LESSON F3.1 -RATIO AND PROPORTION







In Concept 1: Ratios, you will find a section on each of the following:

- How to Use a Ratio to Compare Two Quantities
- The Definition of Equivalent Ratios
- How to Use a Ratio to Represent a Rate

CONCEPT 1: RATIOS

How to Use a Ratio to Compare Two Quantities

A ratio is a way to compare two quantities using division.

For example, a salsa recipe calls for 3 onions and 5 peppers. The ratio of 3 onions to 5 peppers is 3 to 5. This ratio can also be written by using a colon, 3:5, or by using a fraction, $\frac{3}{5}$.

In general, the ratio of the number **a** to the number **b** can be written in the following ways:

using words using a colon using a fraction **a** to **b a** : **b** $\frac{\mathbf{a}}{\mathbf{b}}$

Since division by zero is undefined, the second number, b, cannot be 0.

Most of the time a ratio is written as a fraction. Since a fraction represents division, this is a reminder that a ratio compares two numbers using division.

Remember, the fraction $\frac{3}{4}$ means $3 \div 4$.

cups of tomatoes to cups of cilantro.

Sometimes a ratio may compare more than two numbers. You usually use colons to represent these ratios.

For example, suppose there are 5 apples, 6 oranges, and 2 bananas in a bag. The ratio of apples to oranges to bananas is 5:6:2.

In her salsa, Maria uses 2 cups of tomatoes for each cup of cilantro. Find the ratio of

Since there are 2 cups of tomatoes for 1 cup of cilantro, the ratio of cups of tomatoes

Example 1

Example

2

1.

2.

The ratio $\frac{2}{1}$ could be written as 2.

You may find these

for this section.

Examples useful while doing the homework

> But here, the 1 is left in the denominator of the ratio to represent the number of cups of cilantro, the second quantity in the ratio.

 $\frac{number of cups of tomatoes}{number of cups of cilantro} = \frac{2}{1}$

The ratio is $\frac{2}{1}$.

to cups of cilantro is:

What is the ratio of 2.5 cups of flour to 1.5 cups of sugar?

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Since there are 2.5 cups of flour for 1.5 cups of sugar, the ratio of number of cups of flour to number of cups of sugar is:

 $\frac{number of cups of flour}{number of cups of sugar} = \frac{2.5}{1.5}$

Notice that the ratio contains decimal numbers. To clear the decimals:

• Multiply the ratio by 1, written as $\frac{10}{10}$. • Simplify the ratio. $\frac{2.5}{1.5} \times \frac{10}{10} = \frac{25}{15}$ $= \frac{5}{3}$

3.	What is the ratio of 2 cars to 1 boat to 4 bicycles?		Example 3
	Since more than 2 quantities are being compared,		
	use colons to write the ratio.		
	The ratio of cars to boats to bicycles is 2 : 1 : 4.		
 4.	There are 20 students in a class. 12 students are bo	bys and 8 students are girls.	Example 4
	Write the ratio of the number of girls in the class t	o the total number of students.	L. J.
	The ratio of the number of girls in the class		
	to the total number of students is:	the number of girls the total number of students	
	There are 8 girls in the class, and there are		
	20 students in the class.		
	So the ratio of the number of girls to the total		
	number of students is:	$\frac{8}{20}$	
		20	

The Definition of Equivalent Ratios

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A salsa recipe calls for 3 onions and 5 peppers per batch. The table below compares the number of onions to the number of peppers for different size batches.

SALSA	1 Batch	2 Batches	$\frac{1}{2}$ Batch
Number of Onions	3	6	1.5
Number of Peppers	5	10	2.5
Ratio of Onions to Peppers	$\frac{3}{5}$	$\frac{6}{10}$	$\frac{1.5}{2.5}$

The ratios $\frac{6}{10}$ and $\frac{1.5}{2.5}$ can be simplified to $\frac{3}{5}$: $\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$

 $\frac{1.5}{2.5} = \frac{1.5 \times 2}{2.5 \times 2} = \frac{3}{5}$

So, the fractions $\frac{3}{5}$, $\frac{6}{10}$, and $\frac{1.5}{2.5}$ are equivalent fractions.

They are also called equivalent ratios.

No matter the size of the batch, the ratio of onions to peppers is always 3 to 5.

In general, to find a ratio equivalent to a given ratio:

• Multiply or divide the numerator and denominator of the given ratio by the same number.

Ratios can be used to compare quantities. For example, ratios can be used to compare amounts of money, quantities of time, lengths, or weights. In order to use a ratio to do such comparisons, the quantities being compared need to have the same units of measurement.

For example, to write a ratio comparing 2 dollars to 1 cent, you must first write both quantities using the same unit.

Since 1 dollar = 100 cents, 2 dollars = 200 cents.

So, the ratio of 2 dollars to 1 cent becomes the ratio $\frac{200 \text{ cents}}{1 \text{ cent}}$ or $\frac{200}{1}$.

In order to write this ratio, 2 dollars was changed to 200 cents. But it would also have been correct to change cents into dollars. Here's how:

You can write 1 cent as part of a dollar, like this: \$0.01.

Now you are working with decimals instead of whole numbers.

The ratio of 2 dollars to 1 cent becomes the ratio $\frac{2 \text{ dollars}}{0.01 \text{ dollars}}$ or $\frac{2}{0.01}$.

To get rid of the decimal, multiply the numerator and denominator by 100:

$$\frac{2}{0.01} = \frac{2 \times 100}{0.01 \times 100} = \frac{200}{1}$$

So, the ratio of 2 dollars to 1 cent is still 200 to 1.

In general, if you'd like to work with a ratio of whole numbers instead of decimals, write both quantities using the "smaller" unit. (For example, use cents instead of dollars, minutes instead of hours, inches instead of feet, etc.)

You may find these Examples useful while doing the homework for this section.	Example 5	5.	A recipe calls for 3 onions for every 5 peppers. If you want to n 18 onions, how many peppers do you need?	nake the receipe using
			To find how many peppers you need:	
			• Write the ratio of onions to peppers.	$\frac{3}{5}$
			• Find an equivalent fraction with 18 as the numerator.	$\frac{3}{5} = \frac{18}{?}$
			Multiply the numerator and denominator by 6.	$\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$
			• The denominator of the equivalent fraction is the number of peppers you need.	30

So, you need 30 peppers.



