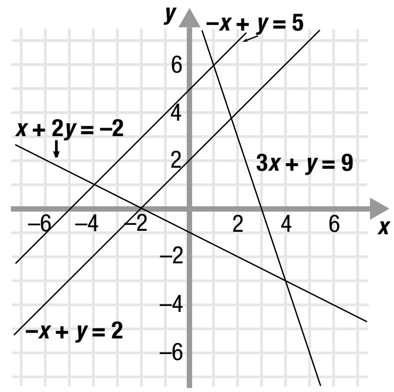


Figure 5.1.2

9. Raymond weighs 180 pounds and wants to lose some weight before his high school reunion. He figures he can lose 2 pounds a week if he sticks to a strict diet. If his reunion is in 14 weeks and he really sticks to his diet, will he be able to get down to his goal weight of 150 pounds? If so, how long will it take him? If not, how much longer will he have to stay on his diet? Graph the system to help you answer the questions.

$$\begin{aligned} y &= 180 - 2x \\ y &= 150 \end{aligned}$$

10. Katelyn has $50 in her bank account and saves $10 per week. Caesar has $200 in his bank account and withdraws $20 per week. When will Katelyn and Caesar have the same amount of money and how much will each have? Graph the system to help you answer the question.

$$\begin{aligned} y &= 50 + 10x \\ y &= 200 - 20x \end{aligned}$$

Use Figure 5.1.3 to answer questions 11 and 12.

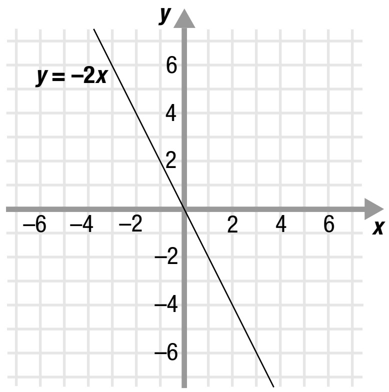


Figure 5.1.3

11. Draw a line on the grid in Figure 5.1.3 so that the system of equations has no solution.

12. Draw a line on the grid in Figure 5.1.3 so that the system of equations has one solution at the point (2, –4).

Solution by Algebra

13. Use the substitution method to solve this system:

$$x + 2y = 4$$
$$2x + 3y = 7$$

14. Use the elimination method to solve this system:

$$x + y = 7$$
$$x - y = 1$$

15. Use the elimination method to solve this system:

$$4x - y = -8$$
$$3x + 2y = 5$$

16. Use the substitution method to solve this system:

$$4x + y = 14$$
$$3x + 5y = -15$$

17. Use the substitution method to solve this system:

$$2x + y = 5$$
$$6x + 3y = 15$$

18. Use the elimination method to solve this system:

$$x + 7y = 31$$
$$x - 9y = -1$$

19. Use the elimination method to solve this system:

$$3x + y = 5$$
$$6x + 2y = 9$$

20. Use the substitution method to solve this system:

$$3x + y = 6$$
$$6x + 2y = 10$$

21. When renting a compact car, you have a choice of paying a flat rate of $25 per day with unlimited mileage or you can pay $15 per day and 20¢ per mile. How many miles can you drive before the cost of paying for mileage is the same as getting unlimited mileage? Use the substitution method or the elimination method to solve the system below to get the answer.

$$y = 25$$
$$y = 15 + 0.20x$$

22. The monthly rate for phone service can be paid for in one of two ways. One choice is to pay a measured rate of $4.45 per month and $.03 a minute for each local call. The other choice is to pay a flat rate of $8.35 per month. How many minutes of local calls can you make before the cost for measured rate service is the same as the cost for flat rate service? Use the substitution method or the elimination method to solve the system below to get the answer.

$$y = 4.45 + 0.03x$$
$$y = 8.35$$

23. Solve this system:

$$x - 4y = -31$$
$$3x + 2y = 5$$

24. Solve this system:

$$12x - 3y = 132$$
$$6x + 5y = 14$$

Explore

25. A system of two linear equations has the solution (2, 4). The slope of one of the lines is twice the slope of the other line. If the equation of one of the lines is $y = 3x - 2$, what are the two possible equations of the other line?

