LESSON 2.4 – LINEAR INEQUALITIES

OVERVIEW

Here's what you'll learn in this lesson:

Solving Inequalities

a. Recognizing solutions of linear inequalities

b. Graphing solutions of inequalities in one variable

c. The addition and subtraction principles for solving a linear inequality

d. The multiplication and division principles for solving a linear inequality

e. Combining the addition, subtraction, multiplication, and division principles

f. Solving problems using inequalities

Most of the problems you have solved have only had one solution. However, some problems have more than one solution. For example, if you know your business expenses, you can find what the value of your sales must be in order for you to make a profit. Another example is, to get an A in Dr. Gold's course you have to have an average greater than or equal to 94% on all of your tests.

In this lesson, you will learn to solve linear inequalities—algebraic statements that have more than one solution.

EXPLAIN

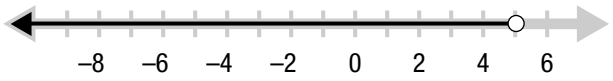
SOLVING INEQUALITIES

Summary

Using the Number Line

When you compare algebraic expressions, you use signs such as $=$, $<$, $>$, $\leq$, or $\geq$. Unlike linear equations which typically have only one solution, linear inequalities can have an infinite number of solutions.

For example, the linear inequality $x < 5$ has infinitely many values of x which make it true: $x = 4, 0, -\frac{2}{3}, \ldots$. Although you can't list all of the solutions, you can represent them on the number line:



The open circle at the point $x = 5$ shows that $x = 5$ is not a solution of the inequality $x < 5$.

Another linear inequality is $x \geq -4$. The solutions of this inequality can also be graphed on the number line:

The closed circle at the point $x = -4$ indicates that $x = -4$ is a solution to the inequality $x \geq -4$.

Two inequalities can sometimes be combined to form a compound inequality.

For example, the two inequalities $-4 \leq x$ and $x < 5$ can be combined to form the compound inequality $-4 \leq x < 5$.

Once again the solution can be graphed on the number line:

Solving Inequalities

Solving a linear inequality is similar to solving a linear equation. You can perform the same operations (addition, subtraction, multiplication, division) on both sides of the inequality. However, remember that **when you multiply or divide an inequality by a negative number, the direction of the inequality is reversed.** Here are the operations you can perform to solve an inequality:

Operation	Numeric Example	Algebraic Example
Add the same number to both sides.	$4 < 12$ $4 + 5 < 12 + 5$ $9 < 17$	$x - 2 < 3$ $x - 2 + 2 < 3 + 2$ $x < 5$
Subtract the same number from both sides.	$4 < 12$ $4 - 1 < 12 - 1$ $3 < 11$	$x + 3 < 5$ $x + 3 - 3 < 5 - 3$ $x < 2$
Multiply both sides by the same **positive** number.	$4 < 12$ $2 \cdot 4 < 2 \cdot 12$ $8 < 24$	$\frac{1}{3}x < 5$ $3 \cdot \frac{1}{3}x < 3 \cdot 5$ $x < 15$
Divide both sides by the same **positive** number.	$4 < 12$ $\frac{4}{2} < \frac{12}{2}$ $2 < 6$	$6x < 18$ $\frac{6x}{6} < \frac{18}{6}$ $x < 3$
Multiply both sides by the same **negative** number.	$4 < 12$ $-2 \cdot 4 > -2 \cdot 12$ $-8 > -24$	$-\frac{1}{2}x < 4$ $(-2) \cdot \left(-\frac{1}{2}x\right) > (-2) \cdot 4$ $x > -8$
Divide both sides by the same **negative** number.	$4 < 12$ $\frac{4}{-2} > \frac{12}{-2}$ $-2 > -6$	$-3x < 9$ $\frac{-3x}{-3} > \frac{9}{-3}$ $x > -3$

Solving Compound Inequalities

To solve a compound inequality, you can either split it into its component inequalities and solve each separately, or you can perform the same operations on all parts of the compound inequality.