	How to Use a Ratio to I	Represent a Rate				
	Often a ratio is used to compare quantiti sometimes called rates . Here are two ex	es that have very different units. These ratios are amples.				
	• Suppose you travel 60 miles for each h distance, 60 miles, to the time, 1 hour.	nour you drive. You can use a ratio to compare the				
The word per tells you that you use	The ratio of 60 miles to 1 hour is $\frac{60 \text{ m}}{1 \text{ ho}}$	iles our . This rate is usually read 60 miles per hour.				
<i>division</i> to compare miles to hours.	• Suppose you buy a 5 pound bag of app cost, \$3.00, to the weight, 5 pounds.	• Suppose you buy a 5 pound bag of apples for \$3.00. You can use a ratio to compare your cost, \$3.00, to the weight, 5 pounds.				
	The ratio of 3 dollars to 5 pounds is $\frac{1}{5}$	$\frac{\$3.00}{\text{pounds}} = \frac{\$3.00 \div 5}{5 \text{ pounds} \div 5} = \frac{\$0.60}{1 \text{ pound}}.$				
	By writing the ratio with denominator That is, the apples cost \$0.60 per pour	1, you see that the apples cost \$0.60 for 1 pound.				
Notice, in each example, the rate was	These examples suggest a way to use a r	ratio to find a rate. Here's how:				
written as a ratio with denominator 1.	• Write the ratio.					
	• Find an equivalent ratio with 1 in the d	lenominator.				
	• Read the rate.					
You may find these Examples useful while	9. Suppose daisies cost \$3 per dozen. What is the cost per daisy?					
for this section.	Here's one way to find the cost per	Here's one way to find the cost per daisy:				
	• Write the ratio.	$\frac{\text{the cost of the daisies}}{\text{the number of daisies}} = \frac{\$3.00}{12 \text{ daisies}}$				
	• Find an equivalent ratio with 1 in					
	the denominator.	$=\frac{\$3.00\div12}{12\ daining\ 12}$				
		12 daistes -12				
		$=\frac{\$0.25}{1 \text{ daisy}}$				
	• Read the rate.	\$0.25 per daisy				
	So, the cost is \$0.25 per daisy.					
Example 1						
Evanihie i	10. Suppose you earn \$254 for working	g 40 hours. Find your pay rate in dollars per hour.				
	Here's one way to find the pay rate	:				
	• Write the ratio.	$\frac{\text{the pay for 40 hours}}{40 \text{ hours}} = \frac{\$254}{40 \text{ hours}}$				
	• Find an equivalent ratio with 1 in					
	the denominator.	$=\frac{\$254 \div 40}{40 \text{ hours} \div 40}$				
		$=\frac{\$6.35}{1 \text{ hour}}$				
	• Read the rate.	\$6.35 per hour				
	So, the pay rate is \$6.35 per hour.					



In Concept 2: Proportions, you will find a section on each of the following:

- How to Solve a Proportion
- How to Set Up a Proportion
- How to Set Up and Solve a Proportion with Similar Triangles

Here's another way to write the proportion $\frac{4}{18} = \frac{2}{9}$.

- Use colons to write each ratio. 4 : 18 = 2 : 9
- Replace the equals sign with two colons. 4:18::2:9

CONCEPT 2: PROPORTIONS

How to Solve a Proportion

A proportion is a statement that shows one ratio equal to another ratio.

For example, a menu is planned so there are 2 grams of fat for every 9 grams of carbohydrates. That is, the ratio of grams of fat to grams of carbohydrates is $\frac{2}{9}$.

If the entire meal ends up having 4 grams of fat, then it has 18 grams of carbohydrates. Here, the ratio of grams of fat to grams of carbohydrates is $\frac{4}{18}$.

The ratios $\frac{4}{18}$ and $\frac{2}{9}$ are equivalent fractions. $\frac{4}{18} = \frac{2}{9}$

The equation $\frac{4}{18} = \frac{2}{9}$ is an example of a proportion.

A proportion is made up of four numbers. If you know three of the numbers in the proportion then you can find the fourth number.

Here's one way to find a missing value in a proportion if the other three values are known:

- Find the cross products of the proportion and set them equal to each other. (Two ratios are equal if their cross products are equal.)
- Solve the resulting equation for the missing value. That is, get the missing value by itself on one side of the equation.

 $\frac{2}{9} = \frac{x}{11}$ You may find these Example 11 11. Find the missing number, x, that makes this proportion true. Examples useful while doing the homework *Here's one way to find the missing number, x, that makes this proportion true:* for this section. • Find the cross products and $\frac{2}{9} \frac{x}{11}$ $x \cdot 9 = 2 \cdot 11$ set them equal to each other. 9x = 22• Solve the equation for x. $\frac{9x}{9} = \frac{22}{9}$ To get x by itself, divide both sides of the equation by 9. $x = \frac{22}{0}$ A dot, \cdot , is used to represent $x = 2\frac{4}{2}$ multiplication. Rewrite as a mixed numeral. *So*, $x = 2\frac{4}{9}$.