26. Which of the following systems of equations have no solutions?

$$x - y = 7$$
$$3x - y = 9$$

$$x + y = 3$$
$$2y = -2x + 9$$

$$2x - 3y = 5$$
$$x = 2y + 1$$

$$x + y = 3$$
$$2x - 5y = 6$$

27. Find the vertices of the triangle formed by the lines whose equations are shown below.

$y = x + 4 \qquad y = -2x + 4 \qquad \frac{1}{2}x + y = -5$

28. A system of two linear equations has the solution (0, –2). The slope of one of the lines is three times the slope of the other line. If the equation of one of the lines is $y = 3x - 2$, what are the two possible equations of the other line?

29. Which of the following systems of equations have exactly one solution in Quadrant II?

$\begin{aligned} y &= x + 3 \\ x + y &= -1 \end{aligned} \qquad\qquad \begin{aligned} 3x + 2y &= 6 \\ x + 4y &= -2 \end{aligned}$

$\begin{aligned} y &= x + 2 \\ y &= -\frac{1}{2}x - 4 \end{aligned} \qquad\qquad \begin{aligned} x + y &= -6 \\ 3x + y &= -12 \end{aligned}$

30. Find the vertices of the triangle formed by the lines whose equations are shown below.

$y = -2x + 3 \qquad 2y - 5x = 10 \qquad 5y = 2x - 15$

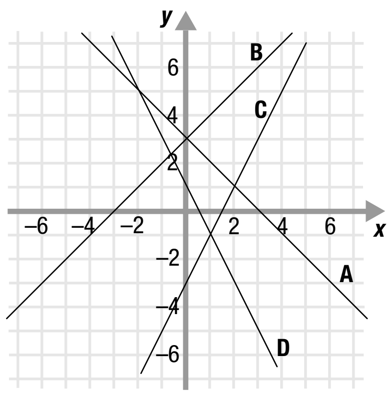
APPLY

Practice Problems

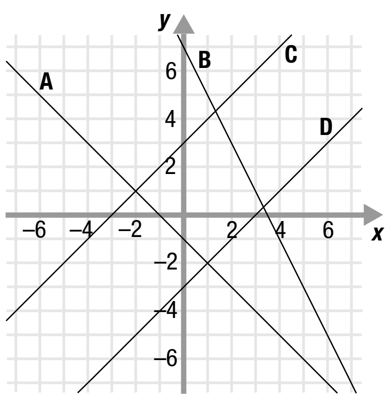
Here are some additional practice problems for you to try.

Solution by Graphing

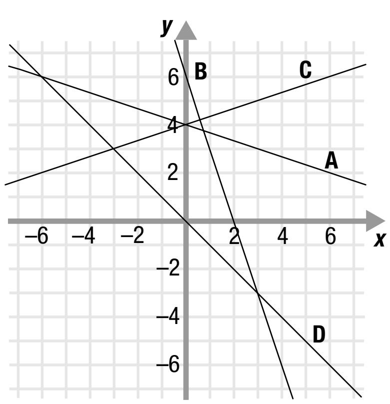
1. Which two lines form a system whose solution is (2, 1)?



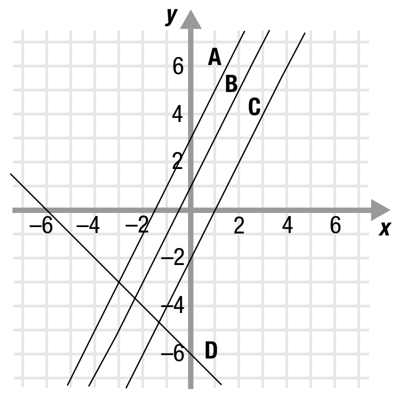
2. Which two lines form a system whose solution is (1, –2)?



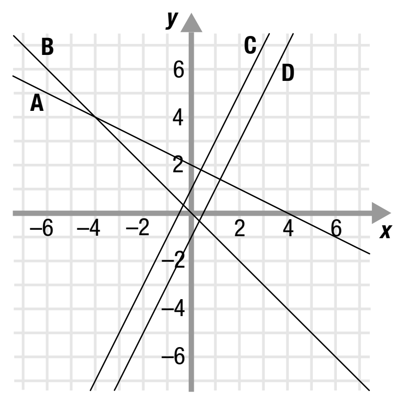
3. Which two lines form a system whose solution is (–3, 3)?



