8.1a
1/2

$$(uv)' = uv' + vu'$$ product rule in reverse.

$$\Rightarrow \int (uv)' = \int [uv' + vu']$$

$$\Rightarrow uv = \int uv' + \int vu'$$

$$\Rightarrow \int uv' = uv - \int vu'$$

OR

$$\int u\,dv = uv - \int v\,du.$$

Ex 1: $\int \underbrace{x}_{u} \underbrace{\cos(x)\,dx}_{dv} = x\sin(x) - \int \sin(x)\,dx$

$= x\sin(x) + \cos(x) + C.$

$u \downarrow \quad dv \downarrow$

$du = 1\,dx \quad v = \sin(x)$

Ex 2: $\int x^2 \cos(3x)\,dx$

Ex 3: $\int \ln(x)\,dx$

Ex 4: $\int t^3 e^t\,dt.$

8.1a
4/2

Ex 5: $\int e^x \cos(x)\,dx$

$u = \cos(x) \rightarrow du = -\sin(x)\,dx$

$dv = e^x dx \rightarrow v = e^x$

$= e^x \cos(x) + \int e^x \sin(x)\,dx$

$u = \sin(x) \rightarrow du = \cos(x)\,dx$

$dv = e^x dx \rightarrow v = e^x$

$= e^x \cos(x) + e^x \sin(x) - \int e^x \cos(x)\,dx$

so $\int e^x \cos(x)\,dx = e^x \cos(x) + e^x \sin(x) - \int e^x \cos(x)\,dx$

$\Rightarrow 2\int e^x \cos(x)\,dx = e^x(\cos(x) + \sin(x))$

$\Rightarrow \int e^x \cos(x)\,dx = \frac{1}{2} e^x(\cos(x) + \sin(x)) + C.$

8.1b
1/1

Integration by parts (b).

$$(uv)' = u'v + v'u$$

$$\Downarrow$$

$$\int u\,dv = uv - \int v\,du$$

formula for integration by parts.

Ex 1: $\int x\cos(4x)\,dx$

Ex 2: $\int e^{2\theta}\sin(3\theta)\,d\theta$

Ex 3: $\int \sin^{-1}(x)\,dx$

Ex 4: $\int te^{-t}\,dt$ Try on your own.

Ex 5: $\int_0^1 x\sinh(x)\,dx$

Ex 6: $\int_1^4 \ln(\sqrt{x})\,dx$

Ex 7: $\int_0^{\pi/4} x\sec^2(x)\,dx$ Try in groups.

Ex 8: $\int x^n e^x\,dx$ reduction formula.