Example 12	12. Find the	e missing number, <i>x</i> , that makes this proportion true.	$\frac{3}{x} = \frac{5}{14}$
	Here is	one way to find the missing number, x, that makes thi	s proportion true:
	• Find t	he cross products and	$\frac{3}{x}$ $\frac{5}{14}$
	set the	em equal to each other.	$5 \cdot x = 3 \cdot 14$
	• Solve	the equation for x.	5x = 42
	To get the eq	t x by itself, divide both sides of uation by 5.	$\frac{5x}{5} = \frac{42}{5}$ $x = \frac{42}{5}$
	Rewrit	te as a mixed numeral.	$x = 8\frac{2}{5}$
	So, x =	$8\frac{2}{5}$.	
Example 13	13. Find the	e missing number, x, that makes this proportion true.	$\frac{6.4}{10} = \frac{16}{x}$
	Here's d	one way to find the missing number, x, that makes this	s proportion true:
	• Find t	he cross products and	$\frac{6.4}{10}$ $\frac{16}{x}$
	set the	em equal to each other.	$16 \cdot 10 = 6.4 \cdot x$
	• Solve	the equation for x.	160 = 6.4x
	To get	x by itself, divide both sides of the equation by 6.4.	$\frac{160}{6.4} = \frac{6.4x}{6.4} \\ \frac{160}{6.4} = x$
	Simpli	ify.	25 = x
	So, $x = x$	25.	
Example 14	14. Find the	e missing number, <i>x</i> , that makes this proportion true.	$\frac{\frac{2}{7}}{\frac{10}{10}} = \frac{x}{\frac{15}{15}}$
	Here's a	one way to find the missing number, x, that makes this	s proportion true:
	• Find t	he cross products and	$\frac{2}{7}$ x
	set the	em equal to each other.	$10 15$ $x \cdot 10 = \frac{2}{2} \cdot 15$
			$x \cdot 10 = \frac{2}{7} \cdot \frac{15}{1}$
	• Solve	the equation for x.	$10x = \frac{30}{7}$
	To get equati	<i>x</i> by itself, multiply both sides of the for by $\frac{1}{10}$.	$\frac{1}{10} \cdot 10x = \frac{1}{10} \cdot \frac{30}{7}$
			$x = \frac{30}{70}$
	Simplify). <u>3</u>	$x = \frac{3}{7}$
	50, x =	7.	

How to Set Up a Proportion

Here's an example of an application that can be solved using a proportion.

The nutritional label on a certain can of soda says there are 150 calories in 12 ounces of the soda. How many calories are there in 7 ounces of the soda?

This proportion can be used to answer the question: $\frac{12 \text{ ounces}}{150 \text{ calories}} = \frac{7 \text{ ounces}}{x \text{ calories}}$

Here are some other ways to set up a proportion to answer the question:

12 ounces	_ 150 calories	7 ounces	x calories
7 ounces	x calories	12 ounces	150 calories

Here's the result of cross multiplying each of these proportions:

$150 \cdot 7 = 12 \cdot x$	$x \cdot 12 = 7 \cdot 150$
1050 = 12x	12x = 1050

And here's the result of cross multiplying the first proportion:

$$\frac{12 \text{ ounces}}{150 \text{ calories}} = \frac{7 \text{ ounces}}{x \text{ calories}}$$
$$7 \cdot 150 = 12 \cdot x$$
$$1050 = 12x$$

In each case you get the same result. So, you can use any of the proportions to answer the question.

In general, given a proportion with one missing number, say x, and another proportion also involving x, you can determine if the two proportions give the same value for x. Here's how:

- In the given proportion, cross multiply.
- In the other proportion, cross multiply.
- Compare the equations.



$$9x = 22$$

• In the other proportions, cross multiply.

$\frac{2}{x} = \frac{9}{11}$	$\frac{11}{x} = \frac{9}{2}$	$\frac{2}{11} = \frac{9}{x}$
$9 \cdot x = 2 \cdot 11$	$9 \cdot x = 11 \cdot 2$	$9 \cdot 11 = 2 \cdot x$
9x = 22	9x = 22	99 = 2x

• Compare the equations.

The given proportion and the first two choices each give the same equation. But the equation 99 = 2x is different from the other equations.

So $\frac{2}{11} = \frac{9}{x}$ will not give the same value for x as the given proportion.

Example 16	16. Which proportion below w	ill not give the same value f	For x as the proportion $\frac{10}{7} = \frac{3}{x}$?
	$\frac{7}{x} = \frac{10}{3}$	$\frac{3}{10} = \frac{7}{x}$	$\frac{x}{7} = \frac{3}{10}$
	To determine which proposed same value for x as $\frac{10}{7} = 10$	rtion will not give the $\frac{3}{x}$:	
	• In the given proportion, a	cross multiply.	$\frac{10}{7} = \frac{3}{x}$ $3 \cdot 7 = 10 \cdot x$
			21 = 10x
	• In the other proportions,	cross multiply.	
	$\frac{7}{x} = \frac{10}{3}$	$\frac{3}{10} = \frac{7}{x}$	$\frac{x}{7} = \frac{3}{10}$
	$10 \cdot x = 7 \cdot 3$	$7 \cdot 10 = 3 \cdot x$	$3 \cdot 7 = x \cdot 10$
	10x = 21	70 = 3x	21 = 10x
	• Compare the equations.		
	The given proportion and But the equation $70 = 3x$	d the first and third choices of is different from the other e	each give the same equation. quations.
	So the proportion $\frac{3}{10} = \frac{7}{x}$ as the proportion $\frac{10}{7} = \frac{3}{x}$	will not give the same valu	e for x
Example 17	17. Suppose there are 116 calc12.5 ounces of juice.	ories in 8 ounces of juice. Fin	nd the number of calories, <i>x</i> , in
	One way to find the numbe	er of calories in 12.5 ounces	of juice is to use a proportion:
	• Write the ratio of 8 ounce	es of juice to 116 calories.	
	Write the ratio of 12.5 or	unces of juice to x calories.	0 10 5
	Set the two ratios equal t	o each other.	$\frac{8 \text{ ounces}}{116 \text{ calories}} = \frac{12.5 \text{ ounces}}{x \text{ calories}}$
	• Solve the proportion.		
	Cross multiply.		$12.5 \cdot 116 = 8 \cdot x$
			1450 = 8x
	Solve for x. Divide both s	sides of the equation by 8.	$\frac{1450}{8} = \frac{8x}{8}$
			181.25 = x

So, there are 181.25 calories in 12.5 ounces of juice.

Example 18

18. On a map, $\frac{3}{8}$ of an inch represents 100 yards. Find the actual distance, *x*, from point A to point B if the distance on the map between these two points is 3 inches.

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One way to find the distance from point A to point B is to use a proportion:

• Write the ratio of $\frac{3}{8}$ inches to 100 yards.Write the ratio of 3 inches to x yards.Set the two ratios equal to each other.• Solve the proportion.Cross multiply. $3 \cdot 100 = \frac{3}{8} \cdot x$ $300 = \frac{3}{8} x$ • Solve for x. Multiply both sides of the equation $\frac{8}{3} \cdot 300 = \frac{8}{3} \cdot \frac{3}{8}x$

800 = x

So, the actual distance from point A to point B is 800 yards.

