LESSON 3.1 – INTRODUCTION TO GRAPHING

OVERVIEW

Here's what you'll learn in this lesson:

Plotting Points

a. The xy-plane

b. The x-axis and y-axis

c. The origin

d. Ordered pairs

e. The x-coordinate (abscissa), the y-coordinate (ordinate)

f. Plotting ordered pairs of numbers

g. Labeling the four quadrants

h. Determining the quadrant in which a point lies

i. The signs of the coordinates in each quadrant

Rise and Run

a. Subscript notation

b. Geometric interpretation of rise and run

c. Algebraic definition of rise and run

The Distance Formula

a. Pythagorean Theorem

b. The distance formula

c. The equation of a circle

Many different types of people use graphs to display many kinds of information. Nurses, sportswriters, car mechanics, engineers, bookkeepers, and scientists all use graphs in their work.

Sometimes they use graphs as a way of recording what they see. Sometimes they use graphs as a way of reviewing what has happened. Other times they use graphs to predict the future.

In this lesson you will learn about the most widely used graphing system—the Cartesian coordinate system. You will learn how to plot points and how to compare the horizontal and vertical change between points. You will also learn how to calculate the distance between two points and find the equation of a circle.

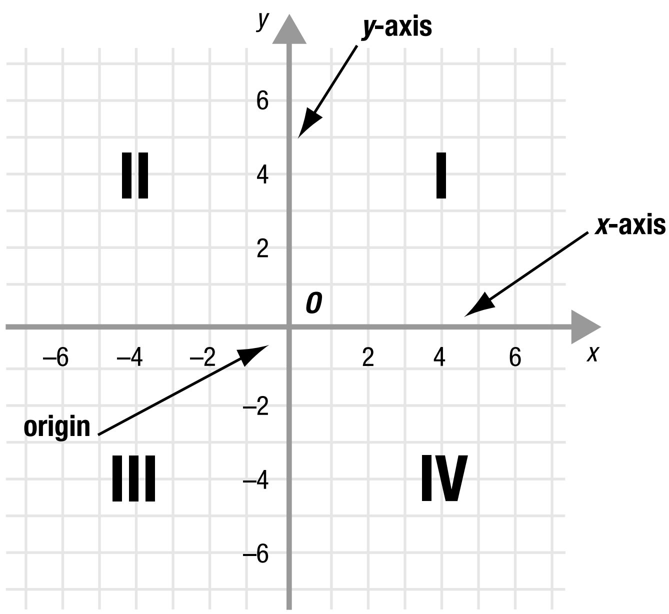
EXPLAIN

PLOTTING POINTS

Summary

The Cartesian Coordinate System

The French mathematician René Descartes realized that if he placed two number lines at right angles to each other he would be able to specify the position of any point in the plane using two reference numbers called coordinates. This arrangement of the two number lines is called the Cartesian coordinate system in honor of Descartes.



- Often, the horizontal number line is called the *x*-axis and the vertical number line is called the *y*-axis.
- The *xy*-plane consists of the *x*-axis and the *y*-axis and all of the points in the plane.
- The origin is usually denoted by the letter *O*.
- The numbers along the *x*-axis get larger as you move toward the right. The numbers along the *y*-axis get larger as you move upward.
- The *x*- and *y*-axes divide the plane into four regions called quadrants. These are labeled with the Roman numerals I, II, III, IV in a counter-clockwise direction beginning in the upper right.
- In Quadrant I, x and y are both positive. In Quadrant II, x is negative and y is positive. In Quadrant III, x and y are both negative. In Quadrant IV, x is positive and y is negative.
- Points on the axes do not lie in a quadrant.

The horizontal axis and vertical axis are not always labeled with x and y. Sometimes it is helpful to use labels which better represent the data.

An easy way to remember which axis is the y-axis is to notice that the y-axis runs up and down, just like the bottom of the Y.

Finding the Coordinates of Points

The coordinates of a point are an ordered pair because the order in which the coordinates are written is important.

To find the position of a point in the *xy*-plane you need two numbers. These two numbers are called the *x*-coordinate and the *y*-coordinate. They can be written as an ordered pair (*x*, *y*).

To find the coordinates of a point:

1. Draw a vertical line from the point to the *x*-axis. The line intersects the *x*-axis at the *x*-coordinate.
2. Draw a horizontal line from the point to the *y*-axis. The line intersects the *x*-axis at the *y*-coordinate.

The coordinates are written as an ordered pair: (*x*-coordinate, *y*-coordinate).

This method of finding the coordinates of the point *P* is illustrated in Figure 3.1.2.

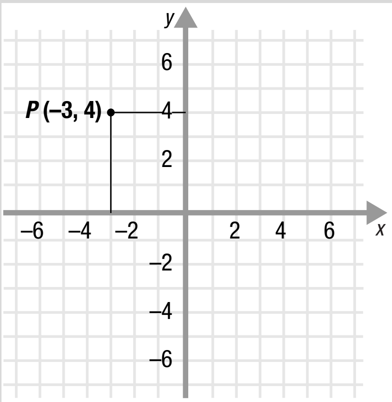


Figure 3.1.2

Here are some tips for finding the coordinates of points which lie on an axis:

- A point which lies on the *x*-axis has coordinates of the form (*x*, 0).
- A point which lies on the *y*-axis has coordinates of the form (0, *y*).
- The origin lies on both axes so it has coordinates (0, 0).

Plotting Points

If the coordinates of a point are not whole numbers, you may only be able to estimate where to plot the point.

If a point has negative coordinates, remember to include the negative sign(s) when you write the coordinates.

You will find that drawing lines through the x- and y-coordinates of a point is often helpful when you first start plotting points. After some practice you may want to try plotting points without drawing these lines.

Once you know the *x*-coordinate and *y*-coordinate of a point, you have enough information to plot the point in the *xy*-plane.

To plot a point:

1. Draw a vertical line through the *x*-coordinate of the point.
2. Draw a horizontal line through the *y*-coordinate of the point.
3. Plot the point where the lines intersect.

As an example, *Q*(2, 4) is plotted in Figure 3.1.3.

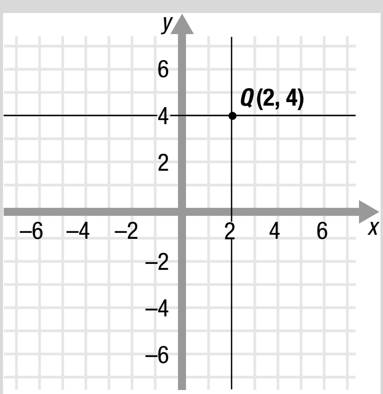


Figure 3.1.3

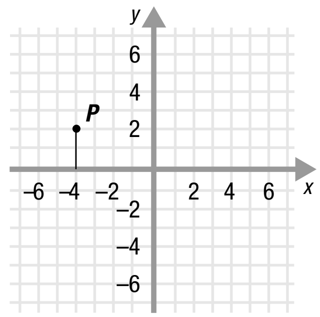
Sample Problems

1. Find the coordinates of the point P below.

   ☑ a. Draw a vertical line from P to the x-axis. This line meets the x-axis at the point $x = -4$.

   ☐ b. Draw a horizontal line from P to the y-axis. This line meets the y-axis at the point $y =$ ______.

   ☐ c. The coordinates of P are (___,___).



2. Plot the point $Q(0, -4)$.

   ☐ a. Draw a vertical line through 0, the x-coordinate of the point Q.

   ☐ b. Draw a horizontal line through –4, the y-coordinate of the point Q.

   ☐ c. The lines intersect at the point Q. Label the point Q.

3. Plot the point $R(5, -2.5)$.

   ☐ a. Draw a vertical line through ______ ______________________________.

   ☐ b. Draw a horizontal line through –2.5, the y-coordinate of R.

   ☐ c. The lines intersect at __________ ______________________________.

4. Plot a point in Quadrant I.

   ☐ a. Shade in Quadrant I.

   ☐ b. Plot a point anywhere in that quadrant.

