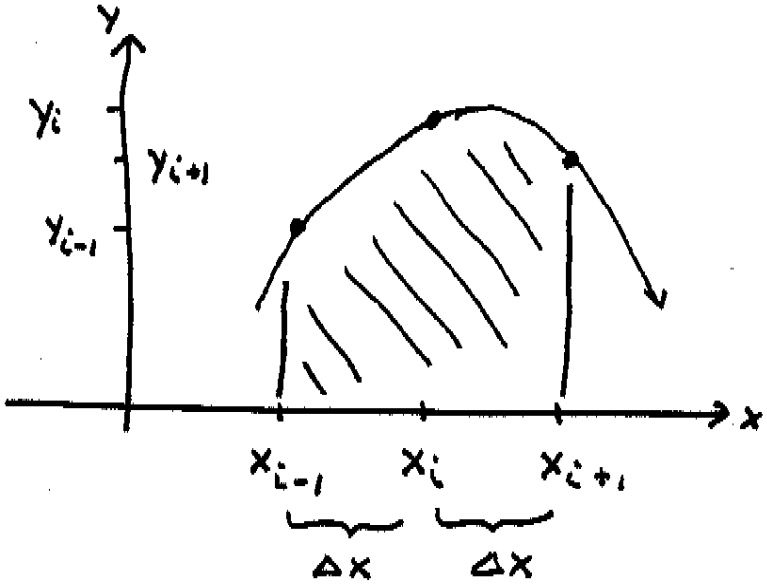
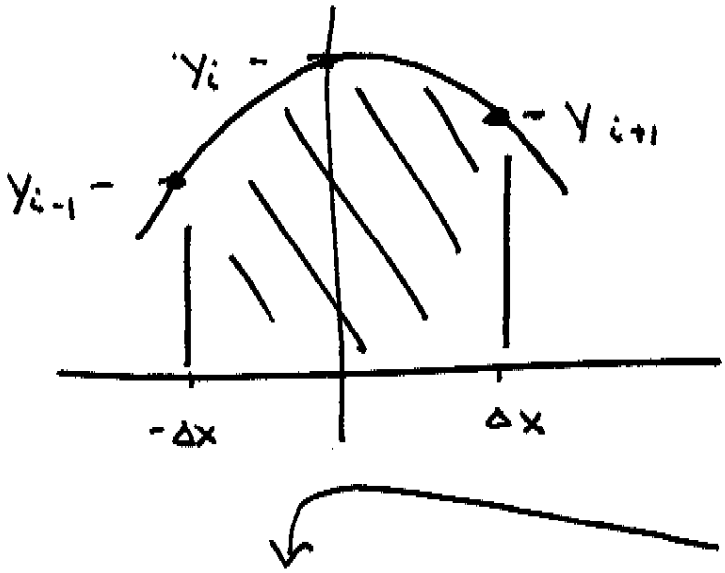
Approximating Integrals - Simpson's Rule.

Find the area under the parabola that goes thru the 3 points:



shift L/R



SAME AREA

parabola: $y = ax^2 + bx + c$

$$\begin{cases} y_{i-1} = a(-\Delta x)^2 + b(-\Delta x) + c \\ \boxed{y_i = a(0)^2 + b(0) + c} \\ y_{i+1} = a(\Delta x)^2 + b\Delta x + c \end{cases}$$

$$y_{i-1} = a(\Delta x)^2 - b(\Delta x) + y_i$$

$$y_{i+1} = a(\Delta x)^2 + b(\Delta x) + y_i$$

$$y_{i+1} - y_{i-1} = 2b(\Delta x) \Rightarrow b = \frac{y_{i+1} - y_{i-1}}{2(\Delta x)}$$

$$\hookrightarrow \quad y_{i+1} = a(\Delta x)^2 + \frac{y_{i+1} - y_{i-1}}{2(\Delta x)} \cdot (\Delta x) + y_i$$

$$\Rightarrow \quad 2y_{i+1} = 2a(\Delta x)^2 + y_{i+1} - y_{i-1} + 2y_i$$

$$\Rightarrow \quad a = \frac{y_{i+1} + y_{i-1} - 2y_i}{2(\Delta x)^2}$$

$$\text{so} \quad y = \underbrace{\left(\frac{y_{i+1} + y_{i-1} - 2y_i}{2(\Delta x)^2}\right)}_{a} x^2 + \underbrace{\left(\frac{y_{i+1} - y_{i-1}}{2(\Delta x)}\right)}_{b} x + \underbrace{y_i}_{c}$$

Integrating from $-\Delta x$ to Δx term by term...