4. Which two lines form a system that has no solution?



5. Which two lines form a system that has no solution?



6. Graph each equation to find the solution of this system:

$$\begin{aligned} x - 4y &= 8 \\ 2x + y &= -2 \end{aligned}$$

7. Graph each equation to find the solution of this system:

$$\begin{aligned} 2x + y &= 4 \\ 3x - 4y &= 6 \end{aligned}$$

8. Graph each equation to find the solution of this system:

$$\begin{aligned} x + y &= 3 \\ 2x - y &= 3 \end{aligned}$$

9. Graph each equation to find the solution of this system:

$$\begin{aligned} x + 3y &= -6 \\ x - 3y &= 0 \end{aligned}$$

10. Graph each equation to find the solution of this system:

$$\begin{aligned} x - y &= -4 \\ x + 2y &= -1 \end{aligned}$$

11. Graph each equation to find the solution of this system:

$$\begin{aligned} 3x - 2y &= -6 \\ -6x + 4y &= 9 \end{aligned}$$

12. Graph each equation to find the solution of this system:

$$\begin{aligned} 4x - y &= 8 \\ -8x + 2y &= -16 \end{aligned}$$

13. Graph each equation to find the solution of this system:

$$\begin{aligned} 2x - y &= 4 \\ -4x + 2y &= 6 \end{aligned}$$

14. Graph each equation to find the solution of this system:

$$\begin{aligned} x - y &= 4 \\ 2x + 3y &= -2 \end{aligned}$$

15. Graph each equation to find the solution of this system:

$$\begin{aligned} 2x + y &= -6 \\ 3x - y &= 1 \end{aligned}$$

16. Graph each equation to find the solution of this system:

$$\begin{aligned} x + y &= 2 \\ 2x + 3y &= 8 \end{aligned}$$

17. Graph each equation to find the solution of this system:

$$\begin{aligned} x + 2y &= 4 \\ -2x - 4y &= -8 \end{aligned}$$

18. Graph each equation to find the solution of this system:

$$\begin{aligned} -2x + 3y &= -6 \\ 6x - 9y &= -18 \end{aligned}$$

19. Graph each equation to find the solution of this system:

$$\begin{aligned} x - 3y &= 6 \\ -2x + 6y &= -12 \end{aligned}$$

20. Graph each equation to find the solution of this system:

$$\begin{aligned} x + y &= 4 \\ 2x - y &= 5 \end{aligned}$$

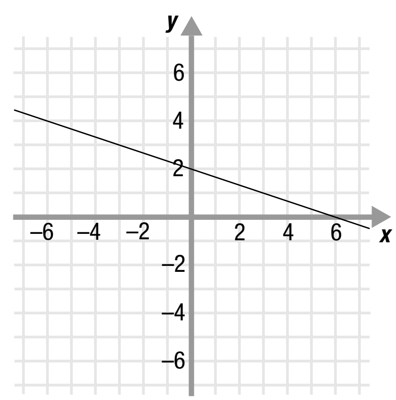
21. Graph each equation to find the solution of this system:

$$\begin{aligned} x + y &= 4 \\ 5x - 2y &= -1 \end{aligned}$$

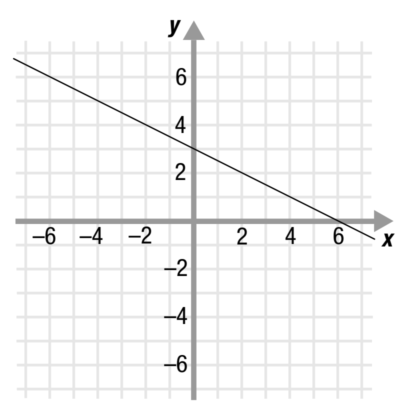
22. Graph each equation to find the solution of this system:

$$\begin{aligned} 2x + 2y &= 8 \\ x - 3y &= 8 \end{aligned}$$

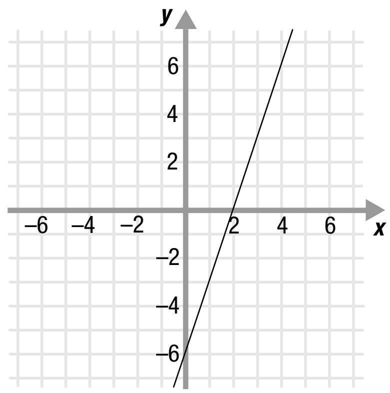
23. Draw a line on the grid below so that the system of equations has no solution.