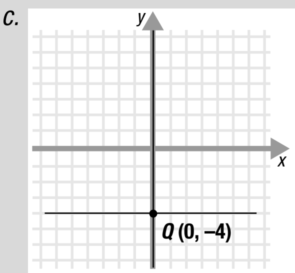
Answers to Sample Problems

b. 2

c. (–4, 2)

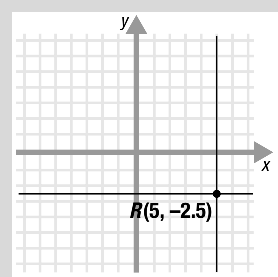
a. This line is the same as the y-axis.

b., c.



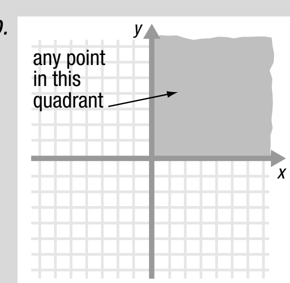
a. Draw a vertical line through 5, the x-coordinate of R.

b.



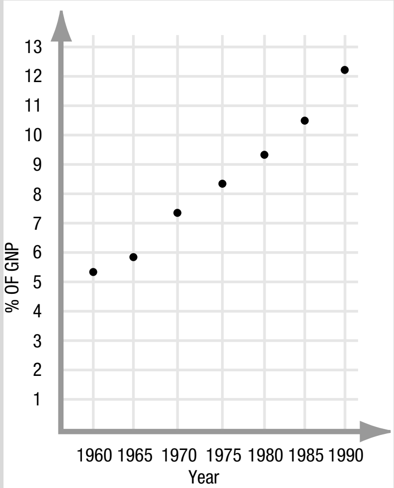
c. The lines intersect at the point R(5, –2.5).

a., b.

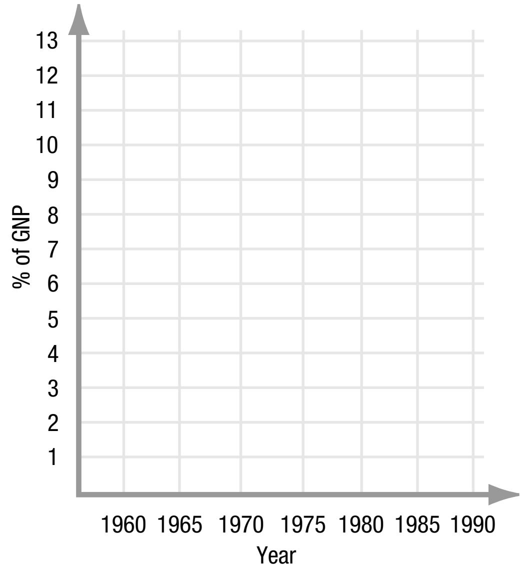


Answers to Sample Problems

a., b., c., d., e., f., g.



5. Health care costs in the United States have been taking an increasing percentage of the Gross National Product (GNP) in recent decades. Use the information in the table below to plot the ordered pairs (year, % of GNP) on the grid provided.



Year	% of GNP
1960	5.3
1965	5.9
1970	7.3
1975	8.3
1980	9.2
1985	10.5
1990	12.2

Plot the point:

- ☐ a. (1960, 5.3)
- ☐ b. (1965, 5.9)
- ☐ c. (1970, 7.3)
- ☐ d. (1975, 8.3)
- ☐ e. (1980, 9.2)
- ☐ f. (1985, 10.5)
- ☐ g. (1990, 12.2)

RISE AND RUN

Summary

Defining Rise and Run

Looking at the relationship between two points often provides more information than looking at each point in isolation. Given two points in the *xy*-plane, for example, you can move from one point to the other with one vertical and one horizontal movement.

- The vertical change as you move from one point to the other is called the rise.
- The horizontal change as you move from one point to the other is called the run.

The rise can be positive or negative.

Likewise, the run, can be positive or negative.

Finding Rise and Run

One way to find the rise and the run in moving from one point to another is by drawing lines on a graph.

To find the rise and the run between two points, P_1 and P_2:

1. Count the number of units of vertical change it takes to move from P_1 to P_2. This is the rise.
2. Count the number of units of horizontal change it takes to move from P_1 to P_2. This is the run.

This method is illustrated in Figure 3.1.4 moving from the point $P_1(-3, 1)$ to $P_2(4, 5)$.

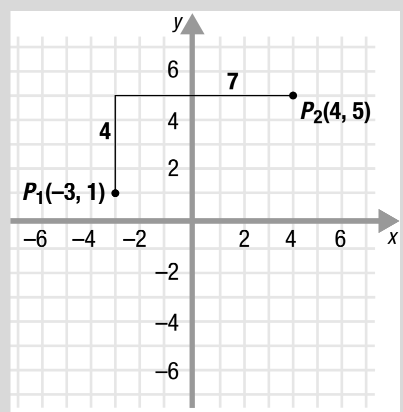


Figure 3.1.4

As you can see, the rise is 4 and the run is 7.

Another way to find the rise and the run in moving from one point to another is by using algebra.

To find the rise and run between two points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

1. Subtract the *y*-coordinates of the points to find the rise, since rise is the vertical change from one point to another along the *y*-axis: **rise =** $\boldsymbol{y_2 - y_1}$
2. Subtract the *x*-coordinates of the points to find the run, since run is the horizontal change from one point to another along the *x*-axis: **run =** $\boldsymbol{x_2 - x_1}$

You can find the rise and the run from $P_1(-3, 1)$ to $P_2(4, 5)$ by using this method:

$$\begin{aligned} \text{rise} &= y_2 - y_1 \\ &= 5 - 1 \\ &= 4 \end{aligned} \qquad \begin{aligned} \text{run} &= x_2 - x_1 \\ &= 4 - (-3) \\ &= 7 \end{aligned}$$

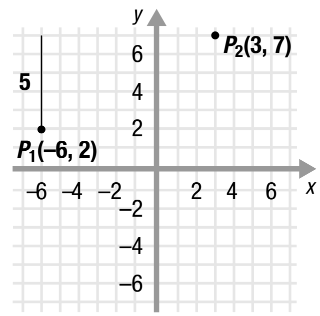
Sample Problems

1. Use the graph to find the rise and the run in moving from $P_1(-6, 2)$ to $P_2(3, 7)$ with one vertical movement and one horizontal movement.

   ☑ a. Start at P_1 and "rise" until you are at the level of P_2.

   ☐ b. Then "run" to the right horizontally to P_2.

   ☐ c. The rise is the length of the vertical line, which is 5 units. The run is the length of the horizontal line, which is ______ units.



2. Find the rise and the run from $P_1(-4, -7)$ to $P_2(3, -1)$ using the algebraic definitions.

   ☐ a. rise $= y_2 - y_1$
   $= (-1) - (-7)$
   $=$______

   ☐ b. run $= x_2 - x_1$
   $=$ ______ $-$ ______
   $=$ ______

3. Find the rise and the run from $P_1(3, -2)$ to $P_2(-5, -1)$.

   ☐ a. rise $= y_2 - y_1$
   $=$ ______ $-$ ______
   $=$ ______

   ☐ b. run $= x_2 - x_1$
   $=$ ______ $-$ ______
   $=$ ______

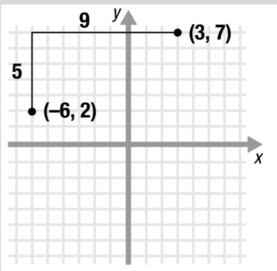
4. Find the rise and the run from $P_1(4, 7)$ to $P_2(-2, 3)$.

   ☐ a. rise $= y_2 - y_1$
   $=$ ______ $-$ ______
   $=$______

   ☐ b. run $= x_2 - x_1$
   $=$ ______ $-$ ______
   $=$ ______

Answers to Sample Problems

b.



c. 9

a. rise = 6

b. run = 3 – (–4)
= 7

a. rise = –1 – (–2)
= 1

b. run = –5 – 3
= –8

a. rise = 3 – 7
= –4

b. run = –2 – 4
= –6

THE DISTANCE FORMULA

Summary

Now that you can plot points in the Cartesian coordinate system, you can learn how to calculate the distance between any two points in this coordinate system. But first you'll learn how to find the distance between two points on the number line.

