7.7
1/2

7.7 : Numerical Integration

Face it, finding antiderivatives is a pain.

Ex 1: $\int_0^1 \sin(x^2)\,dx = \sqrt{\frac{\pi}{2}}\ \text{FresnelS}\left[\sqrt{\frac{2}{\pi}}\right]$

≈ 0.3102683

a) L_{10}

b) R_{10}

c) M_{10}

Ex 2: $\int_0^2 e^{-x^2}\,dx = \sqrt{\frac{\pi}{4}}\ \text{Erf}[1]$

≈ 0.7468241

a) L_8, R_8, and M_8.

~~Ex~~ The Trapezoidal Rule.

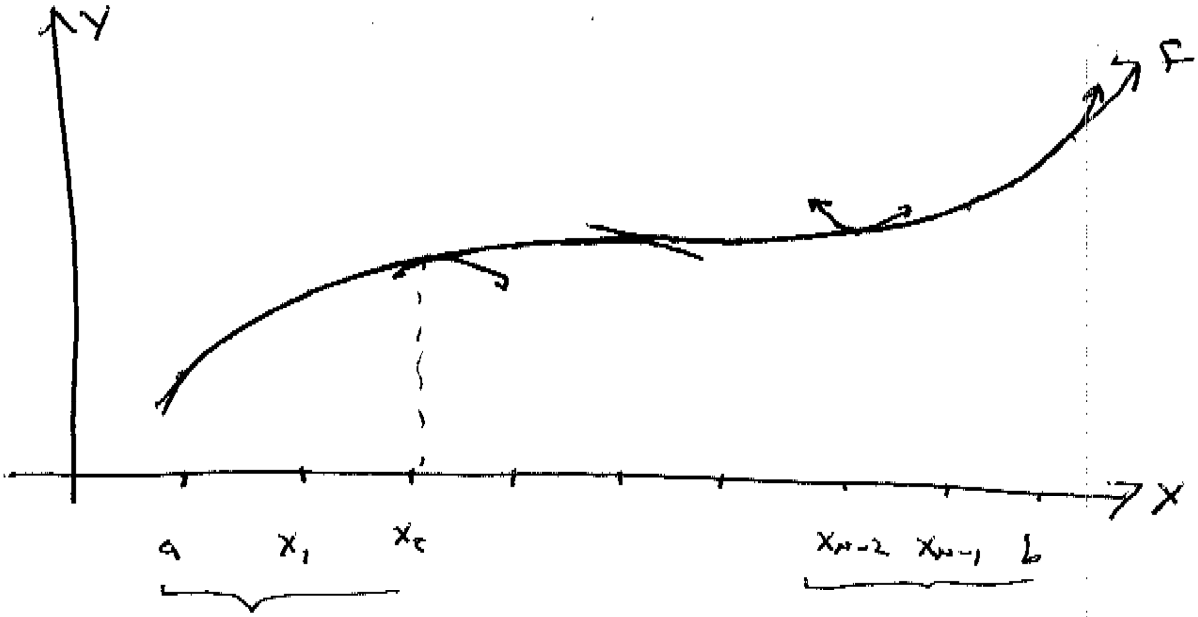
$$T_n = \frac{L_n + R_n}{2} = \frac{f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)}{2}$$

Ex 3: a) approx $\int_0^1 \sin(x^2)\,dx$ w/ T_{10}

b) approx $\int_0^2 e^{-x^2}\,dx$ w/ T_8.

Simpson's Rule

The previous approx techniques use lines to approx curves. An alternative is to approx w/ quadratics taking 2 pts at a time.



$$\frac{\Delta x}{3}\left(f(x_0)+4f(x_1)+f(x_2)\right)+\dots+\frac{\Delta x}{3}\left(f(x_{n-2})+4f(x_{n-1})+f(x_n)\right)$$

$$=\frac{\Delta x}{3}\left[f(x_0)+4(f(x_1))+2f(x_2)+4(f(x_3))+\dots+4f(x_{n-1})+f(x_n)\right]$$

w/ coefficients $1, 4, 2, 4, 2, \dots, 4, 1$.

Ex4: a) approx $\int_0^1 \sin(x^2)\,dx$ w/ S_{10}

b) approx $\int_0^2 e^{-x^2}\,dx$ w/ S_8

Note that $S_{2N}=\frac{1}{3}T_N+\frac{2}{3}M_N$

Ex5: a) approx $\int_0^1 \sin(x^2)\,dx$ w/ S_{20}

b) approx $\int_0^2 e^{-x^2}\,dx$ w/ S_{16}.

Tomorrow: Errorbounds.