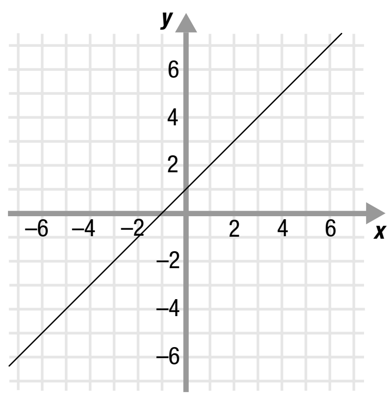
24. Draw a line on the grid below so that the system of equations has an infinite number of solutions.



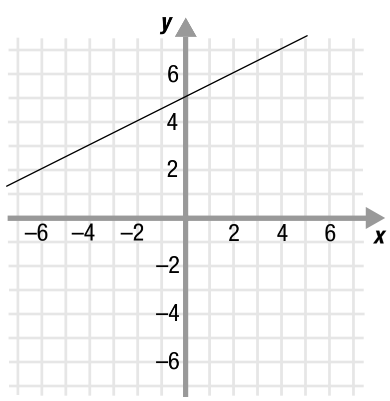
25. Draw a line on the grid below so that the system of equations has no solution.



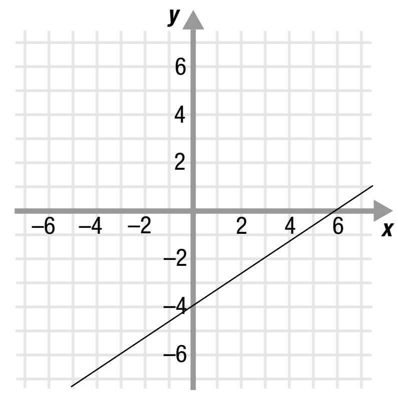
26. Draw a line on the grid below so that the system of equations has one solution at the point (1, 2).