Finding the Distance Between Two Points on the Number Line

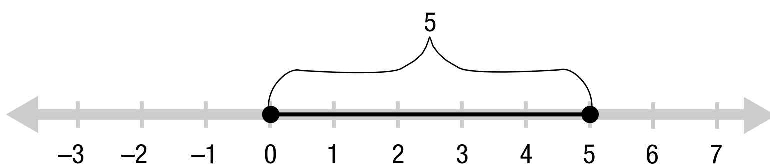
If a and b are any two points on the number line, then the distance between a and b is $|a - b|$ or $|b - a|$.

To find the distance between two points on the number line:

1. Subtract the coordinates of the points (in either order).

2. Take the absolute value of this difference.

You take the absolute value because absolute value gives a nonnegative number and distance can never be negative. Since you take the absolute value, it doesn't matter whether you find $a - b$ or $b - a$.



For example, to find the distance between 0 and 5:

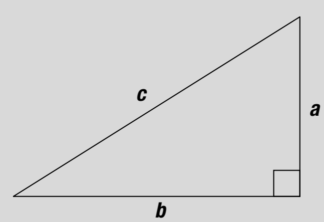
1. Subtract. $0 - 5 = -5$
2. Take the absolute value. $|-5| = 5$

So the distance from 0 to 5 is 5 units.

Now you can find the distance between any two points on the number line. But to be able to find the distance between any two points in the plane, (0, 0) and (3, 4) for example, you need to learn the Pythagorean Theorem.

The Pythagorean Theorem

The Pythagorean Theorem relates the lengths of the sides of a right triangle. It says that if a and b are the lengths of the legs and c is the length of the hypotenuse, then $c^2 = a^2 + b^2$.



A right triangle has one angle that measures 90°. The symbol ⊾ in the corner of the triangle indicates that the angle measures 90°.

If you know the lengths of two of the sides of a right triangle, you can use the Pythagorean Theorem to find the length of the third side. Here's how:

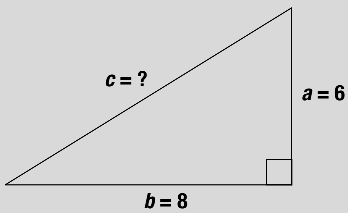
1. Substitute the values for the lengths of the two sides in the Pythagorean Theorem.

2. Solve for the remaining value.

For example, to find c if $a = 6$ and $b = 8$:

1. Substitute $a = 6$ and $b = 8$.

$$c^2 = a^2 + b^2$$
$$c^2 = 6^2 + 8^2$$

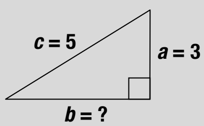


2. Solve for c.

$$c^2 = 36 + 64$$
$$c^2 = 100$$
$$c = \pm\sqrt{100}$$
$$c = \pm 10$$
$$c = 10$$

Choose the positive value of c, since c is a distance.

We always take the positive root of 100 since this quantity represents a distance.



As another example, to find b if $a = 3$ and $c = 5$:

1. Substitute $a = 3$ and $c = 5$.

$$c^2 = a^2 + b^2$$
$$5^2 = 3^2 + b^2$$

2. Solve for b.

$$25 = 9 + b^2$$
$$25 - 9 = b^2$$
$$16 = b^2$$
$$\pm\sqrt{16} = b$$
$$b = \pm 4$$
$$b = 4$$

Choose the positive value of b, since b is a distance.

Finding the Distance Between Two Points Using the Pythagorean Theorem

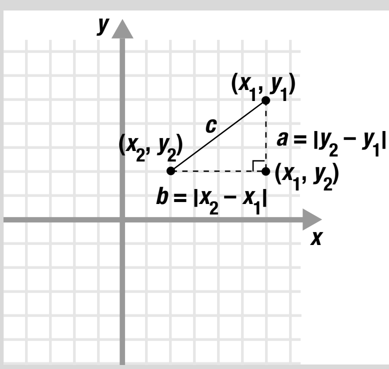


Figure 3.1.5

You can use the Pythagorean Theorem to find the distance between any two points.

To find the distance c between any two points (x_1, y_1) and (x_2, y_2) using the Pythagorean Theorem (refer to Figure 3.1.5):

1. Form a right triangle as follows:
   a. Draw a vertical line segment from (x_1, y_1) to (x_1, y_2). Label this a.
   b. Draw a horizontal line segment from (x_2, y_2) to (x_1, y_2). Label this b.
2. Find the lengths of segments:
   a. The length of the vertical segment a is $|y_2 - y_1|$ units.
   b. The length of the horizontal segment b is $|x_2 - x_1|$ units.
3. Use the Pythagorean Theorem, $c^2 = a^2 + b^2$, to find the length of the hypotenuse, c. The value of C is the distance between the two points.

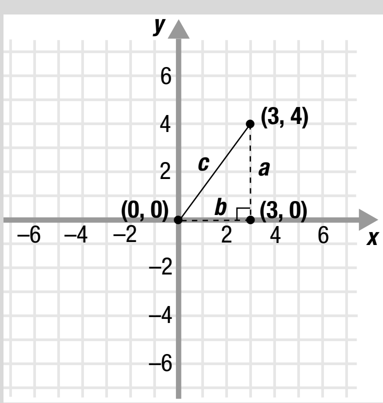


Figure 3.1.6

For example, to find the distance c between (0, 0) and (3, 4):

1. Form a right triangle. — See Figure 3.1.6.
2. Find the lengths of the segments. — $a = |0 - 4| = |-4| = 4$, $b = |0 - 3| = |-3| = 3$
3. Find c, the distance between (0, 0) and (3, 4).

$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 4^2$$
$$c^2 = 9 + 16$$
$$c^2 = 25$$
$$c = 5$$

So the distance from (0, 0) to (3, 4) is 5 units.

The Distance Formula

Instead of drawing a triangle to find the distance between two points, you can just use the distance formula.

From your work with the Pythagorean Theorem you know that $c^2 = a^2 + b^2$, where $a = |y_2 - y_1|$ and $b = |x_2 - x_1|$. So, c^2, the square of the distance between any two points (x_1, y_1) and (x_2, y_2) is:

$$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \text{ and } c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This equation is called the distance formula.

To find the distance between any two points using the distance formula:

1. Identify x_1, y_1, x_2, and y_2.
2. Substitute these values into the distance formula.
3. Simplify.

It doesn't matter which point you call (x_1, y_1) and which one you call (x_2, y_2). Just don't change it once you've decided.

For example, to find the distance between (5, 0) and (–2, –8):

1. Identify x_1, y_1, x_2, and y_2.

$x_1 = 5$
$y_1 = 0$
$x_2 = -2$
$y_2 = -8$

2. Substitute.

$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$c = \sqrt{(-2 - 5)^2 + (-8 - 0)^2}$

3. Simplify.

$c = \sqrt{(-7)^2 + (-8)^2}$
$c = \sqrt{49 + 64}$
$c = \sqrt{113}$

The Equation of a Circle

You can use the square of the distance formula $(x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2$, to find the equation of a circle.

Suppose the center of a circle is at the point (h, k) and its radius is r (where $r > 0$). If (x, y) is any point on the circle, then using the square of the distance formula you can get the equation of the circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

The radius, r, of a circle must be nonnegative.

For example, the equation of the circle shown in Figure 3.1.7 whose center is at (2, 1) and whose radius is 3 is:

$$(x - 2)^2 + (y - 1)^2 = 3^2$$

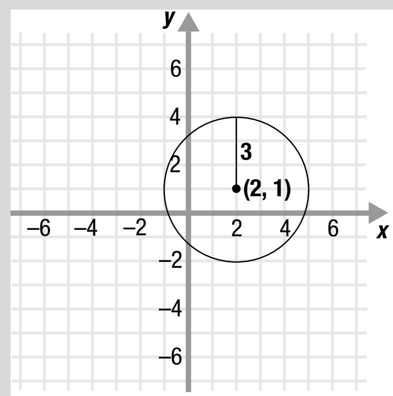


Figure 3.1.7

You can also find the center and radius of a circle if you know the equation of the circle.

For example, if the equation of a circle is $(x + 4)^2 + (y - 3)^2 = 2^2$, its center is at (–4, 3) and its radius is 2. See Figure 3.1.8.

