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$$\text{Ex 1: } \frac{1}{x+2} + \frac{1}{x-3} = \frac{x-3 + x+2}{(x+2)(x-3)} = \frac{2x-1}{x^2-x-6}$$

So

$$\int \frac{2x-1}{x^2-x-6} dx = \int \left[\frac{1}{x+2} + \frac{1}{x-3} \right] dx = \ln|x+2| + \ln|x-3| + C$$

$$= \ln|(x+2)(x-3)| + C.$$

$$\text{Ex 2: } \int \frac{x^2+2x}{x-3} dx$$

use poly. long div. to write
w/ deg of num. less than the
deg. of den.

$$\begin{array}{r} x+5 \\ x-3 \overline{) x^2+2x+0} \\ \underline{-(x^2-3x)} \\ 5x+0 \\ \underline{-(5x-15)} \\ 15 \end{array}$$

$$= \int \left[x+5 + \frac{15}{x-3} \right] dx$$

$$= \frac{x^2}{2} + 5x + 15 \ln|x-3| + C.$$

$$\text{Work on your own: } \int \frac{1-x^2}{x+7} dx = \int \left[-x+7 - \frac{48}{x+7} \right] dx$$

$$\begin{array}{r} -x+7 \\ x+7 \overline{) -x^2+0x+1} \\ \underline{-(-x^2-7x)} \\ 7x+1 \\ \underline{-(7x+49)} \\ -48 \end{array}$$

$$= -\frac{x^2}{2} + 7x - 48 \ln|x+7| + C$$

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$$\underline{\text{Ex 3:}} \int \frac{x-1}{x^2+9x+20} dx$$

$$\frac{x-1}{x^2+9x+20} \Rightarrow \frac{x-1}{\underbrace{(x+5)(x+4)}} = \frac{A}{(x+5)} + \frac{B}{x+4}$$

product of unique
linear factors.

$$\Rightarrow x-1 = A(x+4) + B(x+5)$$

$$\Rightarrow x-1 = Ax + 4A + Bx + 5B$$

$$\Rightarrow \begin{cases} A + B = 1 \\ 4A + 5B = -1 \end{cases} \Rightarrow B = -5 \text{ and } A = 6.$$

$$= \int \left[\frac{6}{x+5} - \frac{5}{x+4} \right] dx$$

$$= 6 \ln|x+5| - 5 \ln|x+4| + C.$$

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$$\text{Ex 4: } \int \frac{1-2x^2}{x^3+4x^2+4x} dx = \int \frac{1-2x^2}{x(x^2+4x+4)} dx = \int \frac{1-2x^2}{x(x+2)^2} dx$$

product of linear terms w/ some repetition.

$$\frac{1-2x^2}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$\Rightarrow 1-2x^2 = A(x+2)^2 + Bx(x+2) + Cx$$

$$\Rightarrow 1-2x^2 = Ax^2 + 4Ax + 4A + Bx^2 + 2Bx + Cx$$

$$\Rightarrow \begin{cases} A+B = -2 \\ 4A+2B+C = 0 \\ 4A = 1 \end{cases} \quad \begin{cases} B = -\frac{9}{4} \\ C = \frac{7}{2} \\ A = \frac{1}{4} \end{cases}$$

$$= \int \left[\frac{1}{4} \cdot \frac{1}{x} - \frac{9}{4} \cdot \frac{1}{x+2} + \frac{7}{2} \cdot \frac{1}{(x+2)^2} \right] dx$$

$$= \frac{1}{4} \ln|x| - \frac{9}{4} \ln|x+2| - \frac{7}{2} \cdot \frac{1}{x+2} + C.$$

Work on your own: $\int \frac{5x}{x^2+6x+9} dx = \int \frac{5x}{(x+3)^2} dx$

$$\frac{5x}{(x+3)^2} = \frac{A}{x+3} + \frac{B}{(x+3)^2} = \int \left[\frac{5}{x+3} + \frac{15}{(x+3)^2} \right] dx$$

$$\Rightarrow 5x = A(x+3) + B$$

$$\Rightarrow 5 = A$$

$$0 = -3A + B \Rightarrow B = 15$$

$$= 5 \ln|x+3| - 15 \cdot \frac{1}{x+3} + C.$$

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$$\underline{\text{Ex 5:}} \quad \int \frac{x-1}{x^3+3x} dx = \int \frac{x-1}{x(x^2+3)} dx$$

contains an irreducible
quadratic factor.

