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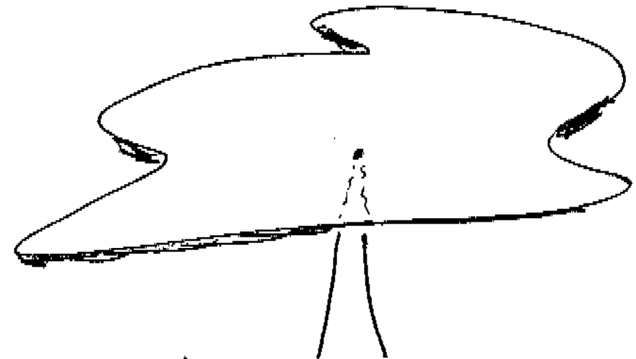
# 9.3: The Centroid

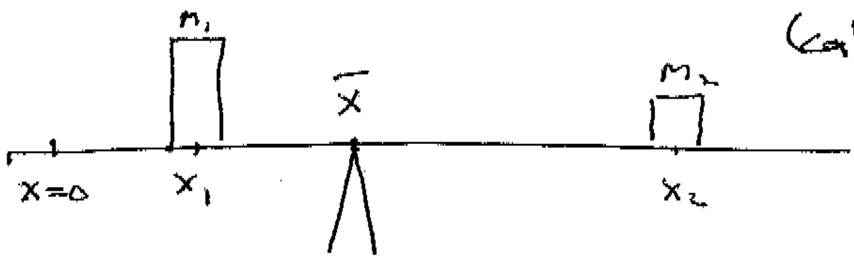
The big picture.

Find the center of mass of a plate w/uniform density.

Plate.



the balancing point is the center of mass. (called the centroid).




The teeter-toter.

where $$\bar{X} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

In general: If $N$ particles with respective masses $m_1, m_2, \ldots, m_N$ are located on the x-axis w/ x-coordinate

$$\bar{X} = \frac{\sum_{i=1}^{N} m_i x_i}{M}$$

$M$ ← total mass.

NOTE: This is just a weighted average.

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The <u>moment</u> is the the tendency of the system (plate) to rotate.

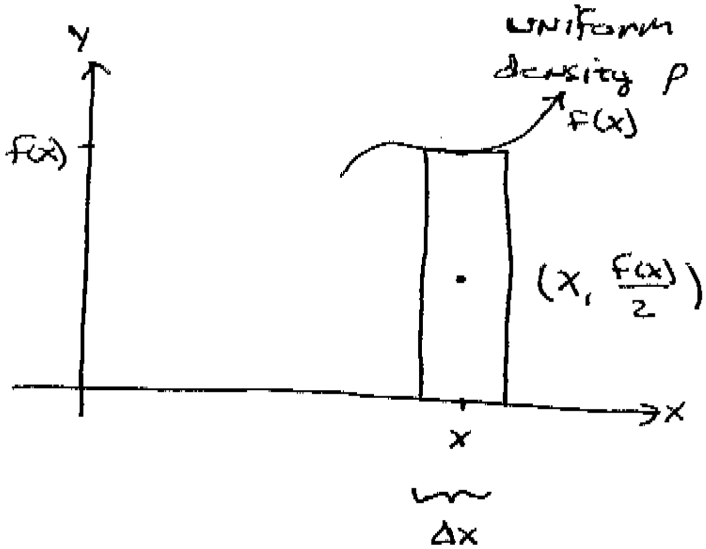
So, the moment on the teeter toter about $x=0$ is the tendency of the system to rotate about $x=0$. It is calculated by $M = \sum_{i=1}^{N} x_i m_i$ (mass * distance)

OR on the x, y plane.

$$M_x = \sum_{i=1}^{N} m_i y_i \longleftarrow$$

$$M_y = \sum_{i=1}^{N} m_i x_i \longleftarrow$$

opposite of the given axis.




The mass is $\rho f(x) \Delta x$,

$$M_y = \overbrace{\rho f(x) \Delta x}^{\text{mass}} \cdot \overbrace{x}^{\text{dist.}}$$

$$M_x = \overbrace{\rho f(x) \Delta x}^{\text{mass}} \cdot \overbrace{\frac{f(x)}{2}}^{\text{dist}}$$

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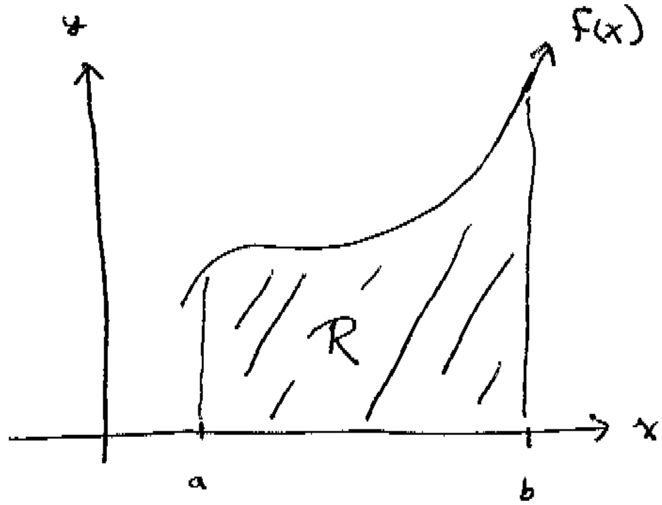


Find the centroid of the region $R$. w/ uniform density $\rho$.

$$\text{mass of } R = \int_a^b \rho f(x)\,dx = \rho \int_a^b f(x)\,dx.$$

$$M_y = \int_a^b \rho f(x) \cdot x\,dx = \rho \int_a^b x f(x)\,dx$$

$$M_x = \int_a^b \frac{\rho}{2} [f(x)]^2 dx = \rho \int \frac{[f(x)]^2}{2} dx.$$

The center of mass of $R$ is at $(\bar{x}, \bar{y})$

where

$$\bar{x} = \frac{M_y}{M} = \frac{\rho \int x f(x)\,dx}{\rho \int_a^b f(x)\,dx} = \frac{1}{A} \int x f(x)\,dx$$

$$\bar{y} = \frac{M_x}{M} = \frac{\rho \int \frac{1}{2} [f'(x)]^2 dx}{\rho \int_a^b f(x)\,dx} = \frac{1}{A} \int \frac{1}{2} [f'(x)]^2 dx.$$

This is where the centroid is located.

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Ex 1: Find the centroid of the region bounded by $y = \sin(x)$, $y = 0$, and on $[0, \pi]$.

$$A = \int_0^{\pi} \sin x \, dx = 2.$$