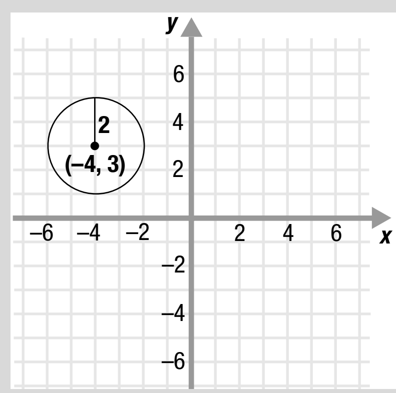


Figure 3.1.8

Why is the x-coordinate of the center equal to –4? Well, $(x + 4)^2$ can be rewritten as $[x - (-4)]^2$.

Answers to Sample Problems

b. 5, 8
3, 6

c. 10

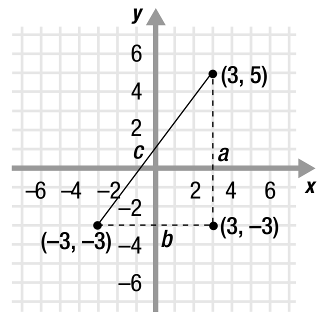
b. –3, –1

c. 5

Sample Problems

1. Use the Pythagorean Theorem to find the distance between (3, 5) and (–3, –3).

   ✔ a. Form a right triangle.



   ☐ b. Find the lengths of the segments.
   $a = |-3 - \_\_\_\_| = \_\_\_\_\_\_$
   $b = |-3 - \_\_\_\_| = \_\_\_\_\_\_$

   ☐ c. Use the Pythagorean Theorem, $c^2 = a^2 + b^2$, to find c, the distance between the points.
   $c = \_\_\_\_\_\_$

2. Using the distance formula, find the distance between (1, 2) and (–3, –1).

   ✔ a. Identify x_1, y_1, x_2, and y_2.
   $x_1 = 1$
   $y_1 = 2$
   $x_2 = -3$
   $y_2 = -1$

   ☐ b. Substitute.
   $c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
   $c = \sqrt{(\_\_\_ - 1)^2 + (\_\_\_ - 2)^2}$

   ☐ c. Simplify.
   $c = \_\_\_\_\_$

EXPLORE

Sample Problems

On the computer, you used the Grapher to move a point around a grid and observed how some of the properties of the point changed. Below are some additional exploration problems.

1. Plot three points in Quadrant I whose x-coordinate is 2.

☐ a. Find Quadrant I and shade it in.

☐ b. Find a point in Quadrant I whose x-coordinate is 2.

☐ c. Plot your point on the graph.

☐ d. Repeat steps (b) and (c) to plot two more points.

2. Plot three points, (x, y), where $y = x + 1$.

☐ a. Pick any three numbers for the x-coordinates, say, $x =$ __0__, $x =$ __–1__, and $x =$ ____.

☐ b. For each value of x, find the value of y.
For example, if $x = 0$, then $y = 0 + 1 = 1$.
The corresponding ordered pair is (0, 1).
Plot that point.

☐ c. If $x = –1$, then $y =$ ____. Plot the corresponding ordered pair (–1, ____).

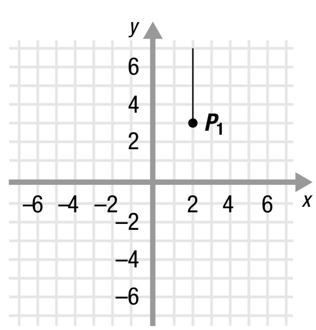
☐ d. If $x =$ ____, then $y =$ ____. Plot the corresponding ordered pair (____,____).

3. Given the point $P_1(2, 3)$, find the coordinates of P_2 if the rise from P_1 to P_2 is 4 and the run is 3.

✔ a. Start at the point (2, 3) and draw a vertical line 4 units long torepresent the rise.

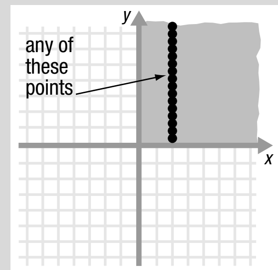
☐ b. From the top of the vertical line, draw a horizontal line 3 units longto represent the run.

☐ c. The horizontal line ends at the point P_2(____,____).

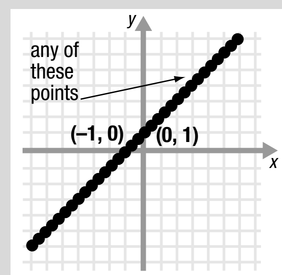


Answers to Sample Problems

a., b., c., d.
Several possible answers are shown.



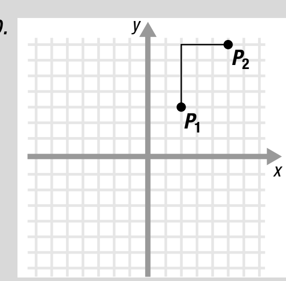
b., c., d.
Several possible answers are shown.



c. 0, 0

d. Answers will vary.

b.



c. (5, 7)

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain
Plotting Points

Use Figure 3.1.9 to answer questions 1 through 8.

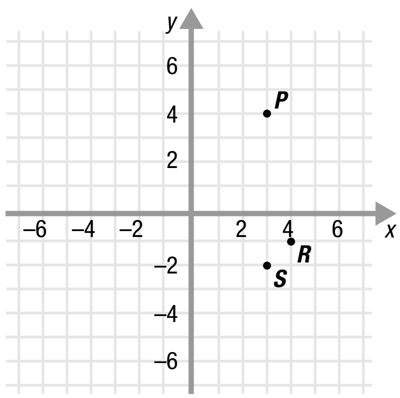


Figure 3.1.9

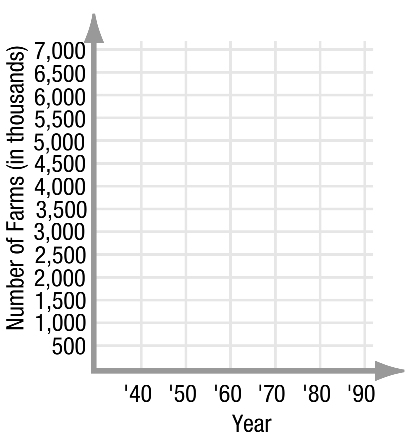


Figure 3.1.10

1. Find the coordinates of point *P*.
2. Plot the point *Q*(–2, 5).
3. In what quadrant does the point *R*(4, –1) lie?
4. Find the coordinates of point *S*.
5. Plot the point *T*(–3, 6).
6. Plot a point which lies in Quadrant III.
7. Plot the point *U*(0, 5).
8. Plot a point in Quadrant I whose *x*-coordinate is 4.
9. For selected years, the number of farms in the United States is listed in the table below. Use this information to plot the ordered pairs (year, number of farms) on the grid in Figure 3.1.10.

Year	Number of Farms (in thousands)	Number of Average Acres per Farm
1940	6,102	175
1950	5,388	216
1960	3,962	297
1970	2,954	373
1980	2,440	426
1990	2,143	461

10. Using the data provided in problem (9), plot the ordered pairs (year, average number of acres per farm) on the grid in Figure 3.1.11.

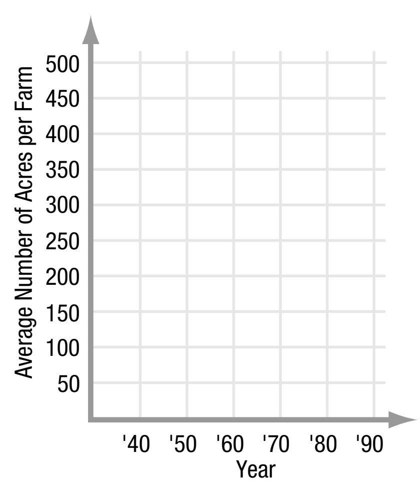


Figure 3.1.11

11. Plot the point $V(-3, 0)$.

12. Plot a point which does not lie in any quadrant.

Rise and Run

13. Draw one vertical and one horizontal line to show the rise and the run in moving from $P_1(1, 3)$ to $P_2(4, 6)$.

14. Plot the points $Q_1(-2, 3)$ and $Q_2(3, 4)$. Draw one vertical and one horizontal line to find the rise and the run in moving from Q_1 to Q_2.

