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B.2: The Definite Integral or FTC.

To find the area under $f(x)$ on $[a, b]$,

$$A = \lim_{N \rightarrow \infty} \sum_{i=1}^N f(x_i) \Delta x$$

Defn: The definite Integral.

If f is cont. on $[a, b]$, then the area under f is given by.

$$\int_a^b f(x) dx = \lim_{N \rightarrow \infty} \sum_{i=1}^N f(x_i) \Delta x$$

where $\Delta x = \frac{b-a}{N}$ and $x_i = a + i \Delta x$.

Ex1: Write $\int_{-3}^3 (4x-7) dx$ as the limit of sums.

How does the definite integral relate to the indefinite?

FTOC Let f be cont on $[a, b]$, then the definite integral exists and

$$\int_a^b f(x) dx = F(b) - F(a)$$

where $F'(x) = f(x)$ on $[a, b]$.

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Ex 2: $\int_0^1 x dx$

Ex 3: $\int_0^1 x^3 dx$

Ex 4: $\int_1^9 \sqrt{x} dx$

Ex 5: $\int_0^5 4\sqrt[3]{x^2} dx$

Ex 6: $\int_2^4 (x^2+2)^3 x dx$

Ex 7: $\int_{-1}^2 x\sqrt{x^2-5} dx$

Ex 8: Suppose that a vending machine service co. models its income by assuming that money flows cont. into the machines w/ annual rate of flow $f(t) = 120e^{0.01t}$ in \$1000's/yr. Find the total income over the 1st 3 years.