How to Set Up and Solve a Proportion with Similar Triangles

Look at the triangles below.



These ratios compare the lengths of the corresponding sides of the triangles.

length of shortest side of small triangle	=	1 inch	=	$\frac{1}{2}$
length of shortest side of large triangle		3 inches		3
length of medium side of small triangle	=	1.5 inches	=	1
length of medium side of large triangle		4.5 inches		3
length of longest side of small triangle	_	2 inches	_	1
length of longest side of large triangle	-	6 inches	_	3

The lengths of the corresponding sides of the two triangles are in the same ratio.

Two such triangles are called similar triangles.

Given similar triangles, if you are missing the length of the side of one of the triangles, here's a way to find that length:

- Write a proportion comparing the lengths of corresponding sides.
- Solve the proportion for the missing length.



Here's how to find the proportion that will help find the length x:

Notice that the first proportion compares 2 to x, which are not lengths of corresponding sides. The same is true for the third proportion. The second proportion compares the lengths of corresponding sides. Here's how:

$$\frac{\text{length of longest side of large triangle}}{\text{length of longest side of small triangle}} = \frac{\text{length of medium side of large triangle}}{\text{length of medium side of small triangle}}$$
$$\frac{x}{3} = \frac{4}{2}$$

So, the second proportion will help you find the length x.



lengths of corresponding sides.	$\frac{10.5}{7} = \frac{x}{5}$
• Solve the proportion.	
Cross multiply.	$x \cdot 7 = 10.5 \cdot 5$
	7x = 52.5
Solve for x. Divide both	$\frac{7x}{52.5}$
sides of the equation by 7.	7 7
	$x = \frac{52.5}{7}$
	x = 7.5
So the loweth wig 75	

So, the length x is 7.5.

21. A man, 6 feet tall, is standing 24 feet from a street light. The length of his shadow produced by the street light is 4 feet. Find the height, *x*, of the street light.

Example 21



You can use similar triangles to find the height of the street light.

Here's how:

• Draw and label two similar triangles.



Notice that even though you aren't given the lengths of two of the sides, you can still find x.

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٠	Write	a

proportion	length of vertical side of large triangle		length of horizontal side of large triangle		
proportion	length of vertical side of small triangle		length of horizontal side of small triangle		
comparing the			20		
lengths of corresp	oonding sides.	$\frac{x}{6} =$	$\frac{28}{4}$		
• Solve the proport	ion.				
Cross multiply.		28 · 6 =	$x \cdot 4$		
		168 =	<i>4x</i>		
Solve for x. Divid of the equation by	e both sides v 4.	$\frac{168}{4} =$	$\frac{4x}{4}$		
		42 =	x		

So, the height of the street light is 42 feet.



This Explore contains two investigations.

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- Inverting a Ratio
- Similar Rectangles

You have been introduced to these investigations in the Explore module of this lesson on the computer. You can complete them using the information given here.

Investigation 1: Inverting a Ratio

The areas in which people live can be catagorized in two different ways: metropolitan and rural. A metropolitan area consists of towns and/or cities where business or industry provides a majority of the jobs for the residents of the area. A rural area may consist of some small towns, and agriculture provides most of the jobs for area residents.

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In a certain region, the ratio of undeveloped land to developed land is $\frac{1 \text{ acre}}{9 \text{ acres}}$ or $\frac{1}{9}$.

1. Write some possible numbers of acres of undeveloped and developed land that satisfy this ratio:

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\frac{\text{acres of undeveloped land}}{\text{acres of developed land}} = \frac{1}{9}
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- 2. Interpret the data in question 1. That is, describe the setting. Could the setting be a metropolitan area? Could it be a rural area?
- 3. What if the values were inverted? That is, consider this ratio:

 $\frac{\text{acres of undeveloped land}}{\text{acres of developed land}} = \frac{9}{1}$

What kind of setting would this ratio represent?

4. Examine some ratios in your own community in this same way. Some possible quantities to compare are the number of bicycles on a school campus to the number of cars on a school campus, the number of homes for sale in a neighborhood to the number of homes not for sale, etc.

Write your ratios.

- 5. Interpret the data. What do the ratios "say" about the situation?
- 6. Invert your ratios.
- 7. Interpret this "new" data. That is, what do the inverted ratios "say" about the situation?

Investigation 2: Similar Rectangles

- 1. A rectangular piece of land has a length of 36 meters and a width of 24 meters. Let the length of each side of a square on a piece of graph paper represent 1 meter. Draw a scale drawing of this piece of land. Label each side with the appropriate measurement.
- 2. The perimeter of a figure is the distance around the figure. For instance, if you wanted to put a fence around the piece of land in question 1, you would want to know its perimeter. Find the perimeter of the piece of land by adding the lengths of the four sides.

Perimeter = _____ meters

3. The area of a figure measures the space inside the boundary of the figure. For example, if you were going to cover the entire piece of land in the rectangle in question 1 with cement, you would want to know its area. One way to find area is to draw a grid on the figure and count the number of 1 by 1 squares it takes to precisely fill the figure. So, find the area of the rectangular piece of land by counting the number of 1 by 1 squares inside the rectangle you drew in question 1.

Area = _____ square meters

4. Now you will use graph paper to draw some other scale drawings of the piece of land described in question 1.

Each small square on the graph paper is 1 unit long by 1 unit wide. The area of each small square is 1 square unit.

If you let 1 unit represent 6 meters, then each square on the graph paper represents a square 6 meters long by 6 meters wide. Figure 1 shows a drawing of the piece of land using this scale. The first row in the chart below shows the length, width, and perimeter (in units), and area (in square units) of this scale drawing.

Now draw three more scale drawings and label them Drawing 2, Drawing 3, and Drawing 4. In Drawing 2, let 1 unit represent 4 meters. In Drawing 3, let 1 unit represent 2 meters. In Drawing 4, let 1 unit represent 1 meter. Use your drawings to fill in the rest of this chart.