15. Use rise = $y_2 - y_1$ and run = $x_2 - x_1$ to find the rise and the run in moving from $R_1(2, 5)$ to $R_2(5, 7)$.

16. Find the rise and the run in moving from $S_1(6, 7)$ to $S_2(2, -4)$ by drawing one vertical and one horizontal line on the graph.

17. Find the rise and the run from $T_1(-1, -4)$ to $T_2(-5, -8)$ by subtracting the appropriate coordinates.

18. Which is greater, the rise from $U_1(-9, -6)$ to $U_2(-1, 5)$ or the rise from $V_1(0, -6)$ to $V_2(10, 2)$?

19. Find the rise and the run from $W_1(-7, 11)$ to $W_2(17, 19)$ by subtracting the appropriate coordinates.

20. Given $P_1(1, 2)$, find the coordinates of P_2 if the rise from P_1 to P_2 is 2 and the run is 5. Use the grid in Figure 3.1.12.

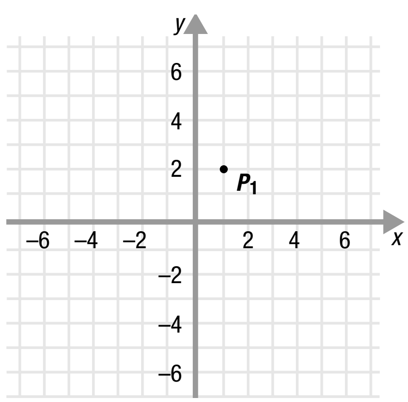


Figure 3.1.12

21. Plotted in Figure 3.1.13 is the federal minimum hourly wage rate (for nonfarm workers) for selected years. Use this information to determine which five-year period had the greatest rise in minimum wage. (You can refer to the table for more accurate numbers.)

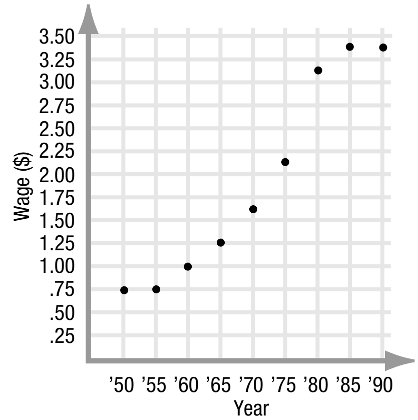


Figure 3.1.13

Year	Wage
1950	$ .75
1955	$ .75
1960	$1.00
1965	$1.25
1970	$1.60
1975	$2.10
1980	$3.10
1985	$3.35
1990	$3.35

22. Use the graph and table in problem (21) to determine which five-year period had the smallest rise in minimum wage.

23. Find the rise and the run from $P_1(-68, -32)$ to $P_2(17, 94)$ by subtracting the appropriate coordinates.

24. Starting at $P_1(-3, -6)$, find the coordinates of P_2 if the rise from P_1 to P_2 is 8 and the run is 7.

The Distance Formula

25. If $a = 5$ and $b = 12$, use the Pythagorean Theorem to find c, the length of the hypotenuse of the right triangle shown in Figure 3.1.14.

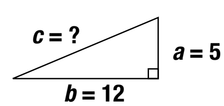


Figure 3.1.14

26. What is the equation of a circle whose center is at (2, 3) and whose radius is 4?

27. Using the distance formula, find the distance between (0, 0) and (5, 2).

28. Use the Pythagorean Theorem to find the distance between (0, 0) and (6, 8). See Figure 3.1.15.

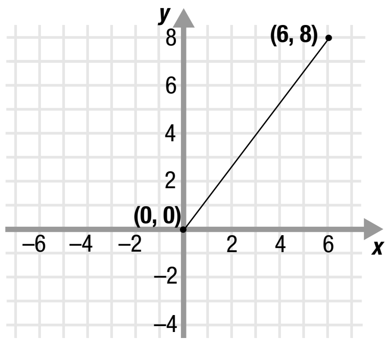


Figure 3.1.15

29. Find the center and the radius of the circle whose equation is $(x + 5)^2 + (y - 7)^2 = 2^2$.

30. Using the distance formula, find the distance between (–2, 4) and (–1, –7).

31. Use the Pythagorean Theorem to find the distance between (–2, –1) and (5, –6). See Figure 3.1.16.

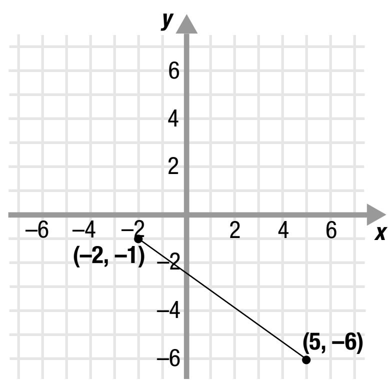


Figure 3.1.16

32. Find the center and the radius of the circle whose equation is: $(x - 6)^2 + (y + 1)^2 = 16$.

33. A fullback takes the ball from his 5 yard line (30 yards from the sideline) to his 45 yard line (50 yards from the same sideline). How many yards did he actually run? (You can express your answer as the square of the distance.) Start by finding *a* and *b* as shown in Figure 3.1.17. Then find *c*.

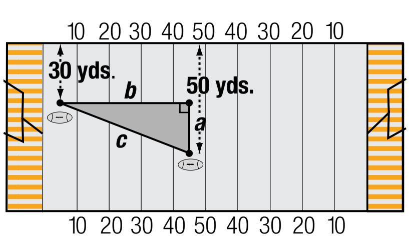


Figure 3.1.17

34. Marilena has been taking a shortcut across a lawn as shown in Figure 3.1.18. If the two lengths of the sidewalk measure 6 ft. and 8 ft., how much distance does Marilena save by taking the shortcut?

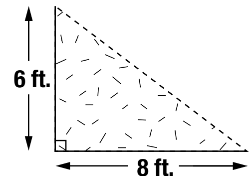


Figure 3.1.18

35. Write the equation of the circle with radius 5 whose center is at (–3, 2).

36. Using the distance formula, find the distance between (–4, 4) and (5, –8).

Explore

37. Plot three points in Quadrant II each of which has a *y*-coordinate equal to 5.

38. Plot three points, (x, y), where $y = x$.

39. Starting at the point $P_1(1, 1)$, if you rise 2 and also run 2, you end up at the point (3, 3). Start again at $P_1(1, 1)$ and plot three other points such that the rise and run are equal to each other.

40. Plot three points in Quadrant IV each of which has an *x*-coordinate equal to 3.

41. Plot three points, (x, y), where $y = x + 2$.

42. Starting at the point $Q_1(-2, -4)$, if you rise 2 and run 1, you end up at the point (–1, –2). Starting at the point $Q_1(-2, -4)$, plot three other points which have a rise which is twice as much as the run.

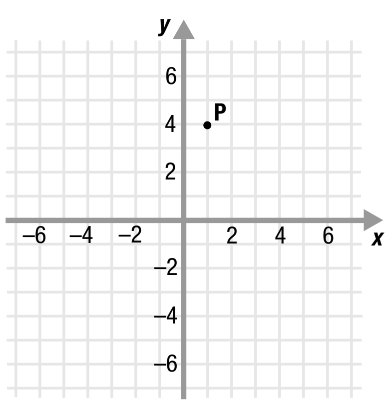
APPLY

Practice Problems

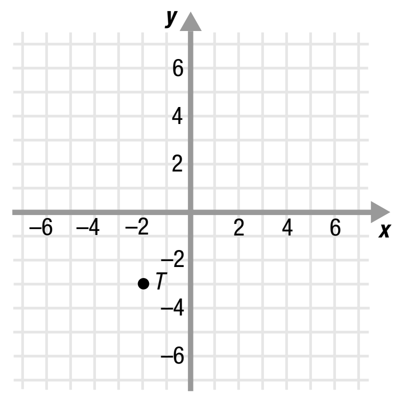
Here are some additional practice problems for you to try.