For example, the compound inequality $-4 \le 3x - 1 < 5$:

		$-4 \le 3x - 1$	and	$3x - 1 < 5$
1.	Add 1.	$-4 + 1 \le 3x - 1 + 1$	and	$3x - 1 + 1 < 5 + 1$
2.	Simplify.	$-3 \le 3x$	and	$3x < 6$
3.	Divide by 3.	$-\frac{3}{3} \le \frac{3x}{3}$	and	$\frac{3x}{3} < \frac{6}{3}$
4.	Simplify.	$-1 \le x$	and	$x < 2$

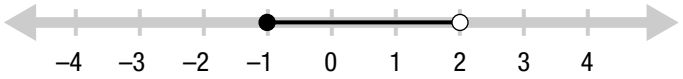
You can see that $-1 \le x$ and $x < 2$. Combining these two inequalities you get $-1 \le x < 2$.

Now solve the same compound inequality, $-4 \le 3x - 1 < 5$, but this time do not break it into two inequalities. Instead, perform the same operations on each part of the inequality.

$$-4 \le 3x - 1 < 5$$

1. Add 1: $-4 + 1 \le 3x - 1 + 1 < 5 + 1$
2. Simplify: $-3 \le 3x < 6$
3. Divide by 3: $-\frac{3}{3} \le \frac{3x}{3} < \frac{6}{3}$
4. Simplify: $-1 \le x < 2$

The solutions of this inequality can be graphed on the number line:



Sample Problems

1. Solve for x: $x + 3 \le 8$.

$$x + 3 \le 8$$

☑ a. Subtract 3. $x + 3 - 3 \le 8 - 3$

☐ b. Simplify. $x \le$ ___

2. Solve for y: $3y - 2 > -5$.

$$3y - 2 > -5$$

☐ a. Add 2. $3y - 2 +$ ___ $> -5 +$ ___

☐ b. Simplify. $3y >$ ___

☐ c. Divide by 3. $\frac{3y}{\text{___}} > \frac{\text{___}}{\text{___}}$

☐ d. Simplify. $y >$ ___

3. Solve for x: $-9 < 5 - 2x \le 6$.

$$-9 < 5 - 2x \le 6$$

☑ a. Subtract 5. $-9 - 5 < 5 - 2x - 5 \le 6 - 5$

☐ b. Simplify. -14 ___ $-2x$ ___ 1

☐ c. Divide by –2. $\frac{-14}{-2}$ ___ x ___ $\frac{1}{-2}$

☐ d. Simplify. 7 ___ x ___ $-\frac{1}{2}$

Answers to Sample Problems

b. 5

a. 2, 2

b. –3

c. $\frac{3y}{3}, \frac{-3}{3}$

d. –1

b. $<, \le$

c. $>, \ge$

d. $>, \ge$

EXPLORE

Sample Problems

On the computer you used the Solver to analyze and solve linear inequalities. Below are some additional exploration problems.

1. Graph the solution of the compound inequality $-4 < 2x - 6 < 8$.

$$-4 < 2x - 6 < 8$$

☑ a. Add 6. $-4 + 6 < 2x - 6 + 6 < 8 + 6$

☐ b. Simplify. _____ < _____ < _____

☐ c. Divide by 2. $\frac{\quad}{\quad} < \frac{\quad}{\quad} < \frac{\quad}{\quad}$

☐ d. Simplify. ___ < x < ___

☐ e. Graph the solution.

2. Graph the solution of the compound inequality $7 \le 5 - \frac{1}{2}x < 12$.

$$7 \le 5 - \frac{1}{2}x < 12$$

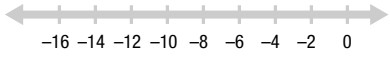
☑ a. Subtract 5. $7 - 5 \le 5 - \frac{1}{2}x - 5 < 12 - 5$

☐ b. Simplify. 2 ___ $-\frac{1}{2}x$ ___ 7

☐ c. Multiply by –2. $(-2) \cdot 2$ ___ $(-2) \cdot (-\frac{1}{2}x)$ ___ $(-2) \cdot 7$

☐ d. Simplify. -4 ___ x ___ -14

☐ e. Graph the solution.