Drawing Number	Length (in units)	Width (in units)	Perimeter (in units)	Area (in square units)
1	6	4	20	24
2				
3				
4				
Actual piece of land	36 meters	24 meters	120 meters	864 square meters



Figure 1

5. Find the ratio in lowest terms of length to width for each of the scale drawings and for the actual piece of land.

Drawing 1:	$\frac{\text{Length (in units)}}{\text{Width (in units)}} =$	$\frac{6}{4}$	=	$\frac{3}{2}$
Drawing 2:	$\frac{\text{Length (in units)}}{\text{Width (in units)}} =$		=	
Drawing 3:	$\frac{\text{Length (in units)}}{\text{Width (in units)}} =$		=	
Drawing 4:	$\frac{\text{Length (in units)}}{\text{Width (in units)}} =$		=	
Actual piece of land:	$\frac{\text{Length (in meters)}}{\text{Width (in meters)}} =$		=	

- 6. For a scale drawing where the length of one square represents 10 meters, what is the ratio in lowest terms of length (in units) to width (in units)?
- 7. Now, find the ratio in lowest terms of the perimeter of each scale drawing to the perimeter of the actual piece of land.

Perimeter of Drawing 1 (in units)	_	_20_	_	1
Perimeter of Actual Piece of Land (in meters)	_	120	_	6
Perimeter of Drawing 2 (in units)	=		=	
remineter of Actual Fiece of Land (in increas)				
Perimeter of Drawing 3 (in units)				
Perimeter of Actual Piece of Land (in meters)	=		=	
Perimeter of Drawing 4 (in units)	_		_	
Perimeter of Actual Piece of Land (in meters)	-		-	

8. For a scale drawing where the length of one square represents 10 meters, what is the ratio in lowest terms of the perimeter of the scale drawing (in units) to the perimeter of the actual piece of land (in meters)?

9. Now, find the ratio in lowest terms of the area of each scale drawing to the area of the actual piece of land.

Area of Drawing 1 (in square units) Area of Actual Piece of Land (in square meters)	=	$\frac{24}{864}$	=	$\frac{1}{36}$
Area of Drawing 2 (in square units) Area of Actual Piece of Land (in square meters)	=		=	
Area of Drawing 3 (in square units) Area of Actual Piece of Land (in square meters)	=		=	
Area of Drawing 4 (in square units) Area of Actual Piece of Land (in square meters)	=		=	

Observe that the ratio of the area of each scale drawing to the area of the actual piece of land is:

Perimeter of Scale Drawing (in units)	\sim	Perimeter of Scale Drawing (in units)
Perimeter of Actual Piece of Land (in meters)		Perimeter of Actual Piece of Land (in meters)

10. For a scale drawing where the length of one square represents 10 meters, what is the ratio in lowest terms of the area of the scale drawing (in square units) to the area of the actual piece of land (in square meters)?

Homework

CONCEPT 1: RATIOS

How to Use a Ratio to Compare Two Quantities

For help working these types of problems, go back to Examples 1-4 in the Explain section of this lesson.

- 1. Write a fraction that expresses the ratio of 2 onions to 7 peppers.
- 2. Write a fraction that expresses the ratio of 6 squash to 11 potatoes.
- 3. Write a fraction that expresses the ratio of 7 girls to 13 boys.
- 4. Write a fraction that expresses the ratio of 7 girls to 20 students.
- 5. Write a fraction that expresses the ratio of 45 new cars to 29 used cars.
- 6. Write a fraction that expresses the ratio of 16 bicycles to 9 tricycles.
- 7. Write a fraction that expresses the ratio of 15 dogs to 20 cats.
- 8. Write a fraction that expresses the ratio of 25 cows to 5 horses.
- 9. Write a fraction that expresses the ratio of 10 full time instructors to 27 part-time instructors.
- 10. Write a fraction that expresses the ratio of 12 inches to 2 inches.
- 11. Write a fraction that expresses the ratio of 15 ounces to 48 ounces.
- 12. Write a fraction that expresses the ratio of 5 cups to 12 cups.
- 13. Write a fraction that expresses the ratio of 2.5 quarts to 15 quarts.
- 14. Write a fraction that expresses the ratio of 44.8 ounces to 12 ounces.
- 15. Use colons (:) to write the ratio of 5 red marbles to 7 white marbles to 17 blue marbles.
- 16. Use colons (:) to write the ratio of 3 ducks to 5 chickens to 2 geese.
- 17. There are 13 oranges and 18 apples in a bowl. Write the ratio of the number of oranges to the number of apples.
- 18. There are 13 oranges and 18 apples in a bowl. Write the ratio of the number of apples to the number of oranges.
- 19. In a class of 20 students, 9 are girls and 11 are boys. Write the ratio of the number of girls to boys.
- 20. In a class of 20 students, 9 are girls and 11 are boys. Write the ratio of the number of boys to girls.
- 21. In a bag of 38 marbles, there are 10 blue marbles, 7 green marbles, 8 black marbles, and 13 red marbles. Write the ratio of the number of blue marbles to the number of red marbles.
- 22. In a bag of 38 marbles, there are 10 blue marbles, 7 green marbles, 8 black marbles, and 13 red marbles. Write the ratio of the number of green marbles to the number of black marbles.
- 23. A recipe calls for $2\frac{1}{4}$ cups of flour and $\frac{3}{4}$ cups of sugar. Write the ratio of the number of cups of flour to the number of cups of sugar.
- 24. A recipe calls for 4 cups of flour and $1\frac{3}{4}$ cups of sugar. Write the ratio of the number of cups of sugar to the number of cups of flour.

The Definition of Equivalent Ratios

For help working these types of problems, go to Examples 5-8 in the Explain section of this lesson.