Plotting Points

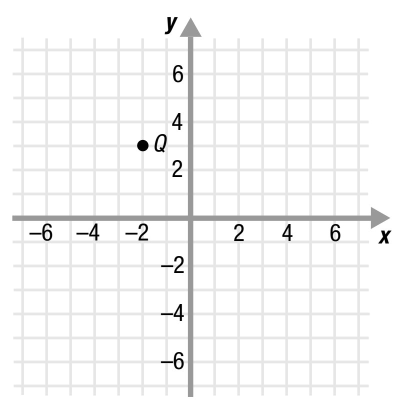
1. Plot the point (3, 5).
2. Plot the point (6, 1).
3. Plot the point (4, 1).
4. Plot the point (–3, 4).
5. Plot the point (–5, 6).
6. Plot the point (–1, 2).
7. Plot the point (–1, –5).
8. Plot the point (–6, –2).
9. Plot the point (–3, –5).
11. Plot the point (4, –4).
12. Plot the point (4, –2).
13. Plot the point (0, –3).
14. Plot the point (2, 0).
15. Plot the point (–3, 0).
16. Find the coordinates of the point P.



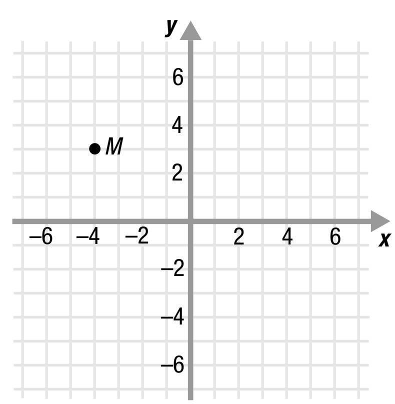
17. Find the coordinates of the point *T*.



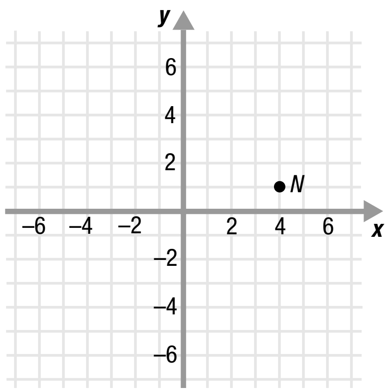
18. Find the coordinates of the point *Q*.



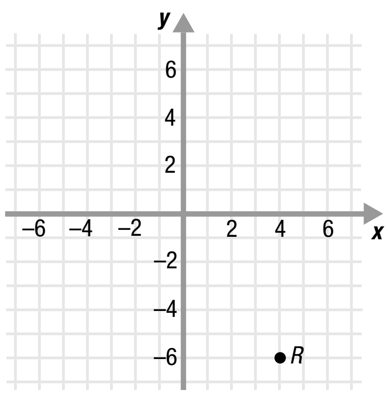
19. Find the coordinates of the point *M*.



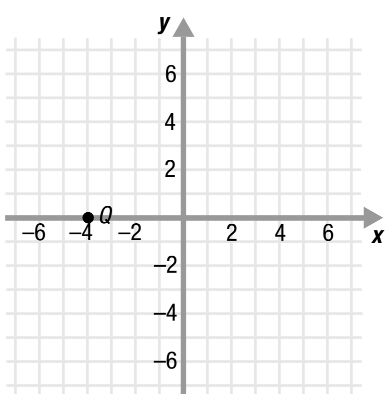
20. Find the coordinates of the point *N*.



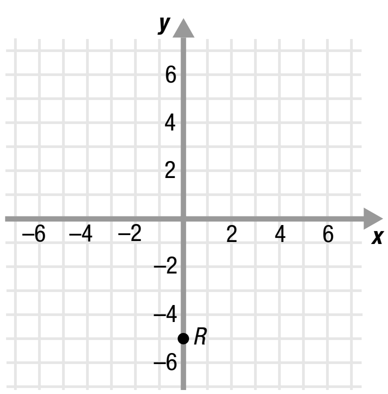
21. Find the coordinates of the point *R*.



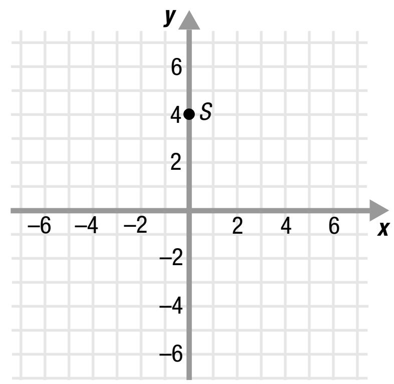
22. Find the coordinates of the point *Q*.



23. Find the coordinates of the point *R*.



24. Find the coordinates of the point *S*.



25. In what quadrant does the point (–4, 3) lie?

26. In what quadrant does the point (–2, –3) lie?

27. In what quadrant does the point (1, –3) lie?

28. In what quadrant does the point (4, 2) lie?

Rise and Run

29. Find the rise and the run in moving from the point (1, 5) to the point (9, 7).

30. Find the rise and the run in moving from the point (12, 8) to the point (25, 17).

31. Find the rise and the run in moving from the point (11, 5) to the point (2, 9).

32. Find the rise and the run in moving from the point (4, 3) to the point (2, –7).

33. Find the rise and the run in moving from the point (0, –6) to the point (8, 5).

34. Find the rise and the run in moving from the point (2, 5) to the point (11, 9).

35. Find the rise and the run in moving from the point (3, –10) to the point (0, –4).

36. Find the rise and the run in moving from the point (–21, –16) to the point (–19, –13).

37. Find the rise and the run in moving from the point (–2, –5) to the point (4, –2).

38. Find the rise and the run in moving from the point (4, 0) to the point (9, 5).

39. Find the rise and the run in moving from the point (8, –1) to the point (0, –7).

40. Find the rise and the run in moving from the point (2, 0) to the point (–5, 2).

41. Find the rise and the run in moving from the point (–5, –9) to the point (8, 2).

42. Find the rise and the run in moving from the point (–10, –4) to the point (1, 8).

43. Find the rise and the run in moving from the point (–4, –4) to the point (6, 3).

44. Find the rise and the run in moving from the point (23, 17) to the point (1, –3).

45. Find the rise and the run in moving from the point (15, –16) to the point (43, 31).

46. Find the rise and the run in moving from the point (11, –7) to the point (35, 24).

47. Find the rise and the run in moving from the point (–13, –29) to the point (0, –7).

48. Find the rise and the run in moving from the point (–85, –57) to the point (0, 3).

49. Find the rise and the run in moving from the point (–27, –14) to the point (0, 12).

50. Which is greater, the rise from $P_1(9, 13)$ to $P_2(21, 17)$ or the rise from $Q_1(-3, -5)$ to $Q_2(4, 16)$

51. Which is greater, the run from $P_1(7, 12)$ to $P_2(19, 13)$ or the run from $Q_1(-1, 5)$ to $Q_2(3, 39)$?

52. Given $P_1(11, 14)$, find the coordinates of P_2 if the rise from P_1 to P_2 is 3 and the run is 9.

53. Given $P_1(8, 9)$, find the coordinates of P_2 if the rise from P_1 to P_2 is 4 and the run is 7.

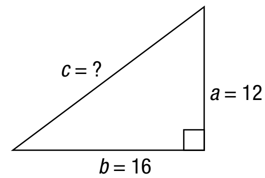
54. Given $P_1(-4, -7)$, find the coordinates of P_2 if the rise from P_1 to P_2 is 6 and the run is 2.

55. Given $P_1(-16, 7)$, find the coordinates of P_2 if the rise from P_1 to P_2 is 13 and the run is 17.

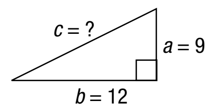
56. Given $P_1(-3, -6)$, find the coordinates of P_2 if the rise from P_1 to P_2 is 6 and the run is 8.

The Distance Formula

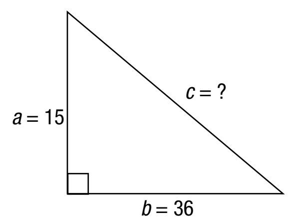
57. If $a = 12$ and $b = 16$, use the Pythagorean Theorem to find c, the length of the hypotenuse of the right triangle shown below.



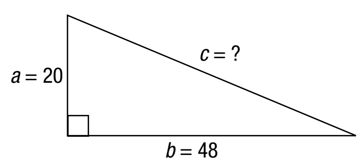
58. If $a = 9$ and $b = 12$, use the Pythagorean Theorem to find c, the length of the hypotenuse of the right triangle shown below.



59. If $a = 15$ and $b = 36$, use the Pythagorean Theorem to find c, the length of the hypotenuse of the right triangle shown below.