27. Draw a line on the grid below so that the system of equations has one solution at the point (–4, 3).



28. Draw a line on the grid below so that the system of equations has one solution at the point (3, –2).



Solution by Algebra

29. Use the substitution method to solve this system:

$$\begin{aligned} x - 2y &= -1 \\ x + y &= 5 \end{aligned}$$

30. Use the substitution method to solve this system:

$$\begin{aligned} x - y &= 4 \\ 3x + y &= 4 \end{aligned}$$

31. Use the substitution method to solve this system:

$$\begin{aligned} 2x + y &= -1 \\ x - y &= 7 \end{aligned}$$

32. Use the substitution method to solve this system:

$$\begin{aligned} 4x - 3y &= -7 \\ x + 2y &= 12 \end{aligned}$$

33. Use the substitution method to solve this system:

$$\begin{aligned} 3x + 5y &= 1 \\ 4x - y &= 9 \end{aligned}$$

34. Use the substitution method to solve this system:

$$\begin{aligned} 3x - 7y &= 4 \\ 2x + y &= -3 \end{aligned}$$

35. Use the substitution method to solve this system:

$$\begin{aligned} 4x + y &= 7 \\ x + 2y &= 2 \end{aligned}$$

36. Use the substitution method to solve this system:

$$\begin{aligned} 4x - 2y &= 7 \\ 4x + y &= -2 \end{aligned}$$

37. Use the substitution method to solve this system:

$$\begin{aligned} 5x + y &= 1 \\ x - 3y &= 2 \end{aligned}$$

38. Use the substitution method to solve this system:

$$\begin{aligned} -3x + 2y &= 8 \\ x + 2y &= -6 \end{aligned}$$

39. Use the substitution method to solve this system:

$$\begin{aligned} 4x - 3y &= -2 \\ -3x + y &= 6 \end{aligned}$$

40. Use the substitution method to solve this system:

$$\begin{aligned} 2x - 3y &= 1 \\ -4x + y &= 7 \end{aligned}$$

41. Use the substitution method to solve this system:

$$\begin{aligned} 3x - y &= 5 \\ -6x + 2y &= -10 \end{aligned}$$

42. Use the substitution method to solve this system:

$$\begin{aligned} x + 5y &= 5 \\ 3x + 15y &= 11 \end{aligned}$$

43. Use the elimination method to solve this system:

$$\begin{aligned} x - y &= 3 \\ x + y &= 5 \end{aligned}$$

44. Use the elimination method to solve this system:

$$\begin{aligned} x + y &= 3 \\ -x + y &= 7 \end{aligned}$$

45. Use the elimination method to solve this system:

$$\begin{aligned} x + y &= 10 \\ x - y &= 2 \end{aligned}$$

46. Use the elimination method to solve this system:

$$\begin{aligned} x - 2y &= -4 \\ x + y &= 2 \end{aligned}$$

47. Use the elimination method to solve this system:

$$\begin{aligned} 3x - y &= 7 \\ x + y &= 5 \end{aligned}$$

48. Use the elimination method to solve this system:

$$\begin{aligned} x + 2y &= 8 \\ x - y &= -1 \end{aligned}$$

49. Use the elimination method to solve this system:

$$\begin{aligned} 3x - 2y &= 4 \\ -6x + 3y &= -15 \end{aligned}$$

50. Use the elimination method to solve this system:

$$\begin{aligned} 4x - 5y &= 12 \\ 6x + 10y &= 18 \end{aligned}$$

51. Use the elimination method to solve this system:

$$\begin{aligned} 5x - 8y &= 10 \\ 3x + 4y &= 6 \end{aligned}$$

52. Use the elimination method to solve this system:

$$\begin{aligned} 2x - 2y &= -1 \\ 3x + 3y &= 2 \end{aligned}$$

53. Use the elimination method to solve this system:

$$\begin{aligned} -3x + 2y &= 3 \\ 4x + 3y &= -2 \end{aligned}$$

54. Use the elimination method to solve this system:

$$\begin{aligned} 2x + 2y &= -1 \\ 5x - 5y &= 1 \end{aligned}$$

55. Use the elimination method to solve this system:

$$\begin{aligned} 3x - 2y &= 5 \\ -9x + 6y &= 12 \end{aligned}$$

56. Use the elimination method to solve this system:

$$\begin{aligned} 2x - 7y &= 0 \\ 6x - 21y &= 0 \end{aligned}$$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. The graph of the linear system below is shown in Figure 5.1.14. Find the solution of the system.

$$\begin{aligned} -x + y &= -2 \\ 3x - 2y &= 8 \end{aligned}$$

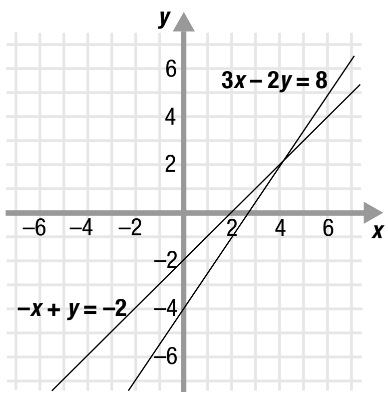


Figure 5.1.4

2. The graph of the linear system below is shown in Figure 5.1.5. Find the solution of the system.

$$\begin{aligned} y &= 2x + 13 \\ x + 2y &= 6 \end{aligned}$$

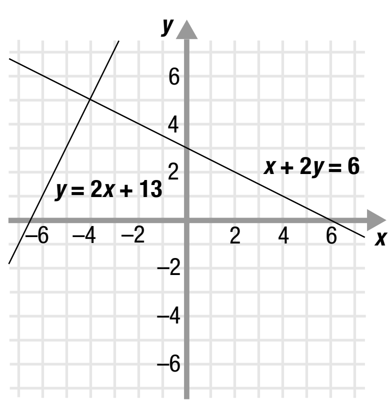


Figure 5.1.5

3. The graph of the linear system below is shown in Figure 5.1.6.

$$\begin{aligned} x + 2y &= -12 \\ -3x + y &= 1 \end{aligned}$$

Circle the statements that are true.