- 25. Write a ratio to compare 21 inches to 3 feet. (Hint: 12 inches = 1 foot)
- 26. Write a ratio to compare 4 yards to 2 feet. (Hint: 3 feet = 1 yard)
- 27. Write a ratio to compare 1.5 quarts to 3 gallons. (Hint: 4 quarts = 1 gallon)
- 28. Write a ratio to compare 7 cups to 11 quarts. (Hint: 4 cups = 1 quart)
- 29. Write a ratio to compare 15 ounces to 3 pounds. (Hint: 16 ounces = 1 pound)
- 30. Write a ratio to compare 2.4 pounds to 9 ounces. (Hint: 16 ounces = 1 pound)
- 31. Write a ratio to compare $\frac{21}{2}$ yards to 5 feet. (Hint: 3 feet = 1 yard)
- 32. Write a ratio to compare $15\frac{3}{4}$ inches to 4 feet. (Hint: 12 inches = 1 foot)
- 33. Write a ratio to compare 36 minutes to 2 hours. (Hint: 60 minutes = 1 hour)
- 34. Write a ratio to compare 18 hours to 48 minutes. (Hint: 60 minutes = 1 hour)
- 35. Write a ratio to compare $\frac{3}{4}$ cup to 3 pints. (Hint: 2 cups = 1 pint)
- 36. Write a ratio to compare $1\frac{2}{3}$ cups to 2 quarts. (Hint: 4 cups = 1 quart)
- 37. Write a ratio to compare 24 seconds to 3 minutes. (Hint: 60 seconds = 1 minute)
- 38. Write a ratio to compare 2 hours to 50 seconds. Hint 3600 seconds = 1 hour)
- 39. Write a ratio to compare 3.75 miles to 1000 yards. (Hint: 1760 yards = 1 mile)
- 40. Write a ratio to compare 4.8 miles to 2400 feet. (Hint: 5280 feet = 1 mile)
- 41. A recipe calls for 3 onions and 7 potatoes. How many onions are needed if 21 potatoes are used?
- 42. A recipe calls for 2 stalks of celery and 5 carrots. How many carrots are needed if 8 stalks of celery are used?
- 43. Monia is filling gift bags for a child's party. In each bag, she wants to include 3 erasers for every 2 pencils. How many erasers does she need if she has 14 pencils?
- 44. Ken is packing boxes of fruit. In each box he packs, he wants to include 3 oranges for every 5 apples. If he has 45 oranges, how many apples does he need to complete his task?
- 45. A bowl contains a total of 28 apples and oranges. The ratio of the number of apples to the number of oranges is 3 to 4. How many apples and how many oranges are in the bowl?
- 46. A bowl contains a total of 14 oranges and bananas. The ratio of the number of oranges to the number of bananas is 2 to 5. How many oranges and how many bananas are in the bowl?
- 47. There are a total of 105 cows and horses in a pen. The ratio of the number of cows to the number of horses is 13 to 2. How many cows and how many horses are in the pen?
- 48. There are a total of 21 chickens and ducks in a pen. The ratio of the number of chickens to the number of ducks is 5 to 2. How many chickens and how many ducks are in the pen?

How to Use a Ratio to Represent a Rate

For help working these types of problems, go to Examples 9-10 in the Explain section of this lesson.

- 49. Ernie loses 18 pounds in 12 weeks. Find his weight loss per week.
- 50. Gladys loses 24 pounds in 6 months. Find her weight loss per month.
- 51. Rachel earns \$225 in 36 hours. Find her rate of pay in dollars per hour.
- 52. Simon earns \$238 in 40 hours. Find his rate of pay in dollars per hour.
- 53. A certain car uses 11 gallons of gas to travel 319 miles. Find the miles traveled per gallon.
- 54. A certain van uses 15 gallons of gas to travel 322.5 miles. Find the miles traveled per gallon.
- 55. A 1500 square foot house costs \$112,500 to build. Find the cost per square foot.
- 56. An 1875 square foot house costs \$178,125 to build. Find the cost per square foot.
- 57. Jody travels 460 miles in 8 hours. Find her rate in miles per hour.
- 58. Jaime travels 675 miles in 9 hours. Find his rate in miles per hour.
- 59. A fish swims 84 feet in 120 seconds. Find its rate in feet per second.
- 60. Jonlyn walks 2 miles in 25 minutes. Find her rate in miles per minute.
- 61. 5 pounds of bananas cost \$1.67. Find the price per pound of bananas.
- 62. 7 pounds of oranges cost \$3.43. Find the price per pound of oranges.
- 63. Kyle cleans the house where he lives. If he can clean 8 rooms in 4 hours, what is his cleaning rate in rooms per hour?
- 64. Kelly cleans stalls for a horse ranch. If she can clean 32 stalls in 8 hours, what is her cleaning rate in stalls per hour?
- 65. A dozen pens cost \$2.79. Find, to the nearest cent, the price per pen.
- 66. A dozen pencils cost \$0.50. Find, to the nearest cent, the price per pencil.
- 67. A box of paper contains 5 reams of paper. If the box costs \$12, what is the price per ream of paper?
- 68. A box of paper contains 5 reams of paper. If the box costs \$16, what is the price per ream of paper?
- 69. In a certain rain storm it rained 8 inches in 10 hours. What is the rate of rainfall in inches per hour?
- 70. In a certain snow storm it snowed 6 feet in 15 hours. What is the rate of snowfall in feet per hour?
- 71. There are 24 problems on a test. If it takes Elizabeth 30 minutes to finish the test, what is her rate in problems per minute?
- 72. There are 52 questions on a test. If it takes Ed 26 minutes to finish the test, what is his rate in questions per minute?

CONCEPT 2: PROPORTIONS

How to Solve a Proportion

For help working these types of problems, go to Examples 11–14 in the Explain section of this lesson.