60. If $a = 20$ and $b = 48$, use the Pythagorean Theorem to find c, the length of the hypotenuse of the right triangle shown below.



61. What is the equation of the circle whose center is at (2, –3) and whose radius is 4?

62. What is the equation of the circle whose center is at (–4, –5) and whose radius is 7?

63. What is the equation of the circle whose center is at (–3, 1) and has radius 5?

64. Write the equation of the circle whose center is at (1, 6) and has radius 10.

65. Write the equation of the circle whose center is at (–3, 7) and has radius 8.

66. Write the equation of the circle whose center is at (3, 10) and has radius 8.

67. Use the distance formula to find the distance between the points (3, 6) and (9, 13).

68. Use the distance formula to find the distance between the points (4, –7) and (–2, 3).

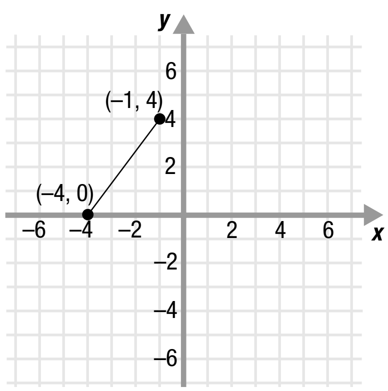
69. Use the distance formula to find the distance between the points (7, 2) and (–8, 3).

70. Use the distance formula to find the distance between the points (–11, –5) and (4, –7).

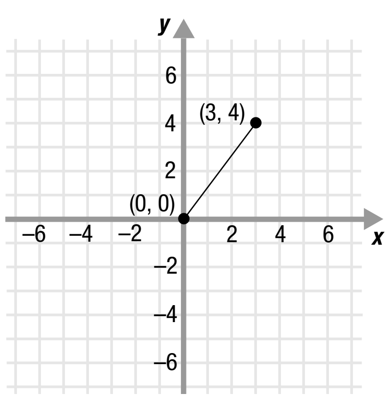
71. Use the distance formula to find the distance between the points (–1, 7) and (–10, –2).

72. Use the distance formula to find the distance between the points (–10, –3) and (4, –2).

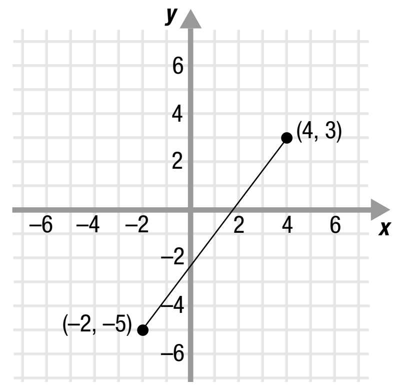
73. Use the Pythagorean Theorem to find the distance between (–4, 0) and (–1, 4).



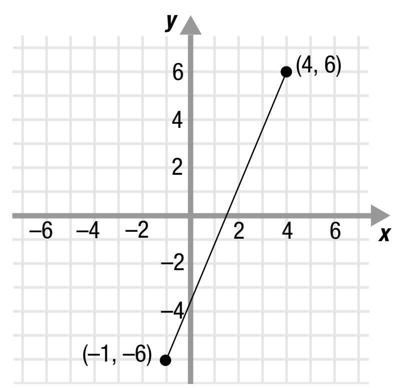
74. Use the Pythagorean Theorem to find the distance between (0, 0) and (3, 4).



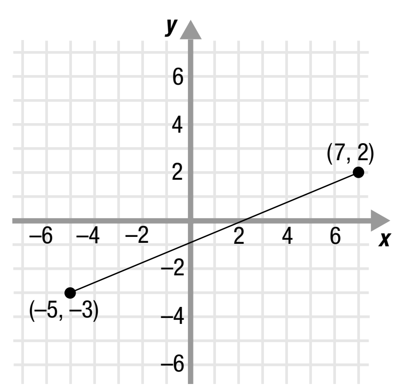
75. Use the Pythagorean Theorem to find the distance between (–2, –5) and (4, 3).



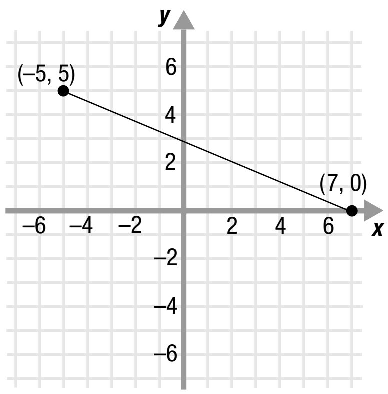
76. Use the Pythagorean Theorem to find the distance between (–1, –6) and (4, 6).



77. Use the Pythagorean Theorem to find the distance between (7, 2) and (–5, –3).



78. Use the Pythagorean Theorem to find the distance between (–5, 5) and (7, 0).



79. Find the center and the radius of the circle whose equation is $(x + 5)^2 + (y + 2)^2 = 3^2$.

80. Find the center and the radius of the circle whose equation is $(x - 10)^2 + (y - 1)^2 = 7^2$.

81. Find the center and the radius of the circle whose equation is $(x + 9)^2 + (y - 12)^2 = 6^2$.

82. Find the center and the radius of the circle whose equation is $(x - 8)^2 + (y + 2)^2 = 25$.

83. Find the center and the radius of the circle whose equation is $(x + 9)^2 + (y - 3)^2 = 121$.

84. Find the center and the radius of the circle whose equation is $(x - 3)^2 + (y + 15)^2 = 144$.

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

Use Figure 3.1.19 to answer questions (1) – (3).

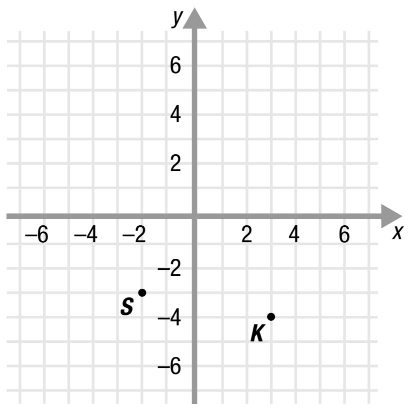


Figure 3.1.19

1. Find the coordinates of point *K*.
2. Plot the point *P*(5, 2.5).
3. In what quadrant does the point *S*(–2, –3) lie?
4. For selected years, average gas mileage for American cars is listed in the table below (rounded to the nearest whole number). Plot the ordered pairs (year, mileage) on the set of axes provided in Figure 3.1.20.

Year	Average Gas Mileage (mpg)
1970	14
1975	15
1980	23
1985	26
1990	27

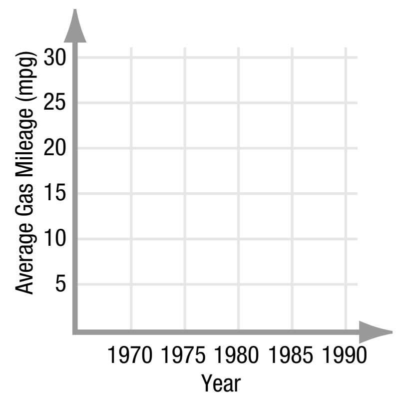


Figure 3.1.20

5. Find the rise and the run in moving from point $P_1(1, -5)$ to $P_2(7, 5)$ by drawing one vertical and one horizontal line on the grid in Figure 3.1.21.

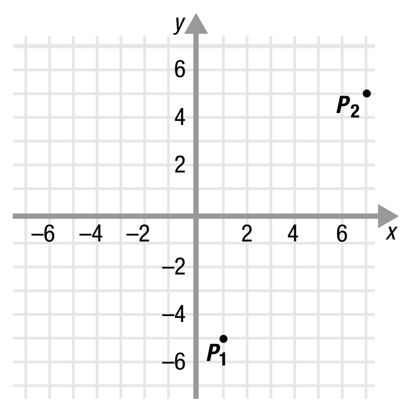


Figure 3.1.21

6. Find the rise and the run from $P_1(-7, -8)$ to $P_2(0, 4)$ by subtracting the appropriate coordinates.
7. Find the rise and the run from $P_1(-12, 7)$ to $P_2(24, 16)$ by subtracting the appropriate coordinates.

8. The average price for a gallon of gasoline is plotted in Figure 3.1.22 for selected years. Use this information to determine which five-year period had the greatest rise in gas prices.

