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Trig Substitution

Ex 1: Find the area of a circle.

$$2\int_{-r}^{r}\sqrt{r^2-x^2}\,dx = 4\int_{0}^{r}\sqrt{r^2-r^2\sin^2\theta}\,r\cos\theta\,d\theta$$

Let $x = r\sin\theta$
$dx = r\cos\theta\,d\theta$

$$= 4\int r^2\cos^2\theta\,d\theta$$

$$= 4r^2\int\frac{1+\cos(2\theta)}{2}\,d\theta$$

$$= 4r^2\left[\frac{1}{2}\theta + \frac{\sin(2\theta)}{4}\right]$$

$$= 4r^2\left[0 - \left(\frac{\pi}{4}+0\right)\right]$$

$$= \pi r^2$$

Substitutions:

$\sqrt{a^2-x^2}$, use $x = a\sin\theta$

$\sqrt{a^2+x^2}$, use $x = a\tan\theta$

$\sqrt{x^2-a^2}$, use $x = a\sec\theta$

Ex 2: $\int x^3\sqrt{x^2+4}\,dx$

Ex 3: $\int\frac{\sqrt{x^2-9}}{x^4}\,dx$

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Ex 4: $\int_0^{2/3} x^3\sqrt{4-9x^2}\,dx$

Ex 5: $\int_0^1 \sqrt{x^2+1}\,dx$

Ex 6: $\int \frac{1}{\sqrt{9x^2+6x-8}}$

Ex 7: (Groups) $\int \frac{\sqrt{1+x^2}}{x}$

Ex 8: (Groups) $\int_0^1 2x\sqrt{1-x^4}\,dx$

$u = x^2 \quad du = 2x\,dx$

$= \int_0^1 \sqrt{1-u^2}\,du.$