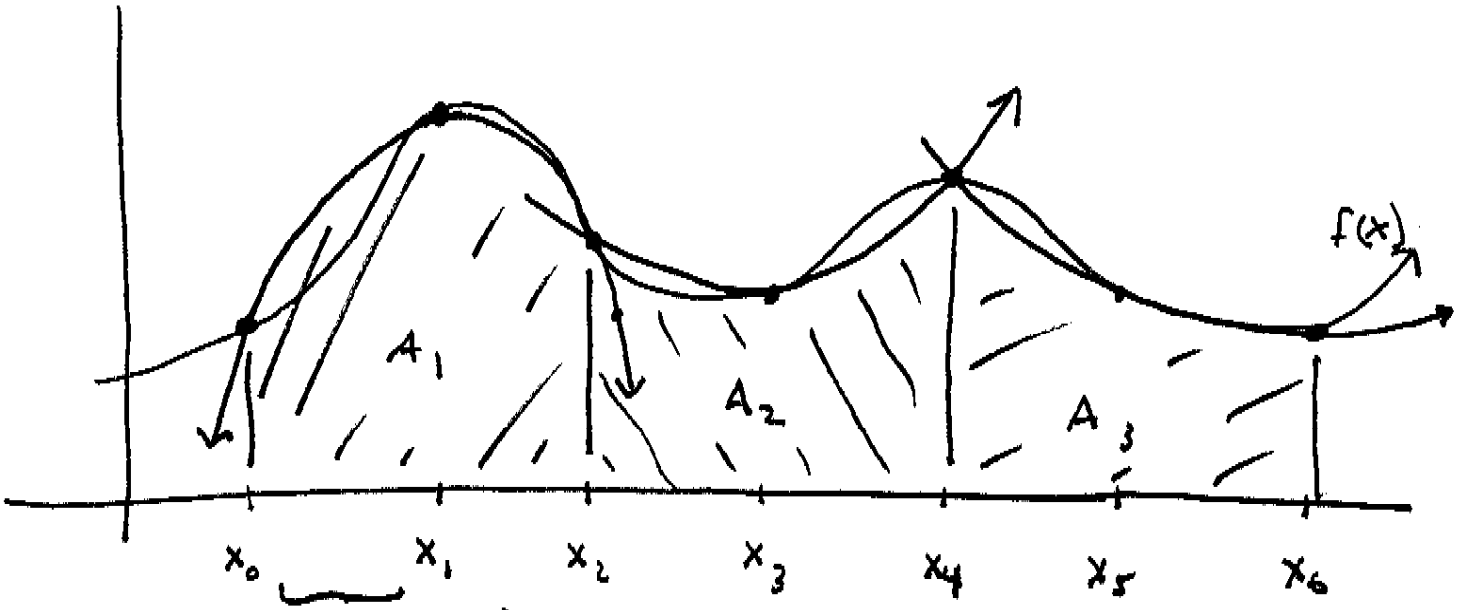
a: Even — b: Odd — c: Even.

$$= 2\int_0^{\Delta x} \left(\frac{y_{i+1} + y_{i-1} - 2y_i}{2(\Delta x)^2}\right) x^2 \, dx + 2y_i \Delta x$$

$$= \frac{2}{3}(\Delta x)^3 \left(\frac{y_{i+1} + y_{i-1} - 2y_i}{2(\Delta x)^2}\right) + 2y_i \Delta x$$

$$= \frac{1}{3} y_{i+1} \Delta x + \frac{1}{3} y_{i-1} \Delta x + \frac{4}{3} y_i \Delta x$$

$$= \frac{\Delta x}{3}(1y_{i-1} + 4y_i + 1y_{i+1})$$



$\Delta x = x_i - x_{i-1}$

$$\int_{x_0}^{x_6} f(x)\,dx \approx A_1 + A_2 + A_3$$

n is even.

$$\approx \frac{\Delta x}{3}(y_0 + 4y_1 + y_2) + \frac{\Delta x}{3}(y_2 + 4y_3 + y_4) + \frac{\Delta x}{3}(y_4 + 4y_5 + y_6)$$

$$= \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6)$$

So the pattern is $\{1, 4, 2, 4, 2, \ldots, 4, 2, 4, 1\}$.

8.7c
1/4

Error Bounds

Trapezoidal Rule:

$$|E_T| \le \frac{k(b-a)^3}{12n^2}$$

Midpoint Rule

$$|E_m| \le \frac{k(b-a)^3}{24n^2}$$

} If $|f''(x)| \le k$ for $a \le x \le b$.

Simpson's Rule

$$|E_s| \le \frac{k(b-a)^5}{180n^4}$$

} If $|f^{(4)}(x)| \le k$ for $a \le x \le b$.

How many operations required for 1 iteration.

S: $\frac{\Delta x}{3}(1y_0 + 4y_1 + 2y_2 + \dots + 4y_{N-1} + y_N)$

calculate $y_0, \dots, y_N$

$N-1$ multiplications.

N additions.

1 division

calc Δx