Answers to Sample Problems

b. *2, 2x, 14*

c. $\frac{2}{2}, \frac{2x}{2}, \frac{14}{2}$

d. *1, 7*

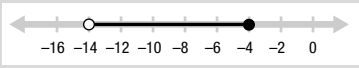
e.

b. ≤, <

c. ≥, >

d. ≥, >

e.



HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain
Solving Inequalities

1. Solve for x: $x - 7 < 2$

2. Solve for x: $15 < 5x$

3. Solve for x: $-3 \leq x + 1 \leq 5$

4. Solve for x: $18 < 2x + 4$

5. Solve for x: $6x < -18$

6. Solve for x: $-5 \leq 2x - 3 < -2$

7. Solve for x: $22 < 6 - 4x$

8. Solve for x: $4 - x < x + 2$

9. Mohammad took $40 out of his savings account to go shopping for a birthday present. He needs $3.00 for parking and $12.50 for gas. Write an inequality to represent the amount he can spend on the present and still have enough money to pay for parking and gas.

10. Donna's new car gets 22 miles per gallon (mpg) in the city and 34 mpg on the highway. Write a compound inequality which represents the number of miles she can drive on 14 gallons of gas.

11. Solve for x: $\frac{3x + 1}{2} - 5 < -1$

12. Solve for x: $\frac{8}{5} < 2 - x < 6$

Explore

13. Graph the solutions of each inequality: $x - 2 \leq 5$ and $x - 2 < 5$. Explain how the solutions of the inequalities differ.

14. Graph the solutions of the compound inequality $-2 < 3x + 7 \leq 10$.

15. Graph the solutions of the compound inequality $1 < -\frac{2}{5}x + 3 < 5$.

16. Graph the solution of each inequality: $4 - 3x \leq -5$ and $4 - 3x < -5$. Explain how the solutions of the inequalities differ.

17. Graph the solutions of the compound inequality $-14 < 2 - 4x < 0$.

18. Graph the solutions of the compound inequality $\frac{1}{2} \geq \frac{2}{3}x - 2 > -\frac{4}{7}$.

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Solving Inequalities

1. Solve for x: $x + 6 \leq 10$
2. Solve for y: $y + 7 \geq 9$
3. Solve for a: $a - 3 > 9$
4. Solve for w: $w - 6 \leq 3$
5. Solve for b: $3b < 18$
6. Solve for a: $4a \leq 36$
7. Solve for c: $5c \geq -25$
8. Solve for m: $-2m \leq 24$
9. Solve for d: $-4d > 5$
10. Solve for k: $-3k < -9$
11. Solve for x: $3x + 7 < 13$
12. Solve for y: $4y + 7 \geq 15$
13. Solve for z: $8z + 15 > 39$
14. Solve for m: $6m - 8 > -32$
15. Solve for a: $5a - 7 < -8$
16. Solve for h: $7h - 12 \leq 37$
17. Solve for x: $9 - x < 1$
18. Solve for x: $7 - x > 2$
19. Solve for p: $18 - p \geq 20$
20. Solve for y: $6 - 3y \geq 9$
21. Solve for z: $5 - 4z < 37$
22. Solve for y: $9 - 6y \leq -45$
23. Solve for y: $-6 \leq y + 5 < 13$
24. Solve for y: $-4 < y - 2 \leq 10$
25. Solve for z: $-15 < z - 14 < 25$
26. Solve for z: $16 \leq 7 - 2z < 23$
27. Solve for x: $15 \leq 8 - 3x \leq 20$
28. Solve for k: $-15 < 8 - 4k \leq -8$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Solve for x: $x - 3 < 4$

