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9.1: Intro to Diff. Eqt.

Population growth.

* Population grows at a rate proportional to the population size. $\frac{dP}{dt} = kP$

* when pop small $\frac{dP}{dt}$, $\frac{dP}{dt} \approx kP$.

but for $P > k$ (capacity), $\frac{dP}{dt} < 0$.

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{k}\right)$$ Logistic Diff Eqt.

Question – what does k look like in real life.

Spring motion

* Hooke's Law $F = -kx$

$$ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$ (2nd order).

Specific vs. General Solutions.

Ex 1: $\frac{dy}{dt} = \sin t$

Ex2: Construct the diff eqt.

a) the roc of the area of a disk over time is proportional to the product of r & the roc of r.

b) the accel. of a particle is inversely prop. to its velocity.

Ex3: Show $y = \tan(t)$ is a soln to $\frac{dy}{dt} = 1 + y^2$. Is this the only soln?

$y = a\tan(t + b) + c$

Ex4: Show $y = \frac{1}{\frac{t^3}{3} + c}$ is a soln to $\frac{dy}{dt} = -t^2 y^2$. (what does c do?)

$c = 0$ a soln.

What does an initial value do? IVP