- 89. On a map, 1 inch represents 5 miles. Find the actual distance, *x*, from point A to point B if these two points are 2.5 inches apart on the map. To answer the question, solve this proportion for *x*: $\frac{1}{5} = \frac{2.5}{x}$
- 90. On a map, 1 inch represents 8 miles. Find the actual distance, *x*, from point A to point B if these two points are 3.25 inches apart on the map. To answer the question, solve this proportion for *x*: $\frac{1}{8} = \frac{3.25}{x}$
- 91. Carl is placing cut-up turkey in freezer storage bags. For every 4 drumsticks he puts in a bag, he puts in 2 wings. If he has 36 drumsticks, how many wings does he have? To answer this question, solve this proportion for x: $\frac{4}{2} = \frac{36}{x}$
- 92. Jane is making a nut mix for a backpacking trip. For every 3 cups of peanuts in her mix, she includes $\frac{1}{2}$ cup of cashews. If she has 21 cups of peanuts, how many cups of cashews does she have? To answer this question, solve this proportion for x: $\frac{3}{\frac{1}{2}} = \frac{21}{x}$

- 93. Steven is on a diet that requires him to eat 3 grams of protein for every 4 grams of carbohydrates. If his lunch contains 24 grams of carbohydrates, how many grams of protein should he include to maintain the proper ratio? To answer this question, solve this proportion for *x*: $\frac{3}{4} = \frac{x}{24}$
- 94. Erica is on a diet that requires her to eat 2 grams of protein for every 5 grams of carbohydrates. If her breakfast contains 12 grams of protein, how many grams of carbohydrates should she include to maintain the proper ratio? To answer this question, solve this proportion for *x*: $\frac{2}{5} = \frac{12}{x}$
- 95. A soup recipe that feeds 4 people calls for 3 cups of broccoli. How many cups of broccoli will be needed to make enough soup to feed 9 people? To answer this question, solve this proportion for x: $\frac{4}{3} = \frac{9}{x}$
- 96. A soup recipe that feeds 6 people calls for 5 cups of potatoes. How many people can be fed with a soup that contains 15 cups of potatoes? To answer this question, solve this proportion for x: $\frac{6}{5} = \frac{x}{15}$

How to Set Up a Proportion

For help working these types of problems, go to Examples 15–18 in the Explain section of this lesson.

97. Which proportion below will not give the same value for x as the proportion
$$\frac{7}{x} = \frac{21}{15}$$
?

$$\frac{x}{7} = \frac{12}{21}$$
 $\frac{x}{21} = \frac{7}{15}$ $\frac{x}{15} = \frac{7}{21}$

98. Which proportion below will not give the same value for x as the proportion $\frac{8}{x} = \frac{16}{10}$?

$$\frac{x}{16} = \frac{8}{10} \qquad \qquad \frac{x}{10} = \frac{8}{16} \qquad \qquad \frac{16}{8} = \frac{10}{x}$$

99. Which proportion below will not give the same value for x as the proportion $\frac{x}{16} = \frac{3}{4}$?

 $\frac{4}{16} = \frac{3}{x} \qquad \qquad \frac{16}{x} = \frac{4}{3} \qquad \qquad \frac{x}{4} = \frac{3}{16}$

100. Which proportion below will not give the same value for x as the proportion $\frac{x}{24} = \frac{8}{7}$?

 $\frac{x}{7} = \frac{8}{24} \qquad \qquad \frac{8}{x} = \frac{7}{24} \qquad \qquad \frac{24}{x} = \frac{7}{8}$

101. Which proportion below will not give the same value for x as the proportion $\frac{10}{3.5} = \frac{x}{4.7}$?

$$\frac{10}{x} = \frac{3.5}{4.7} \qquad \qquad \frac{x}{10} = \frac{4.7}{3.5} \qquad \qquad \frac{3.5}{x} = \frac{10}{4.7}$$

102. Which proportion below will not give the same value for x as the proportion $\frac{4.8}{20} = \frac{x}{5.6}$?

$$\frac{x}{4.8} = \frac{20}{5.6} \qquad \qquad \frac{4.8}{x} = \frac{20}{5.6} \qquad \qquad \frac{x}{4.8} = \frac{5}{2}$$

103. Which proportion below will not give the same value for x as the proportion $\frac{7.2}{x} = \frac{2.1}{15}$?

$$\frac{x}{7.2} = \frac{15}{2.1} \qquad \qquad \frac{x}{2.1} = \frac{7.2}{15} \qquad \qquad \frac{x}{15} = \frac{7.2}{2.1}$$

104. Which proportion below will not give the same value for x as the proportion $\frac{x}{3.8} = \frac{9.2}{12}$?

$$\frac{x}{9.2} = \frac{3.8}{12} \qquad \qquad \frac{x}{9.2} = \frac{12}{3.8} \qquad \qquad \frac{12}{3.8} = \frac{9.2}{x}$$

105. Which proportion below will not give the same value for x as the proportion $\frac{2\frac{1}{2}}{x} = \frac{5}{\frac{1}{5}}$?

$$\frac{x}{\frac{1}{5}} = \frac{5}{2\frac{1}{2}} \qquad \qquad \frac{x}{\frac{1}{5}} = \frac{2\frac{1}{2}}{5} \qquad \qquad \frac{5}{2\frac{1}{2}} = \frac{\frac{1}{5}}{x}$$

106. Which proportion below will not give the same value for x as the proportion $\frac{8}{3} = \frac{x}{\frac{3}{4}}$?

 $\frac{\frac{3}{4}}{3} = \frac{x}{8} \qquad \qquad \frac{x}{8} = \frac{3}{\frac{3}{4}} \qquad \qquad \frac{3}{8} = \frac{\frac{3}{4}}{x}$

107. Which proportion below will not give the same value for x as the proportion $\frac{\frac{12}{5}}{x} = \frac{4}{\frac{5}{2}}$?

	5	12		
4	3	5 x	х	4
12	=	$\underline{4} = \underline{5}$	12	5
5		3	5	3

108. Which proportion below will not give the same value for x as the proportion $\frac{\frac{10}{7}}{6} = \frac{x}{\frac{7}{9}}$? 18 7 7 18

$$\frac{x}{\frac{7}{9}} = \frac{\frac{18}{7}}{6} \qquad \qquad \frac{\frac{7}{9}}{x} = \frac{6}{\frac{18}{7}} \qquad \qquad \frac{\frac{7}{9}}{x} = \frac{\frac{18}{7}}{6}$$

