Key

Group Work 1, Section 8.4
Partial Fractions (Version 3)

1. Compute the following integrals:

(a) $\int \frac{dx}{x+1} = \ln|x+1| + C$

(b) $\int \frac{dx}{x+2} = \ln|x+2| + C$

(c) $\int \frac{dx}{x^2+4} = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C.$

2. Factor $x^4+3x^3+6x^2+12x+8$. (Hint: see above)

$(x+1)(x+2)(x^2+4)$.

3. Compute $\int \frac{20x^2\,dx}{x^4+3x^3+6x^2+12x+8}$.

$20x^3 = A(x+2)(x^2+4) + B(x+1)(x^2+4) + (Cx+D)(x^2+4)$

$0 = 8A + 4B + 2D$

$0x = 2Cx + 4Ax + 4Bx + 3Dx$

$0x^2 = 2Ax^2 + Bx^2 + 3Cx^2 + Dx^2$

$20x^3 = Ax^3 + Bx^3 + Cx^3$

$\Rightarrow$

1	8	4	0	2	1	0
x	4	4	2	3	1	0
x^2	2	1	2	1	1	20
x^3	1	1	1	0	1	20

$A = 4$

$B = -10$

$C = 6$

$D = 4$

4. Compute $\int \frac{x^4+3x^3+26x^2+12x+8}{x^4+3x^3+6x^2+12x+8}\,dx$. (Hint: what is the degree of the numerator & denominator?)

$= \int 1\,dx + \int \frac{20x^2}{x^4+3x^3+6x^2+12x+8}\,dx$

$= x + A$ //.

Integration Strategies.

Some we know. - those on the table.

Some we don't

- Substitution.
- parts.
- trig. integrals.
- trig. substitution.
- partial fractions

Test 4

No calculators

study old tests.

NOTE ON HYPERBOLICS.

8.3 #14 and $-\operatorname{sech}^{-1}(x)$.

STrategy

1) Do you know it?
2) can you simplify it?
3) will substitution work?
4) what does it look like?

- PARTS
  - LIATE
- Trig Integral
- Trig Substitution
  - ~~8.3~~.
  - can it be avoided?
- Partial Fractions.
  - becomes.

8.3 : 2, 3, 4, 5, 6, 8, [9]..

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} \quad (n \neq -1)$

2. $\int \dfrac{1}{x}\,dx = \ln|x|$

3. $\int e^x\,dx = e^x$

4. $\int a^x\,dx = \dfrac{a^x}{\ln a}$

5. $\int \sin x\,dx = -\cos x$

6. $\int \cos x\,dx = \sin x$

7. $\int \sec^2 x\,dx = \tan x$

8. $\int \csc^2 x\,dx = -\cot x$

9. $\int \sec x \tan x\,dx = \sec x$

10. $\int \csc x \cot x\,dx = -\csc x$

*11. $\int \sec x\,dx = \ln|\sec x + \tan x|$

*12. $\int \csc x\,dx = \ln|\csc x - \cot x|$

*13. $\int \tan x\,dx = \ln|\sec x|$

*14. $\int \cot x\,dx = \ln|\sin x|$

*15. $\int \sinh x\,dx = \cosh x$

*16. $\int \cosh x\,dx = \sinh x$

17. $\int \dfrac{dx}{x^2 + a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$

18. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right)$

*19. $\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\ln\left|\dfrac{x-a}{x+a}\right|$

*20. $\int \dfrac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$