The system has a solution at the point (–2, –5).

The system has only one solution but it is not shown on the graph.

The system has no solution.

The system has an infinite number of solutions.

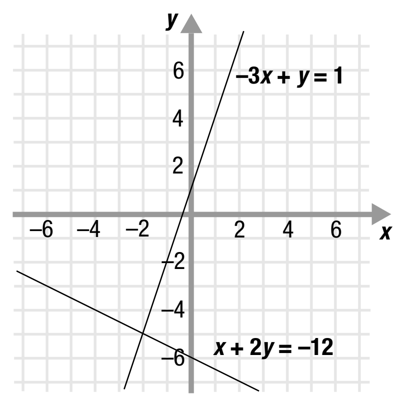


Figure 5.1.6

4. The graph of the linear system below is shown in Figure 5.1.7. Find the solution of the system.

$$\begin{aligned} 2x - 5y &= 10 \\ 4x + 5y &= 20 \end{aligned}$$

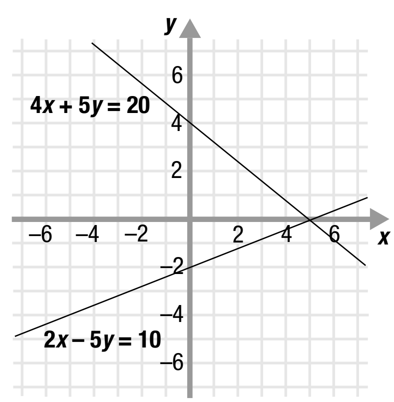


Figure 5.1.7

5. One of the equations in a linear system is $2x + 3y = 6$. Its graph is shown in Figure 5.1.8. If the solution of the system is (–6, 6), which of the following could be the other equation in the system?

$$5x - y = 7$$
$$3x + 4y = 6$$
$$x + 3y = -11$$
$$2x - 4y = 1$$

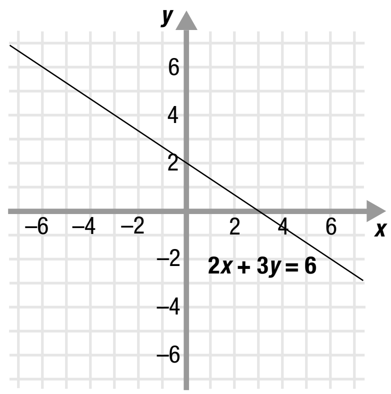


Figure 5.1.8

6. The graph of the linear system below is shown in Figure 5.1.9.

$$3x + 2y = 8$$
$$x - 4y = 12$$

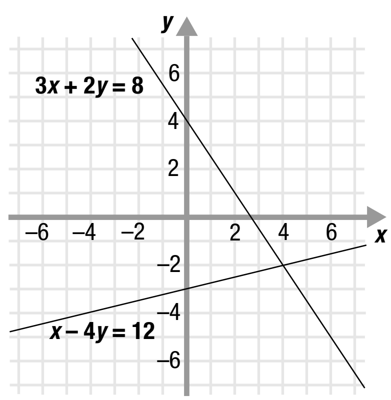


Figure 5.1.9

Which other line passes through the solution of the system?

$$4x - y = 12$$
$$-2x + y = -5$$
$$3x - 3y = 7$$
$$x + y = 2$$

7. Find the solution of this system:

$$x = -1$$
$$y = 4$$

8. The solution of the following linear system is (–2, –3).

$$5x - 3y = -1$$
$$2x + 7y = -25$$

If the first equation is multiplied by 7 and the second equation is multiplied by 3, the result is this system:

$$35x - 21y = -7$$
$$6x + 21y = -75$$

Which of the following statements are true?

The system has an infinite number of solutions.
The system has no solution.
The system has only one solution, the point (–2, –3).
The system has only one solution, the point (–14, –9).

9. Use substitution to solve this linear system:

$$3x - 5y = 11$$
$$2x + y = 29$$

10. Use substitution to solve this linear system:

$$x - 3y = -3$$
$$2x + y = 22$$

11. Use the elimination method to solve this linear system:

$$2x + y = 4$$
$$5x - 2y = 1$$

12. Solve this linear system:

$$7x - 6y = 27$$
$$4x - 5y = 17$$