- 109. A scale drawing of a floor plan uses $\frac{1}{4}$ inch to represent 5 feet. What is the length of a dining room if it is $\frac{3}{4}$ inches long on the scale drawing?
- 110. A scale drawing of a floor plan uses $\frac{1}{2}$ inch to represent 4 feet. What is the length of a living room if it is 3 inches long on the scale drawing?
- 111. A certain car can travel 210 miles on 7 gallons of gasoline. At this rate, how far can the car travel on a full tank of 12 gallons?
- 112. A certain van can travel 154 miles on 11 gallons of gasoline. At this rate, how far can the van travel on a full tank of 20 gallons?
- 113. A recipe calls for 3 onions and 5 peppers. How many onions are needed if 15 peppers are used?
- 114. A recipe calls for 4 cups of tomatoes and 1 cup of celery. How many cups of tomatoes are needed if $\frac{1}{2}$ cup of celery is used?
- 115. Suppose it costs \$5 for 10 pounds of apples. At this rate, how much does it cost for 7 pounds of apples?
- 116. Suppose it costs \$2.07 for 3 pounds of plums. At this rate, how many pounds of plums can you buy for \$5? Round your answer to the nearest tenth of a pound.
- 117. Betty is taking a trip and has traveled 240 miles in 4 hours. At this rate, how long will it take her to complete the remaining 300 miles of the trip?
- 118. Boris has been driving for 6 hours and has traveled 330 miles. At this rate, how far can Boris drive in another 3 hours?
- 119. Rita earns \$540 in a 40 hour pay period. At this rate, how much will Rita earn in 30 hours?
- 120. Brennan earns \$360 in 30 hours. At this rate, how many hours will Brennan have to work to earn \$240?

How to Set Up and Solve a Proportion with Similar Triangles

For help working these types of problems, go to Examples 19–21 in the Explain section of this lesson.

121. The triangles below are similar triangles. Find *x*, the length of the longest side of the large triangle.



122. The triangles below are similar triangles. Find x, the length of the shortest side of the large triangle.



123. The triangles below are similar triangles. Find x, the length of the shortest side of the small triangle.



124. The triangles below are similar triangles. Find x, the length of the medium side of the small triangle.



125. The triangles below are similar triangles. Find *x*, the length of the longest side of the large triangle.



126. The triangles below are similar triangles. Find x, the length of the longest side of the small triangle.



127. The triangles below are similar triangles. Find x, the length of the medium side of the small triangle.

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128. The triangles below are similar triangles. Find *x*, the length of the longest side of the large triangle.



129. The triangles below are similar triangles. Find x, the length of the shortest side of the small triangle.



130. The triangles below are similar triangles. Find *x*, the length of the longest side of the large triangle.



131. The triangles below are similar triangles. Find x, the length of the shortest side of the large triangle.



132. The triangles below are similar triangles. Find x, the length of the medium side of the large triangle.



133. The triangles below are similar triangles. Find x, the length of the shortest side of the small triangle.

.....



134. The triangles below are similar triangles. Find x, the length of the longest side of the small triangle.



135. The triangles below are similar triangles. Find *x*, the length of the medium side of the large triangle.



136. The triangles below are similar triangles. Find x, the length of the shortest side of the large triangle.



137. A man, 6 feet tall, is standing 25 feet from a street light. The length of his shadow produced by the street light is 5 feet. Find the height, *x*, of the street light. Use the similar triangles below to help you.



138. A woman is standing 7.5 feet from a street light that is 22 feet tall. The length of her shadow created by the street light is 2.5 feet. Find the height, *x*, of the woman. Use the similar triangles below to help you.



139. Ariana, who is 4 feet tall, is standing by a tree. The tree is 15 feet tall. How long is the shadow cast by the tree if Ariana's shadow is 6 feet long? Use the similar triangles below to help you.



140. Gabe is standing by a tree. The tree is 20 feet tall and casts a shadow of 12 feet. How tall is Gabe if his shadow is 3 feet long? Use the similar triangles below to help you.



141. Lisa is flying a kite. When 82 feet of string is out, the kite is 20 feet off the ground. Lisa pulls in the string until there is only 20.5 feet of string out. How high is the kite from the ground now? (Assume that the angle the string makes with the ground does not change.) Use the similar triangles below to help you.



142. Dan is flying a kite. When 96 feet of string is out, the kite is 24 feet off the ground. Dan pulls in the string until the kite is 8 feet off the ground. How much string is out now? (Assume that the angle the string makes with the ground does not change.) Use the similar triangles below to help you.



143. Sean has drawn a scale drawing of his backyard showing the location of 3 of his hiding places. His sister, Arlene, knows that the actual distance from the first hiding place to the second is 10 feet. How far is it from the second hiding place to the third? Use the similar triangles below to help you.



144. Liza has drawn a scale drawing of her backyard showing the location of 3 of her hiding places. Her brother, Arty, knows that the actual distance from the first hiding place to the second is 12 feet. How far is it from the second hiding place to the third? Use the similar triangles below to help you.



Evaluate

Take this Practice Test to prepare for the final quiz in the Evaluate module of the computer.

Practice Test

- 1. In a choir consisting of sopranos, altos, tenors, and basses, there are 49 singers. Of this number, 15 are sopranos and 16 are tenors.
 - a. What is the ratio of the number of sopranos to the number of tenors?
 - b. What is the ratio of the number of sopranos to the number of singers?
- 2. In a fruit and nut mix, the ratio of the number of fruits to the number of nuts is 5 to 9.

Select all the choices below that will keep the mix at this same ratio.

- a. Add 5 fruits and 9 nuts to the mix.
- b. Add 5 fruits and 5 nuts to the mix.
- c. Add 9 fruits and 5 nuts to the mix.
- d. Add 10 fruits and 18 nuts to the mix.
- 3. Write a ratio to compare 47 cents to 3 dollars.
- 4. Nancy drove 360 miles in 8 hours. Find the rate that she drove in miles per hour.
- 5. Choose the ratio below that forms a proportion with the ratio $\frac{14}{18}$.

a. $\frac{13}{17}$ b. $\frac{21}{27}$ c. $\frac{9}{7}$ d. $\frac{8}{10}$

- 6. Solve this proportion for x: $\frac{20}{x} = \frac{5}{11}$
- 7. After hiking 5.6 miles, Sharon found that she was $\frac{4}{5}$ of the way along the trail. Use this proportion to find *x*, the length of the trail in miles: $\frac{5.6}{x} = \frac{4}{5}$
- 8. The two triangles shown below are similar triangles. That is, the lengths of their corresponding sides are in the same ratio.

Use this proportion to find x, the missing length: $\frac{70}{x} = \frac{42}{63}$

