Appendix 3: Selected Solutions/Hints

(Some solutions are only partially shown)

Section 1

1. (a) $a_1 = 3 + 2(1)^2 = 5$
$a_2 = 3 + 2(2)^2 = 11$
$a_3 = 3 + 2(3)^2 = 21$
$a_4 = 3 + 2(4)^2 = 35$
and so on where the pattern is: 5, 11, 21, 35, 53, 75

(c) 1, 4, 27, 256, 3125, 46656

(d) $f_1 = (-1)^{1-1}\ \dfrac{1+1}{1^2} = 2$

$f_2 = (-1)^{2-1}\ \dfrac{2+1}{2^2} = -\dfrac{3}{4}$

and so on where the pattern is: 2, -3/4 , 4/9 , -5/16 , 6/25, -7/49 (the pattern is easier to see if the answers are left as fractions!)

2b. (i) An easy pattern: next 3 terms are 5/6, 6/7, 7/8

(ii) $f_n = \dfrac{n}{n+1}$ (n gives you 1,2,3,4,5,6,7 for numerator, and denominator is 1 larger)

3. Start with 5, and then multiply by 3 to get successive terms:

5, 15, 45, 135, 405, 1215

The formula for this sequence is $a_n = 5(3)^{n-1}$. $a_{17} = 5(3)^{16} = 215233605$.
$S_{12} = (5(3)^{12} - 5)/(3-1) = 1328600$

7. - 2, 4, -8, 16, -32, 64
$S_8 = (-2(-2)^8 - (-2))/ (-2-1) = -510/-3 = 170$
$S_{29} = (-2(-2)^{29} - (-2))/ (-2-1) = -357{,}913{,}942$

9. If the loan is L, the remaining new loan balance is L - .12L = .88L. So just multiply by .88 to get the next month's balance. Hence we have a geometric sequence whose ratio is .88. The balance after n payments is $B_n = 4400(.88)^{n-1}$ So after 1 year(12 payments) the balance is $4400(.88)^{11} = \$1{,}078.36$ (By the way, you'd be correct if you said that $B_n = 5000(.88)^n$). If we want to know when we have \$10 left, solve

$$10 = 4400(.88)^{n-1} \Rightarrow 10/4400 = .88^{n-1} \Rightarrow \log(1/440) = (n-1)\log .88$$

$$(n-1)\log .88 \Rightarrow \log(1/440) / \log .88 = n-1 \Rightarrow n - 1 \approx 47.6 \Rightarrow n \approx 48.6 \text{ months}$$

If you similarly solve the equation for \$1, you will get $n \approx 66.6$ months. (From a practical point of view, at some point you just say, "I'll pay off the balance" rather than dragging this out forever.)

Section 2

1. $A = 5000(1 + 0.06/12)^{12(15)} = \$12{,}270.47$

3. (c) $A = 10000(1+0.08/12)^{12(20)} = \$\ 49{,}268.03$
(f) $A= 10000\ e^{0.08(20)} = \$49{,}530.32$

5. $\$600000 = P\left(1 + \dfrac{0.045}{12}\right)^{12(15)} \Rightarrow P = \dfrac{\$600000}{\left(1 + \dfrac{0.045}{12}\right)^{12(15)}} = \$305{,}879.77$

7. $A = \$17{,}425.64$

9. ≈7 yrs 10 months

11. ≈ 128.31 months ≈ 10.7 years

Section 3

1. $r_{eff} = \left(1+\dfrac{0.056}{365}\right)^{365} - 1 \approx 0.0576 = 5.76\%$

3. Solve $0.072 = e^{r} - 1$. (show work) $\Rightarrow r \approx 6.95\%$

7. Solve $\$1\left(1+\dfrac{r}{12}\right)^{12} = \$1e^{0.07}$. (The effect on \$1 for one year should be the same). Taking the 12th root, $1+\dfrac{r}{12} = \left(e^{0.07}\right)^{1/12} \Rightarrow$ you finish the work.

Section 4

1. $FV = 7000\dfrac{(1+0.07/2)^{2(15)} - 1}{0.07/2} = \$361{,}358.74$

3. Solve $54000 = P\dfrac{(1+0.075/12)^{12(10)} - 1}{0.075/12}$. You should get $P = \$303.49$.

5. \$238,129.46

7. \$87,743.68

Section 5

1. $Pmt = 1300\dfrac{0.18/12}{1-(1+0.18/12)^{-12(2)}} = \$64.90$

Since you made 24 payments of \$64.90, the stereo cost 24(64.90) or \$1,557.60. The extra over the \$1,300 is the interest you paid.

3. We need an approximate down payment of \$564.

5. We need to solve the equation below for m. This is another job for logarithms.

$$\$932.73 = \$125,000\left(\frac{\frac{0.078}{12}}{1-\left(1+\frac{0.078}{12}\right)^{-m}}\right)$$

$$\Rightarrow\ \$932.73\left(1-\left(1+\frac{0.078}{12}\right)^{-m}\right) = \$125,000\left(\frac{0.078}{12}\right)$$

$m \approx 316.2$ months (≈26 years 4 months)
(take the log of both sides and solve for *m*.)

316 months at \$932.73 per month gives a total of \$294,742.68, a savings of \$21,147.36. If you use 316.2 for your value of m, you'll get an answer of \$20,960.81 for savings.

7. You will save \$13,388.21. You take out \$13, 388.00 for your down payment. What you still owe is \$30,500-\$13,388 = \$17,112. This will be the loan you take out. Your monthly payment is \$354.39.

Appendix 2: Miscellaneous Problems

1. \$41,447.03

3. $r_{eff} \approx 0.04576\ldots \approx 4.58\%$

5. $FV = 1000\dfrac{(1+0.08/4)^{15(4)}-1}{0.08/4} = \$114,051.54$ This is put in a savings account

(assuming no additional monthly payments)

$A = \$114,051.54e^{0.06(10)} = \$207,815.46$

7. Payment is \$538.52