Handout – Take Home (due Monday)
Dusty Wilson
Math 112 - Spring 2005

Name: Key

Every year of my life I grow more convinced that it is wisest and best to fix our attention on the beautiful and the good, and dwell as little as possible on the evil and false.

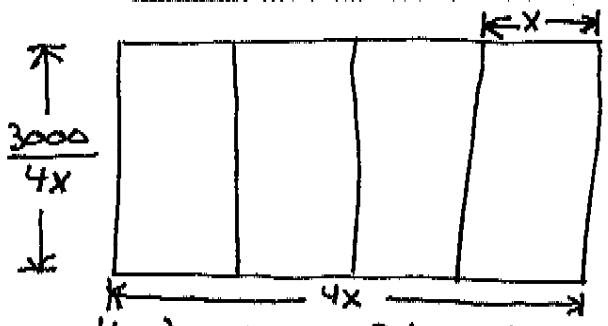
Richard Cecil

Calculators May Be Used

No work = no credit.

Grading: This assignment is valued as two homework assignments.

1.) I need a partitioned four-car garage to contain all my Caterpillar vehicles. Being a mathematician, I want the floor plan of my garage to be as follows. (i.) Form a rectangle that has an area of 3000ft^2. (ii.) Intersect the rectangle with 3 parallel lines in such a way as to form 4 rectangular parking places with equal area. (Note: each of these lines represents a partition between parking spaces). The cost of the exterior walls is \$20 per linear foot and the cost of the interior walls (i.e., partitions) is \$10 per linear foot. What are the *exact* dimensions of the garage that will minimize the total cost of constructing the walls?



$$C(x) = 10(3)\left(\frac{3000}{4x}\right) + 20\left(2\left(\frac{3000}{4x}\right) + 2(4x)\right)$$

$$= \frac{90000}{4x} + \frac{120000}{4x} + 160x$$

$$= \frac{210000}{4x} + 160x$$

$$C'(x) = 160 - \frac{210000}{4x^2}$$

$C'(x) = 0$ when $x = \pm\sqrt{\frac{4(160)}{210000}}$ $\Rightarrow x = \pm\frac{5\sqrt{105}}{2} \approx 18.11$ ft. The optimal dimensions are $5\sqrt{210}$ by $20\sqrt{\frac{30}{7}}$ where dimensions are in feet. (18×41).

2.) The yearly demand for Caterpillar D4C Bulldozers is $p(q) = 100(800 - \frac{1}{2}q^2)$. The average cost of production is $\overline{C}(q) = 100(400 + 2q)$.


Caterpillar D4C

a.) Find the *exact* quantity that will maximize profit. Is this realistic?

$R(q) = q \cdot p(q) = 100q(800 - \frac{1}{2}q^2)$

$P(q) = 80000q - 50q^3 - 40000 - 200q$

$P'(q) = -150q^2 + 79800$

$\Rightarrow q = \pm\sqrt{\frac{79800}{150}} = \pm 2\sqrt{133} \approx 23.065$. We can't sell partial dozers.

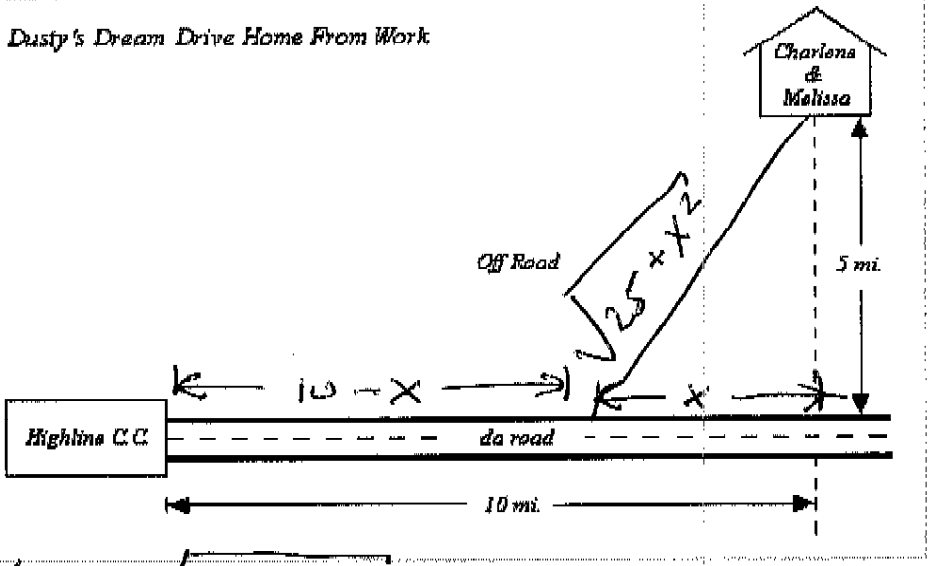
b.) Find the selling price at (i.) the optimal quantity and (ii.) the quantity that corresponds to the highest attainable profit.

$p(2\sqrt{133}) = \$53400$ vs $p(23) = \$53,550$.

c.) What is the maximum attainable profit?

The max attainable profit was \$1,187,050.

3.) Dusty wants to get home to Charlene (Melissa and Faith) as quickly as possible after work. His Caterpillar 797 Off Highway Dump Truck can drive at a rate of 15mph on the road and 5 mph off-road. Given the diagram below, at what distance from Highline Community College should Dusty leave the road in order to arrive home in the least amount of time? What is the minimum time for his commute?



$T = \frac{D}{R}$

$T(x) = \frac{10-x}{15} + \frac{\sqrt{25+x^2}}{5}$

$T'(x) = -\frac{1}{15} + \frac{x}{5\sqrt{25+x^2}}$

solve $T' = 0$. $\frac{1}{15} = \frac{x}{5\sqrt{25+x^2}}$

$\Rightarrow \frac{1}{9} = \frac{x^2}{25+x^2}$

$\Rightarrow 25 + x^2 = 9x^2$

$\rightarrow 25 = 8x^2$

$x = \pm\sqrt{25/8}$

$T_{min} = T\left(\frac{5}{2\sqrt{2}}\right)$

$= \frac{10 - 5/2\sqrt{2}}{15} + \frac{\sqrt{25 + 25/8}}{5}$

CAT 797 Off Highway Truck

The carrying capacity of the CAT 797 is 290yd^3. In order to get a picture of how large my dream truck is: picture the dirt required to fill the courtyard in building 18 (the math building) to a depth of 8 feet. It would all fit!

That would be quite a mess for the janitors…