Year	Price (cents)
1950	26.8
1955	29.1
1960	31.1
1965	31.2
1970	35.7
1975	56.7
1980	119.1
1985	111.5
1990	114.9

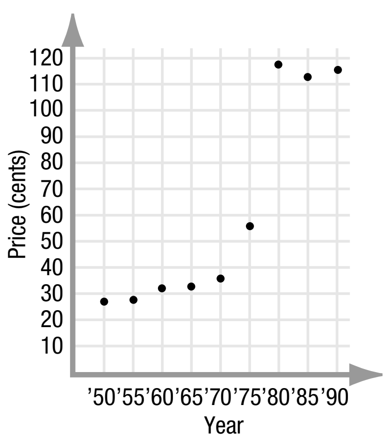


Figure 3.1.22

9. If $a = 9$ and $b = 12$, use the Pythagorean Theorem to find c, the length of the hypotenuse of the right triangle shown in Figure 3.1.23.

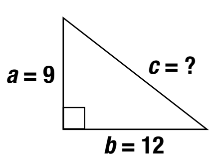


Figure 3.1.23

10. Use the Pythagorean Theorem to find the distance between the points (–3, 1) and (1, –2). See Figure 3.1.24.

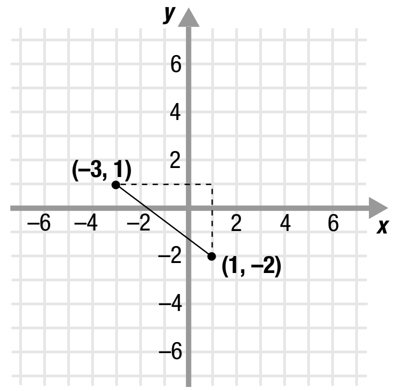


Figure 3.1.24

11. Use the distance formula to find the distance between the points (10, 2) and (–2, –7).

12. Find the radius and the center of the circle whose equation is below.

$$(x - 1)^2 + [y - (-5)]^2 = 2^2$$

13. A point with a negative x-coordinate and a positive y-coordinate lies in which quadrant?

Use Figure 3.1.25 to answer questions (14) – (16).

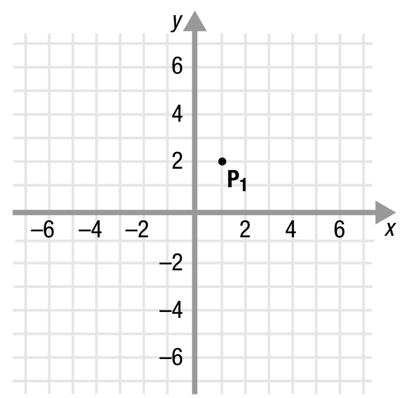


Figure 3.1.25

14. Plot a point in Quadrant III whose x-coordinate is –4.

15. Starting at the point $P_1(1, 2)$, find the coordinates of P_2 if the rise from P_1 to P_2 is 5 and the run is 1.

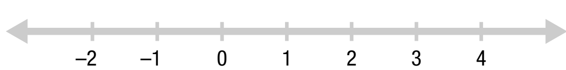
16. Plot a point, (x, y), where $y = x - 1$.

TOPIC 3 CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic. Or you may wish to do these problems to review for a test.

1. Write in lowest terms: $\frac{12}{36}$
2. Evaluate the expression $2xy - 6y + 12$ when $x = -2$ and $y = 4$.
3. Simplify the expression $3(x - 7) + 2(9 - x)$.
4. Plot the points $P_1(1, 2)$ and $P_2(6, 4)$. Draw one vertical and one horizontal line to find the rise and the run from P_1 to P_2.
5. Plot the points $Q_1(-4, -3)$ and $Q_2(-1, 5)$. Draw one vertical and one horizontal line to find the rise and the run from Q_1 to Q_2.
6. Plot three points, (x, y), in Quadrant III where $x = y$. Use the grid in Figure 3.2.
7. Seven years ago, Raoul was as old as Christine is now. If the sum of their ages is 63, how old is each person?
8. Find the rise and the run from $V_1(-54, -37)$ to $V_2(-8, 63)$ by subtracting the appropriate coordinates.
9. Solve for x: $4x + 9 < 13$. Then graph its solution on the number line below.



10. Find: $\frac{3}{4} + \frac{9}{10}$
11. Einstein's famous formula, $E = mc^2$, shows the amount of energy, E, which can be obtained from a particle of mass m. Solve this formula for c.
12. Circle the true statements.

    The equation $x + 3 = x - 7$ has no solution.

    $\frac{5}{6} \neq \frac{2}{4} + \frac{3}{2}$

    $5 \cdot 5 \cdot 5 = 3^5$

    $-3 < -2$

    $|-3| < |-2|$

13. Find: $7[2 - 3(5 - 4) + 1]$
14. Solve for z: $2 < z - 4 \leq 7$. Circle the number below that is not a solution.

    11
    2
    6.1
    7

15. Plot four points, (x, y), where $y = x - 3$. Use the grid in Figure 3.3.
16. One number is 3 more than twice another number. If the sum of the two numbers is –33, what are the two numbers?

17. Circle the true statements.

$\left|-\frac{1}{2}\right| = -\left|\frac{1}{2}\right|$

$16 \div (-2) = (-16) \div 2$

every value of y is a solution of the equation
$4y = 8(y - 2)$

$2^3 \cdot 5^2 = 5^2 \cdot 2^3$

$\frac{6}{9} = \frac{2}{3}$

18. Write the coordinates of the point on the grid in Figure 3.1 that:

a. has an x-value more than –2.

b. has a y-value twice its x-value.

c. has a y-value less than 1.

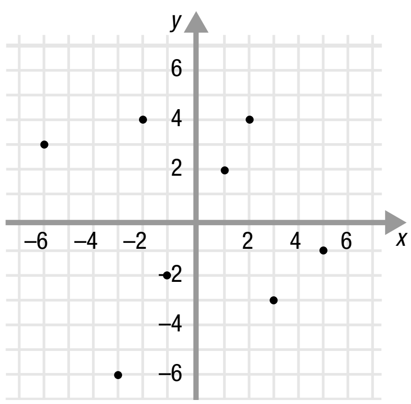


Figure 3.1

19. Find: $\frac{7}{12} - \frac{3}{8}$

20. Find the rise and the run from $T_1(-29, -31)$ to $T_2(14, 26)$ by subtracting the appropriate coordinates.

21. Solve for x: $-5 \le 8 - 3x < 2$

22. Solve for z: $\frac{2}{5}(4z - 1) = \frac{1}{10}(8z + 20)$

23. Plot the point $P(3, -5)$.

24. Plot the point $Q(-4, -6.5)$.

25. Plot the point $R(0, 6)$.

26. Shade in Quadrant I.

27. Solve for x: $7x + 3 = 38$

28. Use the fact that R = {1, 2, 3, 4, 5} and S = {2, 4, 6, 8, 10} to determine if each statement below is true.

a. $2 \in R$

b. $2 \in S$

c. $3 \in R$

d. $3 \in S$

29. Write the equation of the circle with radius 4 whose center is at (2, 3).

30. Use the Pythagorean Theorem to find the distance between the points (–1, –4) and (4, 8).

31. Find: $\frac{18}{25} \div \frac{6}{5}$

32. Circle the true statements.

a. $z = 3$ is a solution of the inequality $2z - 5 < 3$

b. $|0| > |-5|$

c. $0 > -5$

d. $7^4 = 7 \cdot 7 \cdot 7 \cdot 7$

e. $\frac{5}{10} = \frac{1}{2} = \frac{4}{8}$

33. Solve for y: $-3(y + 2) = 6(4 - \frac{1}{2}y)$

34. Use the distance formula to find the distance between the points (–6, 3) and (9, –5).

35. Find the radius and the center of the circle whose equation is $(x + 5)^2 + (y - 1)^2 = 49$.

36. If $R = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $S = \{2, 4, 6, 8, 10\}$, and $T = \{1, 3, 5, 7, 9\}$, then which of the statements below are true?

a. $S \subset R$

b. $R \subset S$

c. $T \subset R$

d. $R \subset T$

e. $S \subset T$

f. $T \subset S$