$$\frac{x-1}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$$

$$\Rightarrow x-1 = A(x^2+3) + (Bx+C)x$$

$$B+A=0 \Rightarrow B=1/3$$

$$\Rightarrow C=1$$

$$3A=-1 \Rightarrow A=-1/3$$

$$= \int \left[-\frac{1}{3} \cdot \frac{1}{x} + \frac{\frac{1}{3}x+1}{x^2+3} \right] dx$$

$$\text{Let } u = x^2+3 \\ du = 2x dx$$

$$= -\frac{1}{3} \ln|x| + \frac{1}{3} \int \frac{x}{x^2+3} dx + \int \frac{1}{x^2+3} dx$$

$$= -\frac{1}{3} \ln|x| + \frac{1}{6} \ln|x^2+3| + \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

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Ex 6: $\int \frac{x^3 - 5x + 1}{x^4 + 4x^2 + 4} dx$

= $\int \frac{x^3 - 5x + 1}{(x^2 + 2)^2} dx$

product of irreducible quadratic terms
not unique.

$$\frac{x^3 - 5x + 1}{(x^2 + 2)^2} = \frac{Ax + B}{x^2 + 2} + \frac{Cx + D}{(x^2 + 2)^2}$$

$$\Rightarrow x^3 - 5x + 1 = (Ax + B)(x^2 + 2) + Cx + D$$

$$\Rightarrow x^3 - 5x + 1 = Ax^3 + 2Ax + Bx^2 + 2B + Cx + D$$

x^3	A	B	C	D	=	1	1	A = 1
x^2	0	1	0	0	1	0	0	B = 0
x	2	0	1	0	1	-5	-5	C = -7
1	0	2	0	1	1	1	1	D = 1

$$\int \frac{x^3 - 5x + 1}{x^4 + 4x^2 + 4} dx = \int \left[\frac{1}{x^2 + 2} + \frac{-7x + 1}{(x^2 + 2)^2} \right] dx$$

$\underbrace{\hspace{10em}}_{\arctan(\frac{x}{\sqrt{2}})}$

$$u = x^2 + 2$$

$$\frac{du}{2} = 2x dx$$

use trig substitution

$$\int \frac{-7x}{(x^2 + 2)^2} dx + \int \frac{1}{(x^2 + 2)^2} dx$$

$$\frac{7}{2} \cdot \frac{1}{x^2 + 2} + \frac{\sqrt{2}}{8} \arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{x}{4(x^2 + 2)}$$

(see attached)

(attached)

scratch work for example 68.4
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$$\text{To find } \int \frac{1}{(x^2+2)^2} dx$$

$$\text{Let } x = \sqrt{2} \tan(\theta)$$

$$dx = \sqrt{2} \sec^2(\theta) d\theta.$$

$$= \int \frac{1}{(2\tan^2(\theta) + 2)^2} \cdot \sqrt{2} \sec^2(\theta) d\theta.$$

$$= \frac{\sqrt{2}}{4} \int \frac{\sec^2(\theta)}{(\tan^2(\theta) + 1)^2} d\theta$$

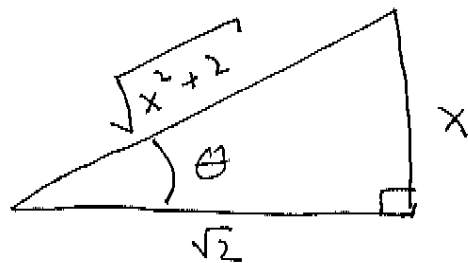
$$= \frac{\sqrt{2}}{4} \int \frac{\sec^2(\theta)}{\sec^4(\theta)} d\theta.$$

$$= \frac{\sqrt{2}}{4} \int \cos^2(\theta) d\theta$$

$$= \frac{\sqrt{2}}{4} \int \frac{1 + \cos(2\theta)}{2} d\theta.$$

$$= \frac{\sqrt{2}}{8} \left[\theta + \frac{\sin(2\theta)}{2} \right]$$

$$= \frac{\sqrt{2}}{8} \left[\theta + \sin(\theta) \cos(\theta) \right]$$



$$= \frac{\sqrt{2}}{8} \left[\arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{x}{\sqrt{x^2+2}} \cdot \frac{\sqrt{2}}{\sqrt{x^2+2}} \right]$$

$$= \frac{\sqrt{2}}{8} \arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{x}{4(x^2+2)}.$$

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Group Work 1, Section 8.4
Partial Fractions (Version 3)

from Teaching
supplements.

1. Compute the following integrals:

(a) $\int \frac{dx}{x+1}$

(b) $\int \frac{dx}{x+2}$

(c) $\int \frac{dx}{x^2+4}$

2. Factor $x^4 + 3x^3 + 6x^2 + 12x + 8$. (Hint: see above)

3. Compute $\int \frac{20x^2 dx}{x^4 + 3x^3 + 6x^2 + 12x + 8}$.

4. Compute $\int \frac{x^4 + 3x^3 + 26x^2 + 12x + 8}{x^4 + 3x^3 + 6x^2 + 12x + 8} dx$. (Hint: what is the degree of the numerator or denominator?)