2. Solve for z: $3z - 7 \leq 5$

3. Solve for x: $7x + 2 < 6x + 5$

4. Solve for y: $9y + 11 > 8y - 3$

5. Solve for x: $9 - 4x \geq -19$

6. Solve for x: $\frac{1}{2}x + 4 \geq x$

7. Solve for z: $10 < 2z + 10 < 20$

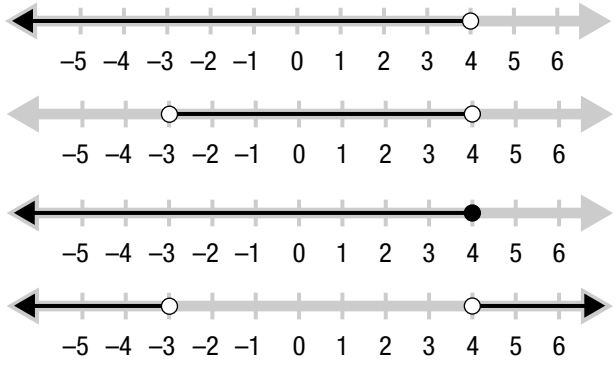
8. At her job, Sonal can choose to work a different number of hours each day, but she must average at least 8 hours per day. This week she worked 10 hours on Monday, 6 hours on Tuesday, 7 hours on Wednesday, and 8 hours on Thursday. How many hours must she work on Friday to maintain or exceed her 8 hour average?

TOPIC 2 CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

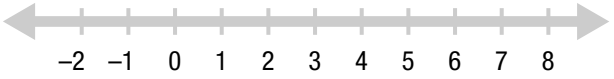
These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic, or you may wish to do these problems to review for a test.

1. Simplify the expression $2x^2y - 5y + 6x^2y + 4x - 3y$.
2. Solve for y: $2y + 5 = 4(\frac{1}{2}y + 3)$
3. Solve for x: $-4 < 4x + 3 < 7$
4. Write using exponents: $3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 17 \cdot 17 \cdot 17$
5. Suppose you have two numbers and the second number is 2 less than 3 times the first. If the sum of the two numbers is 34, what are the numbers?
6. Solve for x: $3(x + 2) = 12$
7. Solve for y: $-1 \leq 6y - 4 < 12$
8. Simplify: $4 \cdot 3^2[7 - (3 + 4)] - 6$
9. Simplify the expression $2(x^2y^2 - 3x) + 4xy - 3(7x + x^2y^2) - 2$.
10. Circle the graph that represents the inequality $x - 7 < -3$.



11. Find: $\frac{5}{9} \div \frac{10}{3}$
12. Simplify the expression $7xy^3 - 4xy^2 - 5x + xy^3 + 3x - 2xy^2$.
13. Solve for y: $5y - 2 \geq 23$, then graph its solution on the number line below.

14. One number is 8 less than 5 times another. If the sum of the two numbers is –2, what are the numbers?
15. Solve for z: $z + 5 = 8$
16. Reduce $\frac{54}{36}$ to lowest terms.
17. Evaluate the expression $xy^2 + 2xy - 3 + 5y$ when $x = 2$ and $y = -3$.
18. Find the GCF of 76 and 57.
19. Bjorn is 3 years older than Ivar was 5 years ago. If the sum of their ages now is 66, how old is each person?
20. Solve for y: $3 < 7 - 2y \leq 6$, then graph its solution on the number line below.



21. The formula to find the area of a circle is $A = \pi r^2$, where A is the area and r is the radius. Solve the formula $A = \pi r^2$ for r.
22. Given the expression $4x^3y - 3 + 2y^2 - 7x + 12$,
    a. what are the terms?
    b. what are the variables?
    c. what are the coefficients?
    d. what are the constants?
23. The length of one side of a square is decreased by 2 meters and the length of an adjacent side is increased by 1 meter. In the resulting rectangle, the length is twice the width. How long was a side of the original square?

24. Evaluate the expression $5x - 3x^2y + 4 - 2y$ when $x = -3$ and $y = 1$.

25. Solve for x: $2x + 1 = -5 + 2(x + 3)$

26. Solve for z: $-8 \leq 3z + 10 \leq 16$, then graph its solution on the number line below.

27. Find: $\frac{2}{3} + \frac{3}{4}$

28. Find the LCM of 16 and 42.

29. Solve for y: $\frac{1}{3}(3 - y) = \frac{5}{6}(3 + y)$

30. Solve for x: $\frac{1}{4}x + 5 = \frac{1}{2}(x - 2)$